Lesson Research Proposal for 5th Year Algebraic Inequalities

For the lesson on 12/01/ 2018
At Maynooth Education Campus
Instructor: Gráinne O'Rourke
Lesson plan developed by: Ruth McCormack, Gráinne O' Rourke,
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Title of the Lesson: Baby it's cold outside

1. Brief description of the lesson

Students tend to have trouble with inequalities and algebra. Thus, algebraic inequalities are a struggle for them. This is why we chose this particular topic. The aim of this lesson is consolidate students' knowledge of how to solve a quadratic equation algebraically and graphically whilst also drawing on students' prior knowledge of Junior Certificate inequalities.

2. Research Theme

At our school,

- We want our students to report on, present and explain the process and outcome of learning activities to a highly competent level.

- We want teachers to engage in planning for assessing all relevant aspects of students' learning using both assessment of learning and assessment for learning.

As a Mathematics department, we will actively support the achievement of these goals by endeavoring to do the following:

(a) Using Blooms Taxonomy and active questioning techniques, teachers will probe students to explain and provide rationale for their choice of methodologies and solutions to questions in class. Key Mathematical terminology will be highlighted throughout the lessons.

(b) Teachers will employ a number of Assessment for Learning techniques. For example, involving students in their own learning by assessing their own work and reflecting on their learning at the end of key lessons.

3. Background & Rationale

a) Factorising poses a problem from its introduction early in Junior cycle, right the way through to the completion of Senior cycle. It has been noted that factorising is a consistent challenge for pupils starting Senior cycle. School tests results show that there is approximately a 20% discrepancy in pupil marks between tests including factorising and those excluding factorising.

b) From the chief examiners report it was highlighted that in some cases, the constant was dropped (temporarily) from the cubic and then the expression was divided by x; in other cases, x was factorised out of the first three terms and then discarded as the candidate worked on with the quadratic part; some even differentiated the cubic to reduce it to a quadratic.

4. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Pupils have studied finding Highest Common Factors, grouping terms, difference of 2 squares and quadratic expressions in Junior Cycle. Pupils have learned how to use the formula: • Evaluate expressions given the value of the variables • Expand and re-group expressions • Use the associative and distributive properties to simplify expressions of the form • $a(bx \pm cy \pm d) \pm \ldots \pm e(fx \pm gy \pm h)$ where a, b, c, d, e, f, g, h ∈ Z • $(x \pm y)(w \pm z)$	• Select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: - $g(x) \leq k$, $g(x) \geq k$, - $g(x) < k$, $g(x) > k$, where $g(x) = ax + b$ and a, b, k ∈ Q - $g(x) < k$, $g(x) > k$, with g(x) $ax^2 + bx + c$ $ax^2 + bx + c$ or $g(x) = \frac{ax+b}{cx+d}$ and a, b, c, d, k ∈ Q, x ∈ R	Pupils will build on this knowledge in future topics: 1. Functions 2. Differentiation

5. Goals of the Unit

- Understand how to use previously learned algebraic and functions skills to solve quadratic inequalities.
- Develop an understanding for multiplying rational inequalities by the square of the denominator.
- Apply the knowledge of solving rational inequalities to solve modular inequalities.
- Develop the skills to prove abstract inequalities.

6. Unit Plan

Lesson	Learning goal(s) and tasks
1 The Research Lesson	Students will apply their algebraic knowledge to solve a word problem. Students' answers will provide the foundation to develop the lesson which will incorporate students' prior knowledge of inequalities and lead to students being able to solve quadratic inequalities. Possibility of ceardaíocht proving that a quadratic (with Real roots) is greater than or equal to 0 for all Real numbers.
2	Rational inequalities
3	Rational inequalities
4	Modular equation
5	Modular inequalities
6	Proof of abstract inequalities

7. Goals of the Research Lesson:

- Apply algebraic skills to solve a quadratic inequality.
- Develop problem solving skills in my students.
- Develop communication skills through pair and group work as well as presenting their solutions.
- Understand the application of Maths in real life.
- Possible extension piece: prove that a quadratic (with Real roots) is greater than or equal to 0 for all Real numbers.

8. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Learning intentions of the lesson will be explained to the class. Students will listen.	Display Powerpoint on Board	Are students motivated?
Student Individual Work The function $f : x \to x^2 - 2x - 8$ gives the predicted temperature, in degrees Celsius, over a 10 hour period from 8pm to 6am. By assigning x = -4 for 8pm. (i) At what times is the temperature 0°C. (ii) At what times is the temperature above 0°C. **Part (i)** Option 1: Solve by finding Factors $x^2 - 2x - 8 = 0$ $(x - 4)(x + 2) = 0$ $x = 4 \text{ or } x = -2$ 10pm and 4am .	Explain the problem while handing out the worksheet to each student.	Check students understand the question posed.

Option 2: Solve using the Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)}}{2(1)}$$

$x = 4$ *or* $x = -2$

10pm and 4am .

Option 3: Solve using Completed Square

$x^2 - 2x - 8 = 0$

$x^2 - 2x + 1 - 1 - 8 = 0$

$(x-1)^2 - 9 = 0 \Rightarrow (x-1)^2 = 9$

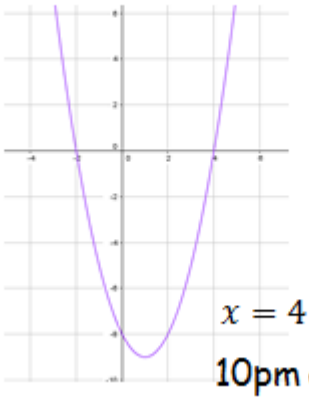
$x - 1 = \sqrt{9}$

$x - 1 = \mp 3$

$x = 3 + 1$ or $x = -3 + 1$

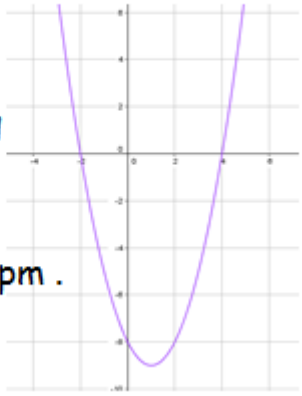
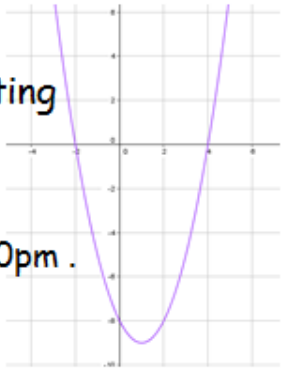
$x = 4$ *or* $x = -2$ 10pm and 4am

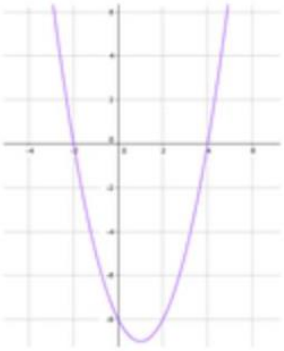
Option 4: Solve by Graphing



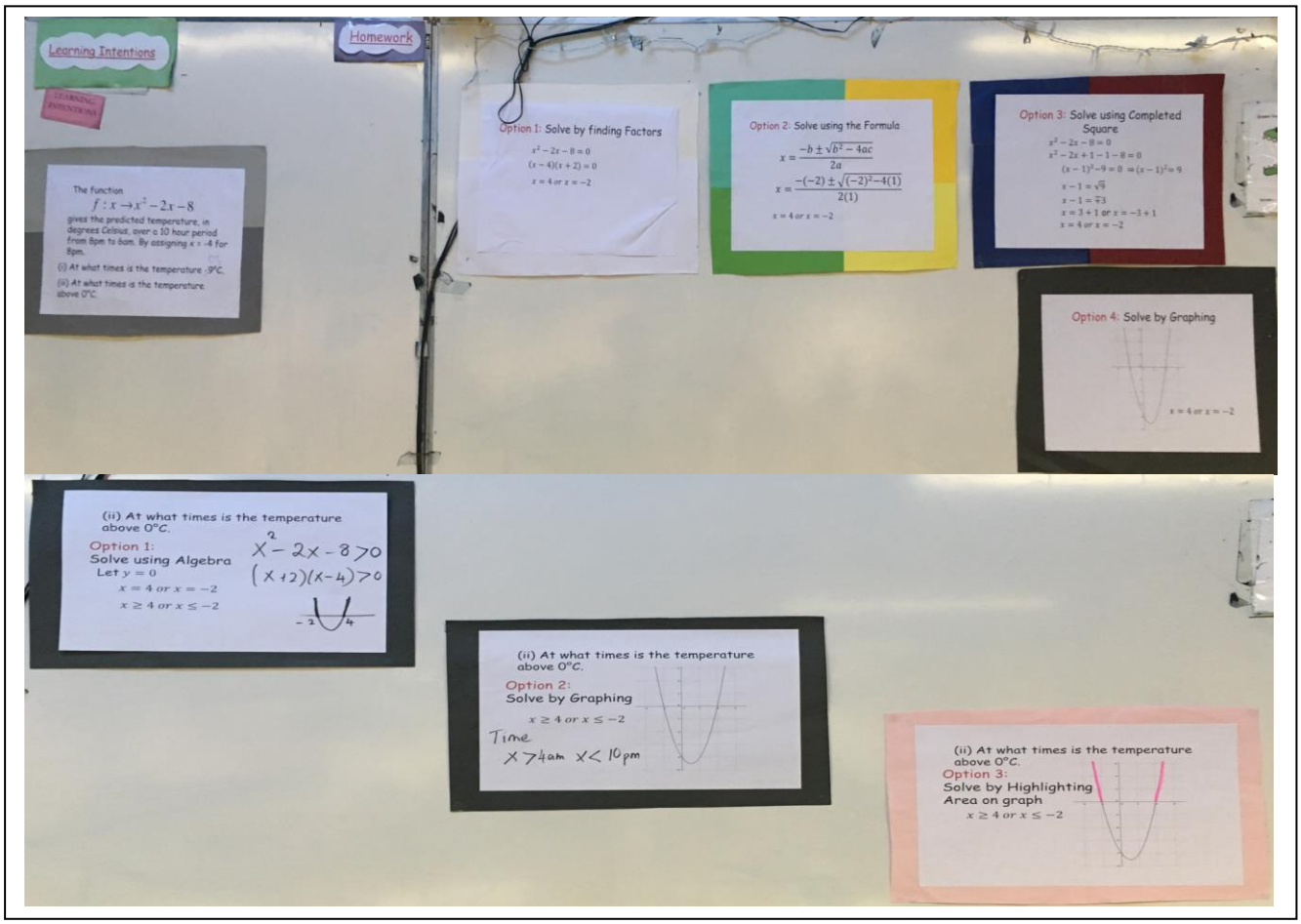
$x = 4$ *or* $x = -2$

10pm and 4am .

Part (ii) Option 1: Solve using Algebra Let $y = 0$ $x = 4 \text{ or } x = -2$ $x \geq 4 \text{ or } x \leq -2$ x ≥ 4am and x ≤ 10pm . (ii) At what times is the temperature above 0°C. Option 2: Solve by Graphing $x \geq 4 \text{ or } x \leq -2$ x ≥ 4am and x ≤ 10pm .  (ii) At what times is the temperature above 0°C. Option 3: Solve by Highlighting Area on graph $x \geq 4 \text{ or } x \leq -2$ x ≥ 4am and x ≤ 10pm . 		
Ceardaíocht Teacher will ask student to look at homework question.	Using appropriate questions the teacher will ensure students know what is required to answer this question.	Teacher will ask students to present their solutions and explain how they got their answer. Misconceptions will be highlighted.

Homework: The temperature is below 0°C from 10pm until 4am, how many ways can you denote this? 		
Summing Up & Reflection Students are asked to reflect on today's lesson by writing down 2 things they did well or learned and 1 thing they could do better or have learned from the lesson.	The teacher will use the layout of the board work to help provide student with a summary of the progression of their learning.	Using post-it notes, students will give their thoughts on the lesson.

9. Board Plan



10. Evaluation

Students engaged well with the activity overall and understood the goal of the lesson. Feedback from students included:

Things you did well	**Things to improve/work on**
I used –b formula well x 4	I wasn't able to convert the answers to time x 2
Used graphing x 10	Reading graphs properly x 3
Factorised the quadratic x 9	Read the question x 2
Solving a quadratic x 3	Drawing graphs x 5
Reading values from graph	Finishing out every question
Found the times x 4	Needs to improve timing x 2
Completed square form used well x 2	Forgot – b formula
Completed the question	Completed square form x 3
Thought of 3 correct methods	How to start the question
No calculation mistakes	Use more methods x 2
Solved in 2 different ways x 2	
I could explain how I got my answers easily	

· This feedback indicated that students enjoyed the process and gained confidence in their own ability to solve part (i) in a number of ways. We as a team agreed that students worked well with their peers and that these interactions with their peers promoted peer to peer learning.

· A misconception identified was that three students reversed the axes of the graph. On reflection, this could be attributed to the fact the question was posed in the form $f:x\rightarrow x^2-2x-8$ instead of (t) as temperature and t is time. This is something that needs to be reinforced by the teacher to the entire class in the next lesson and subsequent graphing lessons.

· Interestingly, students tried more than one method and we believe that this is due to the fact that this class had participated in another lesson study class before Christmas.

· In our opinion, we believe that students enjoyed and benefited from the lesson study approach and as such we feel that regular lesson study classes would benefit all students.

· We anticipated the answers.

11. Reflection

Team Expectations

· We wanted students to present and explain the process and outcome of learning activities to a highly competent level.

· We wanted to observe that students would be able to start the question after a short recap of prior knowledge.

· We expected that all students should be able to solve part (i) of the question using at least two methods and that all students would attempt part (ii) of the question.

Team Observations

· Teachers involved engaged in assessment of students learning using both assessment of learning and assessment for learning.

· Teachers probed students to explain and provide rationale for their choice of methodologies and solutions to questions in class. Key Mathematical terminology was highlighted throughout the lesson.

· Teachers employed a number of Assessment for Learning techniques. For example, involving students in their own learning by assessing their own work and reflecting on their learning at the end of key lessons.

Student Observations by the team

· We observed that all students solved part (i) of the question using at least two methods and not all students attempted part (ii) of the question. Some students also needed additional prompting to solve the question and this was expected by the team as they are 4th Stream class. Students initially attempted the question on their own and when they couldn't progress any further they collaborated well with their peers. We were surprised and impressed that 4 students attempted to use the completed square form as a method.

· Students reported on, presented and explained their solutions to the activity to a highly competent level.

· Approaches used by the students: Students used a number of approaches including algebraic methods, graphical methods and sequencing methods to solve the questions. These solutions were pre-empted by the team.

Students engaged well with the activity overall and understood the goal of the lesson. Feedback from students indicated that students enjoyed the process and gained confidence in their own ability to solve part (i) in a number of ways. We as a team agreed that students worked well with their peers and that these interactions with their peers promoted peer to peer learning.

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Future Study

The team suggested using this Lesson Study with a 1st stream class to investigate if there is a difference in students approaches to the question. Perhaps it would need to be altered to further challenge 1st stream students. This could be achieved by using abstract inequalities as the ceardaíocht question.

Benefits of participating in Lesson Study

One of the main benefits of lesson study is working with colleagues. In a busy school, time is always an issue. However, lesson study gave us the opportunity to use our CPD to have conversations about our students' misconceptions in Maths and their problems with particular topics. Lesson study also provided us with the platform to work together to prepare a lesson designed specifically to challenge our students to work outside their comfort zone and apply their prior knowledge to solve a problem. Finally and perhaps most importantly, our students benefited. Being able to understand and solve the question boosted students confidence and morale. As one student commented 'This is the first time I've left Maths feeling good about myself!'

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