

Lesson Research Proposal for 2nd Year Ordinary Level

For the lesson on 2nd February 2018

At Hartstown Community School, Ms. Brannigan's class

Instructor: Geraldine Brannigan

Lesson plan developed by: Dawn Hutchings-Walsh, Fiona Hartney, Geraldine Brannigan

1. Title of the Lesson: Count like an Egyptian

2. Brief description of the lesson

Students are given a problem and asked to explore different approaches to query the difference between multiplication and addition.

3. Research Theme

At Hartstown Community School, we want our pupils:

- 1) To engage purposefully in meaningful learning activities.
- 2) To enjoy learning, to be motivated to learn and to expect to achieve as learners.

We as teachers want:

- 1) To teach in a creative environment where students feel safe taking risks and approaching problems in different ways
- 2) To create problems that arouse students' interest and motivation
- 3) To develop students' levels of confidence in mathematics so that they understand that ability is not fixed and that their effort will be reflected in their level of achievement.

4. Background & Rationale

a) Why you chose the topic

We have chosen the topic of indices in algebra, specifically the difference between $(a+a)$ and $(a \times a)$. This lesson is aimed at 2nd year students. We chose this topic as from our experience it is fundamental to all students and is often misunderstood and mistakenly applied from first year through to Leaving Certificate. This leads to difficulties across the syllabus.

Algebra is often taught with a procedural approach. We feel students do not understand and/or retain the concept. In particular, students need to understand the difference between addition and multiplication of terms, especially those which include indices. This will have further implications throughout their maths studies.

b) Your research findings

Through discussions within the maths' department and consultation with the maths' department subject inspection, we realize that we often take a procedural approach in teaching algebra particularly. Students are not developing their understanding fully and cannot apply this concept.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<ul style="list-style-type: none">● Recognize and use multiplication of the different number systems and the laws of indices● BMDAS● Simplify algebraic expressions● Multiplication of algebraic expressions through use of area model	<ul style="list-style-type: none">● Simplify algebraic expressions● Remove brackets using Distributive Law● Substitution● Solve linear equations● Solve inequalities	<ul style="list-style-type: none">● Factorizing polynomials● Calculus● Trigonometry● Functions● Area and volume● Statistics

6. Goals of the Unit

- Students need to understand that the laws of the order of operations will carry through to algebra
- Students will apply their prior knowledge of using symbols in primary school and relate this to the concept of variables, constants and coefficients
- Students need to know how to simplify expressions involving addition and subtraction of like terms
- Understand the difference between a^2 and $2a$
- Understand the origins of a^2 and $2a$
- Students need to know how to multiply expressions by applying the Distributive Law, including the use of the laws of indices
- Students need to be able to solve equations and understand the concept of balance in order to use inverse operations.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Review order of operations and link use of symbols to variables, coefficients and constants Students will understand how to use the area model to multiply larger integers.
2	Simplifying expressions <ul style="list-style-type: none"> • Addition and subtraction of like terms
3	Simplifying expressions <ul style="list-style-type: none"> • Addition and subtraction of like terms
4 Research lesson	Introduce multiplication of expressions in a problem-solving context. <ul style="list-style-type: none"> • Use a suitable problem to understand the difference between $(a + a)$ and $(a \times a)$ where • Use the area model to multiply term by expression
5	Distributive law <ul style="list-style-type: none"> • Apply knowledge from previous lesson to simplify expressions using the Distributive Law
6	Distributive law <ul style="list-style-type: none"> • Apply knowledge from previous lesson to simplify expressions using the Distributive Law
7	Solving equations <ul style="list-style-type: none"> • Understand the concepts of balance and inverse operations to solve simple linear equations
8	Solving equations with variables on both sides <ul style="list-style-type: none"> • Students will understand how to apply the concept of balance and inverse operations to solve equations with variables on both sides
9	Solving equations involving multiplication of term by expression <ul style="list-style-type: none"> • Students will understand how to apply the concepts of balance and inverse operations to solve equations involving the multiplication of term by expression
10	Solving equations involving multiplication of term by expression <ul style="list-style-type: none"> • Students will understand how to apply the concepts of balance and inverse operations to solve equations involving the multiplication of term by expression

8. Goals of the Research Lesson:

a) Mathematical Goals

Students will:

- Understand the difference between a^2 and $2a$
- Understand the origins of a^2 and $2a$

b) Key Skills & Statements of Learning

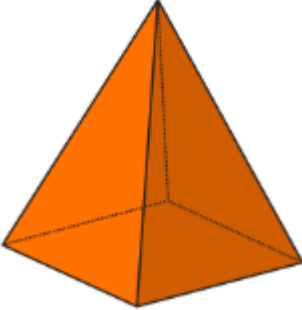
In the planning and design of this lesson the Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will implement and promote JC Key Skills in the following ways:

1. Being Literate: Students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: It will develop a positive disposition towards problem solving.
3. Managing Myself: Student's will have the opportunity to reflect on their own learning.
4. Staying Well: Students' confidence and positive disposition to learning will be promoted.
5. Communicating: Students will present and discuss their mathematical thinking.
6. Being Creative: Students' will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with Others: Students will learn with and from each other.
8. Managing information and thinking: Students will be encouraged to think creatively and critically.

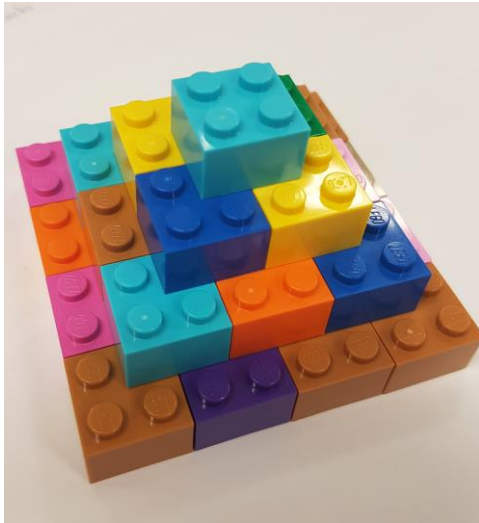
This lesson is also designed to meet the following JC Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The students describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
<p>Introduction 2 Minutes</p> <p>Today we are going to use our mathematical knowledge to solve a problem. We're going to try to solve the problem by ourselves and then we're going to come together as a class and use all your knowledge to discover something new</p>		<p>Can we do it? Yes we can!</p>
<p>Posing the Task</p> <p>In ancient Egypt, a pharaoh used to show how powerful they were by ordering big square-based pyramids to be built in their name.</p> <p>Each pharaoh wanted to build their pyramid one level higher than the last to show that they were, in fact, the greatest.</p> <p>Part1: How many blocks would you need to add if the previous pharaoh's pyramid had a bottom layer made of 16 bricks in order to make your pyramid one layer bigger?</p> <div style="text-align: center;">  </div> <p>Is there a way of working out a rule to tell us how many blocks we need for the 10th layer? Any layer?</p> <p>Part 2: The next pharaoh wants to build his pyramid one layer bigger but isn't sure how big the previous one was. He sends 2 scouts to count the blocks in the bottom layer, each counting two sides.</p> <p>How many faces does each count if the pyramid has 5 layers in total?</p> <p>Is there a way of working out a rule to tell us how many each counted if the pyramid had 10 layers? What if it had any number of layers?</p>	<p>Distribute problem handout and explain the problem</p>	<p>Check for understanding and scaffold questions for groups who may not understand</p>
<p>Part 1: <u>Student response 1:</u></p>	<p>As one makes class rounds, one looks for examples that will lead</p>	<p>Are the students able to handle this problem?</p>

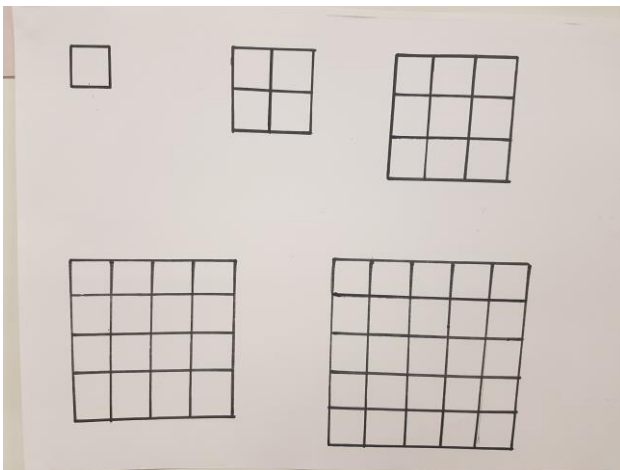
Make model and count number of blocks needed.
25 blocks



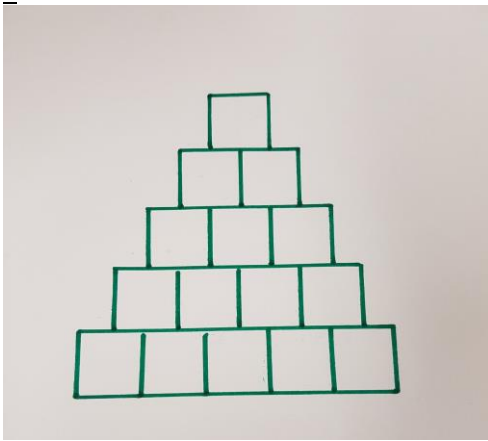
Student response 2:

Diagram

1



2



the students to an understanding of the methods to solve the problem.

If students using the lego bricks are counting the dots as opposed to the bricks, one will encourage them to count the bricks instead.

For students who might have stopped at this point we could ask

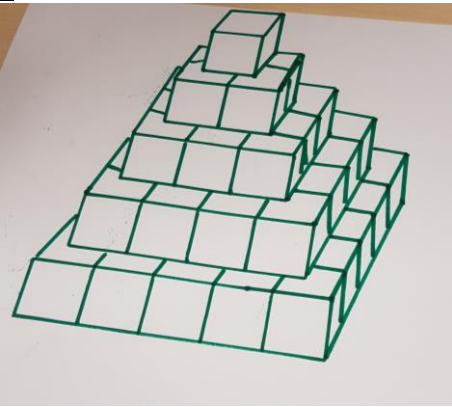
“Could you do the above in a more efficient way”.

Are they using insight and trying different methods?

Students may count the faces of the whole model, rather than those on the bottom layer, or they may only count the faces on one side.

If this is the case, students will be directed to ensure that they have read the question accurately.

3



Student response 3:

Make a table

Layer	Blocks
1	1
2	4
3	9
4	16
5	25

Student response 4:

Use algebra

$$5\text{th layer} = 5^2$$

$$10\text{th layer} = 10^2$$

$$n\text{th layer} = n^2$$

Part 2:

Student response 1:

Count faces from model

10 each

Student response 2:

Count faces/sides in diagram

Student response 3:

Make a table

Layer	Faces
1	2
2	4
3	6
4	8
5	10

Student response 4:

Use algebra

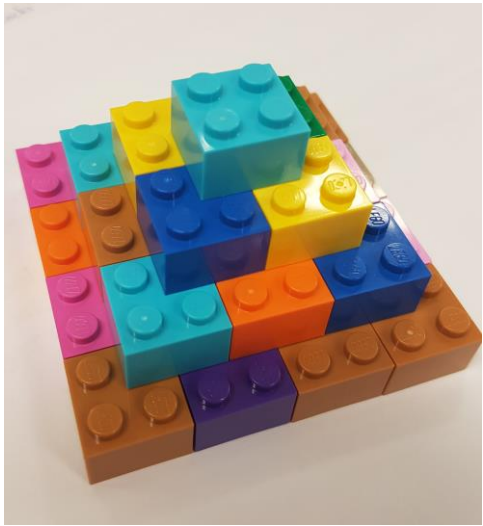
$$5 \times 2 = 10 \quad 5 + 5 = 10$$

$$10 \times 2 = 20 \quad 10 + 10 = 20$$

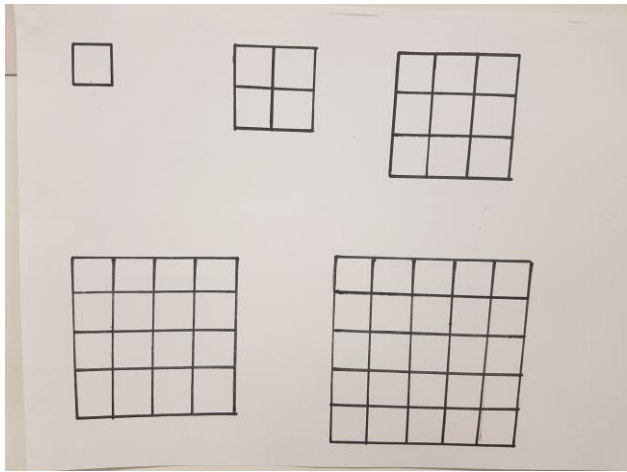
$$n(2) = 2n \quad n + n = 2n$$

Ceardaíocht /Comparing and Discussing

Student response 1:
 Make model and count number of blocks needed.
 25 blocks



Student response 2:
 Diagram
 1



2

Response 1: Please raise your hand if you used this method. Did anyone else get the same method?

Are students able to discuss their solution? During this discussing, what is the level of interaction?

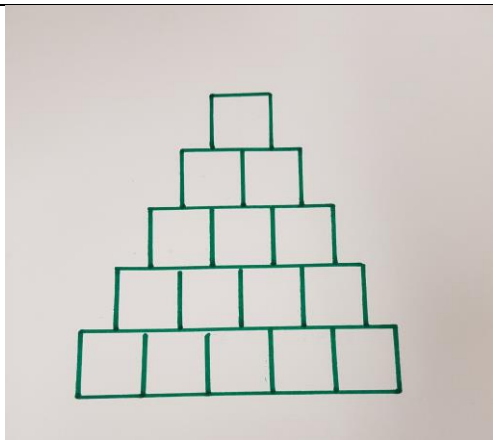
Are students looking at the merits of the various responses?

Do students understand the concept of multiplication and indices? Can they explain it in words?

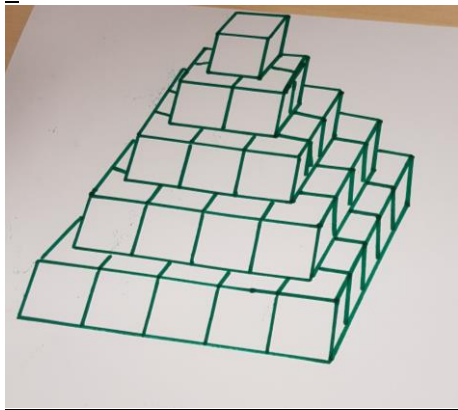
Response 2: Would anybody like to talk about this solution? Why does this diagram represent the problem more clearly than this one? What are the advantages/disadvantages of using this method? Would you prefer it to the first method?

Response 3: Would anybody like to talk about this solution? Do you notice a pattern? What is the relationship between the layers and the blocks? Would you prefer this method rather than method 1 or method 2? If so, why? Can you now explain to me how this method works?

Response 4: Why did you multiply by 5? What do your numbers represent? Do you agree? Can anyone put words on what the numbers represent? What is the advantage of this method?



3



Student response 3:

Make a table

Layer	Blocks
1	1
2	4
3	9
4	16
5	25

Student response 4:

Use algebra

5th layer = $5 \times 5 = 5^2$

10th layer = $10 \times 10 = 10^2$

nth layer = $n \times n = n^2$

Student response 1:

Count faces from model
10 each

Student response 2:

Count faces/sides in diagram

Response 1: Please raise your hand if you used this method. Did anyone else get the same method?

Response 2: Would anybody like to talk about this solution? What are the advantages/disadvantages of using this method? Would you prefer it to the first method?

Are students able to discuss their solution? During this discussing, what is the level of interaction?

Are students looking at the merits of the various responses?

Do students understand the concept of

<p><u>Student response 3:</u> Make a table</p> <table border="1"> <thead> <tr> <th>Layer</th> <th>Faces</th> </tr> </thead> <tbody> <tr><td>1</td><td>2</td></tr> <tr><td>2</td><td>4</td></tr> <tr><td>3</td><td>6</td></tr> <tr><td>4</td><td>8</td></tr> <tr><td>5</td><td>10</td></tr> </tbody> </table> <p><u>Student response 4:</u> Use algebra</p> <table border="1"> <tbody> <tr><td>$5 \times 2 = 10$</td><td>$5 + 5 = 10$</td></tr> <tr><td>$10 \times 2 = 20$</td><td>$10 + 10 = 20$</td></tr> <tr><td>$n(2) = 2n$</td><td>$n + n = 2n$</td></tr> </tbody> </table>	Layer	Faces	1	2	2	4	3	6	4	8	5	10	$5 \times 2 = 10$	$5 + 5 = 10$	$10 \times 2 = 20$	$10 + 10 = 20$	$n(2) = 2n$	$n + n = 2n$	<p><u>Response 3:</u> Would anybody like to talk about this solution? Do you notice a pattern? What is the relationship between the layers and the blocks? Would you prefer this method rather than method 1 or method 2? If so, why? Can you now explain to me how this method works?</p> <p><u>Response 4:</u> Why did you multiply by 2? What do your numbers represent? Do you agree? Can anyone put words on what the numbers represent? What is the advantage of this method? What is different about this solution to the one in the first part of the problem?</p>	<p>multiplication and indices? Can they explain it in words?</p>
Layer	Faces																			
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<p>Summing up & Reflection We learned that:</p> <ul style="list-style-type: none"> • n^2 and $2n$ are not equal in general • n^2 is multiplying a number by itself and often shows area • $2n$ is multiplying a number by 2 and can be used to show length <p>Ask students to write a reflection</p>	<p>Use the layout of the board work to help provide students with a summary of the progression in their learning.</p>	<p>Do the students' reflections represent the teacher's view of the lesson?</p>																		

10. Board Plan

The board plan is organized into four columns with corresponding sticky notes:

- Count/Model (Green):** Features a small red block diagram representing a single layer.
- Diagram (Green):** Displays several grid diagrams representing layers of blocks, showing how the area increases with each layer.
- Table (Orange):** Contains two tables. The first table shows the relationship between the layer number and the total number of blocks. The second table shows the relationship between the layer number and the number of faces on the side.
- Algebra (Orange):** Contains a table that summarizes the data from the previous tables using algebraic notation, showing the area of each layer and the number of faces on the side.

11. Evaluation

- a. To evaluate the lesson, the 3 observers were each assigned an area of the room, each observer had seven or eight students to observe. All observers were given a seating plan and took notes of interactions between students and interactions between teacher and students.
- b. Student engagement and Learning was the research theme of the lesson study so evaluation of this by means of framing the reflection appropriately was important. Students' were asked to reflect on their feelings about the class using post it notes which they stuck to the classroom door on their way out. |A portion of post Lesson discussion centred on this theme.
- c. Students did not complete the task and therefore were unable to reflect on whether they achieved the mathematical goals of the lesson. However, through the notes taken by the observers in the room there was clear evidence that the "link" had been created and all observers were confident that most of their respective groups would have been successful had they sufficient time.

12. Reflection

- a. The students were all highly engaged in the task set by the teacher.
- b. The students' reflections indicated a high level of enjoyment of the lesson.
- c. The students successfully established an understanding of the origins of a^2 and $2a$.
- d. The students did not achieve the second aim of the lesson, to understand the difference between a^2 and $2a$.
- e. The teachers noted, the unease of most of the students to be independent problem solvers this led to a number of sought clarifications which hindered the progress of the lesson.
- f. The students were reluctant to write down any possible solutions without prior confirmation that the solution was correct.
- g. One student, vocalised a possible solution and this shaped the thought process of many of the other students. The student used the only given numerical values and operated without reason and the rest of the class shaped their answers to shape this reasoning.
- h. The students had difficulty constructing the pyramid, this was unexpected.
- i. A number of students progressed to different approaches
- j. The students presented their solutions very well and were competently able to answer follow up questions.
- k. The students were notably more enthusiastic about the second task once assigned
- l. The class did not have sufficient time to complete which was disappointing to all involved.
- m. The students' reflections mostly commented on their enjoyment of the lesson rather than their learning outcome.
- n. The teachers suggested for improving the lesson:
 - i. For this particular group further scaffolding could have been beneficial eg. Giving instruction as to where to start the construction of the pyramid, although it is noted that the balance is hard to strike.
 - ii. The lesson would have been more suitable for a longer class (60 mins or a double period)
 - iii. Encourage their reflections to be more towards their learning.