Lesson Research Proposal for 2nd Year Coordinate Geometry-Distance Formula

For the lesson on date: 01/02/2018
At Deansrath Community College
Instructor: Lorraine Darcy
Lesson plan developed by: Paddy Gallagher, Lorraine Darcy

1. Title of the Lesson: Cover the Distance Faster

2. Brief description of the lesson

Students apply their knowledge and understanding of coordinates, calculating length and Pythagoras' Theorem to investigate the relationship between Pythagoras' Theorem and calculating the distance between two points, with the intension they will derive the distance formula.

3. Research Theme

At our school,

- We want our students to report on, present and explain the process and outcome of learning activities to a highly competent level.

- We want teachers to engage in planning for assessing all relevant aspects of students' learning using both assessment of learning and assessment for learning.

As a Mathematics department, we will actively support the achievement of these goals by endeavoring to do the following:

(a) Using Blooms Taxonomy and active questioning techniques, teachers will probe students to explain and provide rationale for their choice of methodologies and solutions to questions in class. Key Mathematical terminology will be highlighted throughout the lessons.

(b) Teachers will employ a number of Assessment for Learning techniques. For example, involving students in their own learning by assessing their own work and reflecting on their learning at the end of key lessons.

4. Background & Rationale

(a) Why we choose the topic

The lesson is aimed at second year higher level students. Teaching the distance formula of the line brings together previous knowledge of co-ordinates, rise and run (slope) and Pythagoras' Theorem. In previous years we have taught this on a rote-learning basis. Specific issues arise due to a lack of understanding as to where the distance formula stems from, and why it is being used. Students don't often see the links between the distance formula and prior knowledge and so are unable to utilize this for future learning. We aim to step away from this and give deeper understanding of the process of getting the formula.

(b) Our Research Findings:

Through discussions with members of the Mathematics department, we agreed that teaching the distance formula is based on a routine of students learning the formula off by heart without knowing its basis. Due to these deficits, we decided collectively to teach the distance formula using problem-solving techniques and linking prior knowledge.

5. Relationship of the Unit to the Syllabus

Primary school and first year prior learning Outcomes	Learning outcomes for the Second Year Coordinate Geometry Higher Level Course	Related later learning for Junior Cert Higher Level and Leaving Cert
• Use angle and line properties to classify and describe triangles and quadrilaterals • identify, describe and classify vertical, horizontal and parallel lines • plot simple co-ordinates and apply where appropriate use geoboards and squared paper • Navigating the co-ordinate plane • Plotting coordinates • Properties of lines and line segments including midpoint • Construction of a right angle triangle.	In 2nd year students will revisit how to plot points on a co-ordinate plane, finding the slope using rise and run and construction of a right angled triangle. Students should be able to use Pythagoras' Theorem to obtain the length of the sides of a right-angled triangle. Students should find the length of a line segment or distance between two points. Students will create a formula for the distance between two given points (x_1,y_1) and (x_2, y_2) Students will be required to use the equation of a line in the forms $y - y_1 = m(x - x_1)$ $y = mx + c$	In leaving certificate, students will apply the Theorem of Pythagoras to solve problems of a simple nature involving heights and distances in right angled triangles Leaving Cert students will utilize this in co-ordinate geometry and trigonometry Higher level students will use the distance formula in proving trigonometric identities

6. Goals of the Unit

(a) Students will understand that there are numerous ways of arriving at a solution to a given problem. They should see that links exist between all parts of the unit.

(b) Students should be able to manipulate their learning to achieve solutions to real life problems and problems from contextualized situations.

(c) Students should have the confidence to try different approaches to gaining a final outcome or solution.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Revision of first year content: • Plotting points on the XY plane. • Plotting line segments with given points • Constructing right angle triangles on the XY plane . Using visual and concrete resources e.g. whiteboard, geogebra, geometry sets
2	Pythagoras' Theorem: • Get students to construct right angle triangles • Students should measure the sides and find relationships between the given sides
Research Lesson	Students will be able to come up with the distance formula using (x,y) points
4	Students will solve problems based on the distance formula Introduce slopes: • rise over run • have students formulate the slope formula using x and y points Using visual and concrete resources e.g. whiteboard, geogebra, geometry sets
5	Introduce perpendicular slopes and how they are used in solving line problems, tangents and how later we will use for forming equations of lines
6	Formulate the mid-point formula and solve questions using this formula and then have the equation of a line formula introduced as $y-y_1 = m(x-x_1)$
7	Students will solve problems based on the equation of a line formula. They will find the equation of lines when given points only or points and slope
6	Explain how to work backwards to get slopes from given lines, i.e. $y = mx + c$. Students will graphically solve questions where lines cross the axes and cross other given lines.
7	Students will focus on solving simultaneous equations algebraically. Students should discover how symmetry works on X and Y axes and central symmetry through the origin

8. Goals of the Research Lesson:

Mathematical Goals:

Students will

- Understand the use of Pythagoras' Theorem
- Understand the concept of distance
- Derive the formula for distance through collaboration

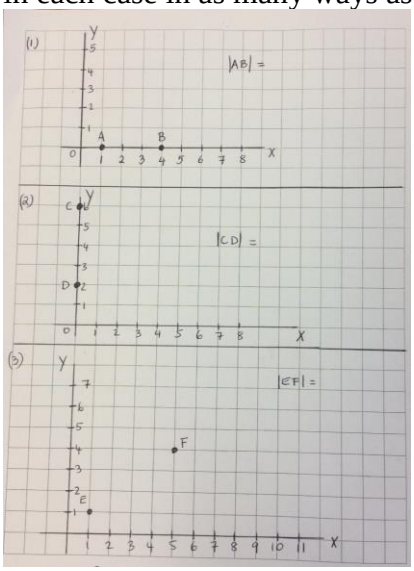
a) Key Skills and Statements of Learning:

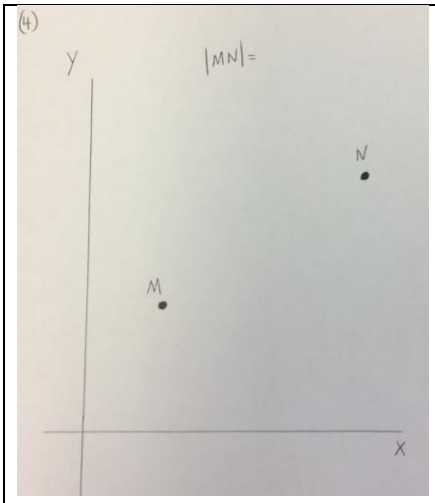
In preparation for implementing the Junior Cycle Specification for Mathematics our Maths department have begun to integrate the development of key skills into Maths lessons.

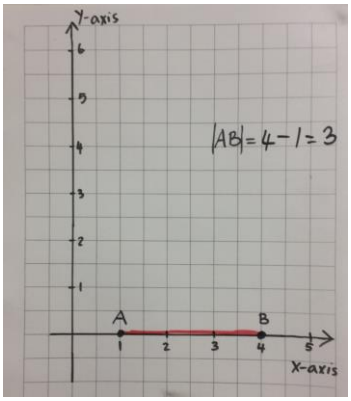
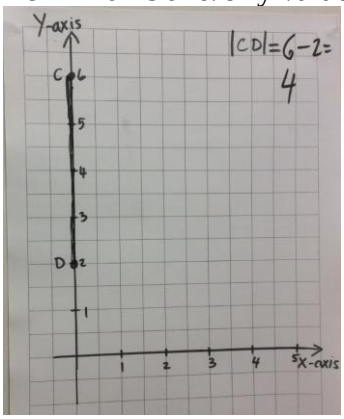
This lesson will address the following key skills:
1. Being Literate: Through Ceardaíocht, students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: By engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.
3. Managing Myself: Students will have the opportunity to reflect on their own learning when the teacher asks them to write a reflection at the end of the lesson.
4. Staying Well: By engaging in tasks which are appropriate to their abilities, students confidence and positive disposition to learning will be promoted.
5. Communicating: During Ceardaíocht, students will present and discuss their Mathematical thinking.
6. Being Creative: Students will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with Others: Students will learn with and from each other by discussing different approaches to solving the problem.
8. Managing Information and Thinking: Students will be encouraged to think creatively and critically.

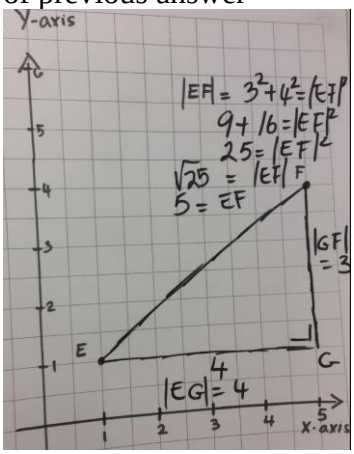
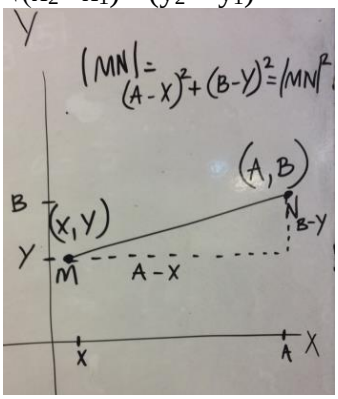
This lesson also meets the following JC Statements of Learning: 1, 15, 16, 17 and 23.

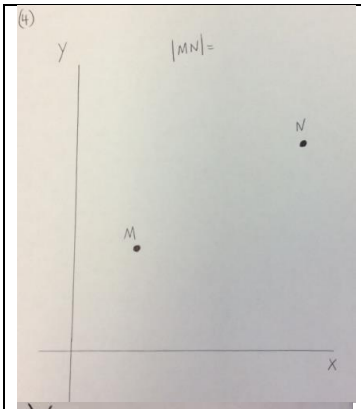
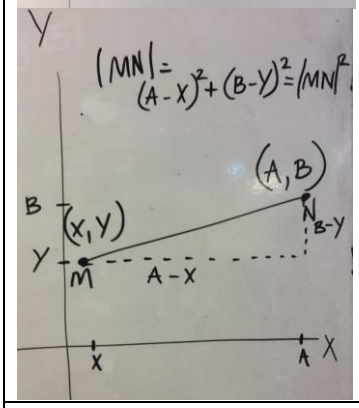
9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Posing the task Find the distance between the two given points in each case in as many ways as possible.  	Task will be written on the board and on handed out worksheets.	Q1. Student will see you can count units on the X-axis to get the distance Students can subtract lower value from upper value (of x) Q2. Students repeat above but in relation to Y-axis and y values Q3 As per Q1, student can get distances/lengths from X-axis and/or x values

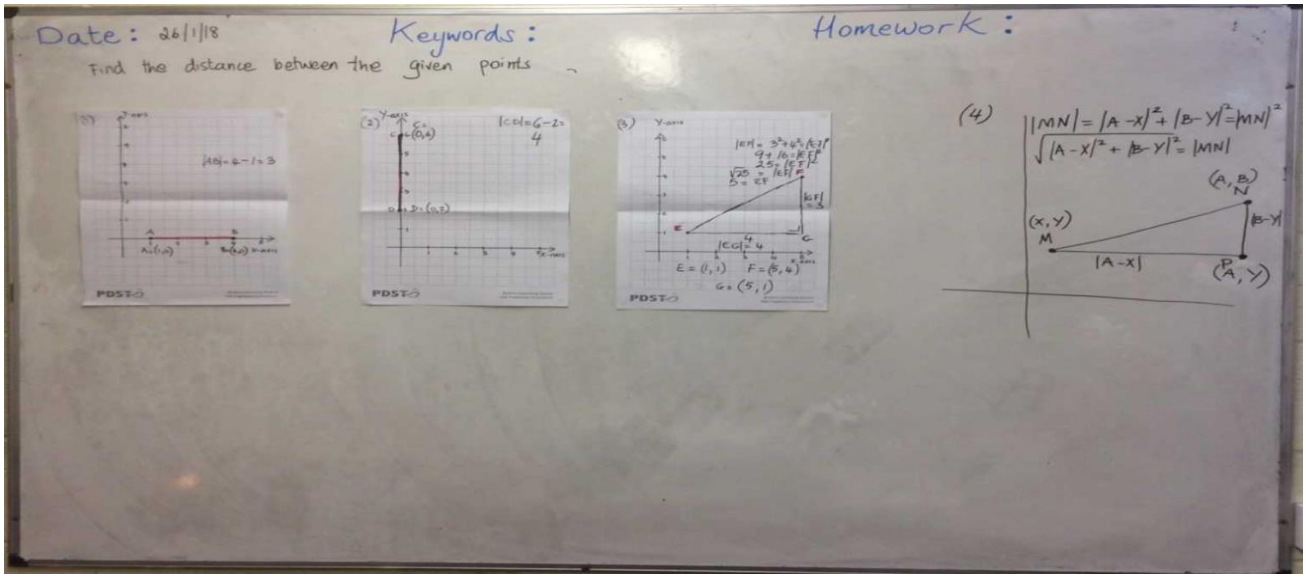
		Q4. As per Q2, student will get distances/lengths from Y-axis and/or y values Q5. Students will create right angled triangles Students can figure out horizontal and vertical distances on the triangle They should square these values to get the hypotenuse squared Find the hypotenuse by getting the square root of previous answer Q6. Students must label given points a and b, (x_1,y_1) and (x_2,y_2) respectively Students will create right angled triangles Students can figure out horizontal and vertical distances on the triangle using x_1, x_2, y_1 and y_2 They should square these distances to get the hypotenuse squared Find the hypotenuse by getting the square root of the previous answer $\sqrt{(x_2 - x_1)^2+(y_2 - y_1)^2}$

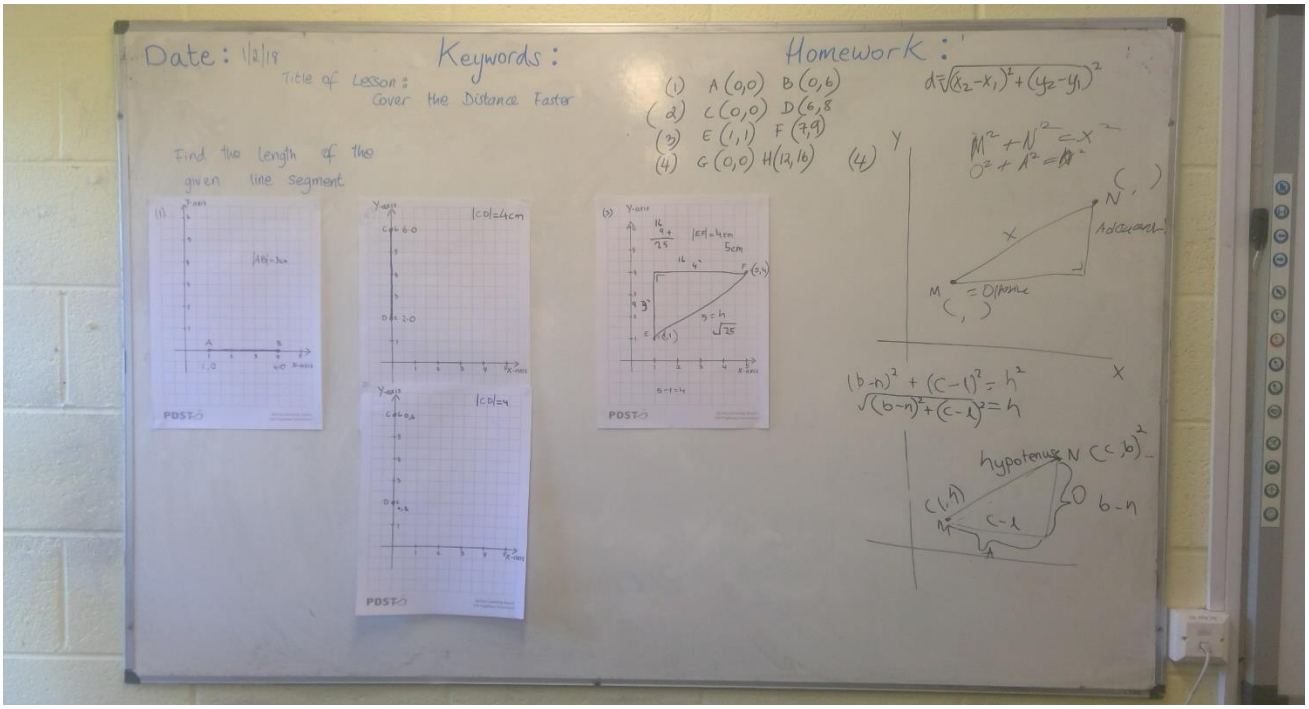
Student Individual Work Give anticipated student solutions, starting with the most likely. Include sample graphs or diagrams, if the reader would need them to understand. Unlikely solutions do not need to be included. Student response(s) Q1. Can we use a ruler Miss? Student will see you can count units on the X-axis to get the distance Students can subtract lower value from upper value (of x) Q2. Students repeat above but in relation to Y-axis and y values Q3 As per Q1, student can get distances/lengths from X-axis and/or x values  Q4. As per Q2, student will get distances/lengths from Y-axis and/or y values  	**Describe how the teacher will handle the different student responses, especially incorrect solutions, students who get stuck, or students who finish early.** What would happen if you used cm and somebody else mm and somebody else used km? Who would be correct? Is there a reason why you all get different answers? Each space on the axis represents one unit…hence, we're all getting the same answers	

Q5. Students will create right angled triangles Students can figure out horizontal and vertical distances on the triangle. They should square these values to get the hypotenuse squared Find the hypotenuse by getting the square root of previous answer  Q6. Students must label given points a and b, (x_1,y_1) and (x_2,y_2) respectively Students will create right angled triangles Students can figure out horizontal and vertical distances on the triangle using x_1, x_2, y_1 and y_2 They should square these distances to get the hypotenuse squared Find the hypotenuse by getting the square root of the previous answer $\sqrt{(x_2 - x_1)^2+(y_2 - y_1)^2}$  	Using what you discovered in Q5, can you see a relationship that may help in solving this question? Why can't the two points both be (x,y)? What is the third point called when you construct your right angled triangle? How do you label it?(x_2,y_1) How do you find the lengths of the triangle sides using these 'new' points?	
Ceardaíocht /Comparing and Discussing Focusing on Q4, the teacher probes and develops their ideas around the relationship between the coordinates and Pythagoras' Theorem to derive the distance formula.	Effective questions to include: "What do you think"? (ask another student(s) other than the presenter) "Why is that"? (Looking	Teacher is looking for students to develop a stronger understanding of the relationship between calculating the distance between two points using

 	for evidence) "Can you explain?" "Did anyone else solve it the same way? Can you explain this method?"	Pythagoras' Theorem. The students drawing links between the two questions. Supporting their answers with mathematical reasoning. Engaging in the discussions about the scenario.
Summing up & Reflection Students are asked to further explore the ideas of the lesson through the homework activity (2nd question) Students are asked to reflect on today's lesson.	The teacher will use the layout of the board work to help provide students with a summary of the progression in their learning.	Using post-it notes surveys students will write down one thing that they thought they did well today in class, one new thing they noticed, any questions they have still at the end of class.

10. Board Plan

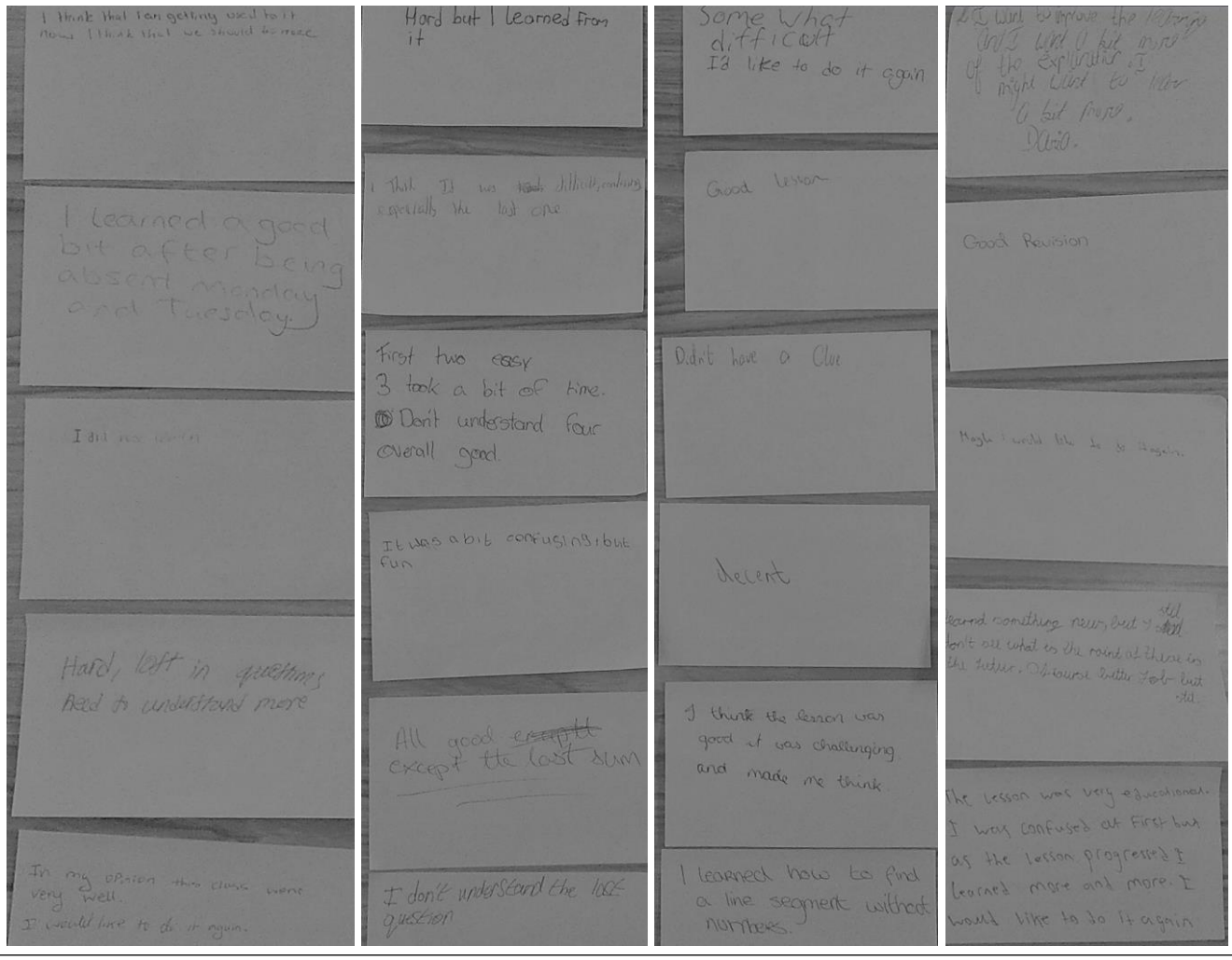




11. Evaluation

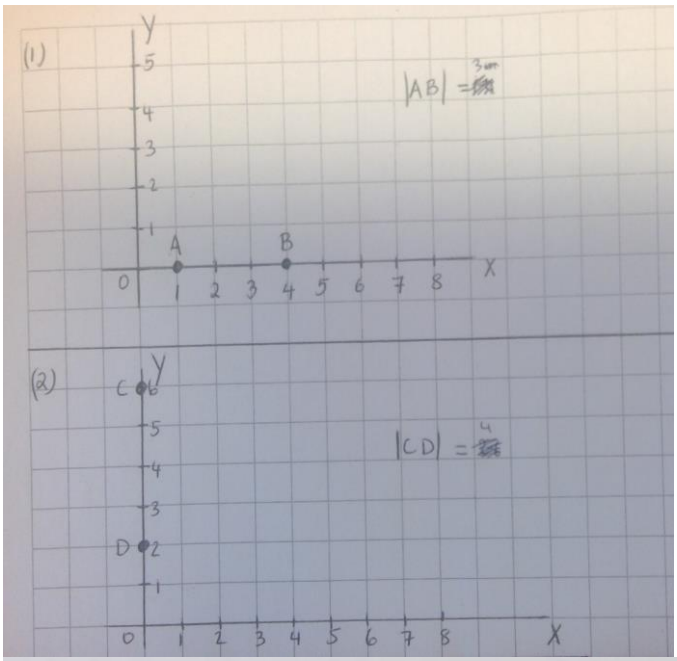
Students engaged well in the activity and understood the goals of the lesson. The students were able to come to the board and explain clearly their methods and reasons. During the 'Ceardaíocht' students were further facilitated towards the final outcome. During the discussions the students used the previous questions as a guide.

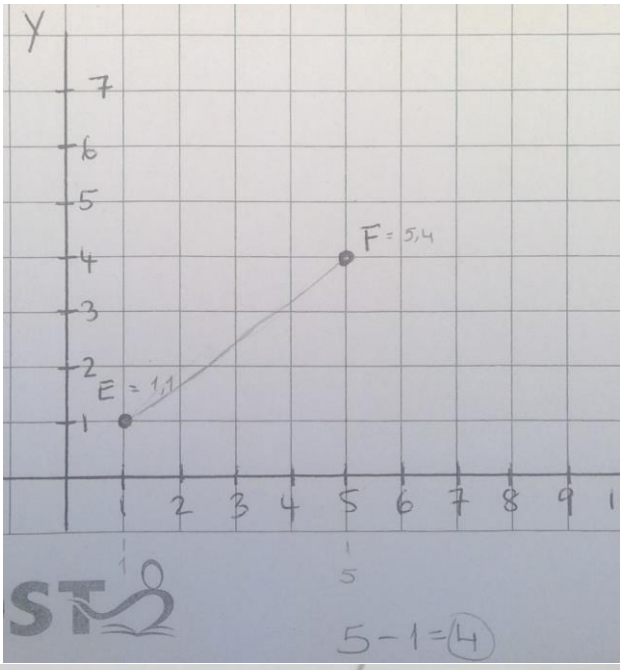
Student Feedback

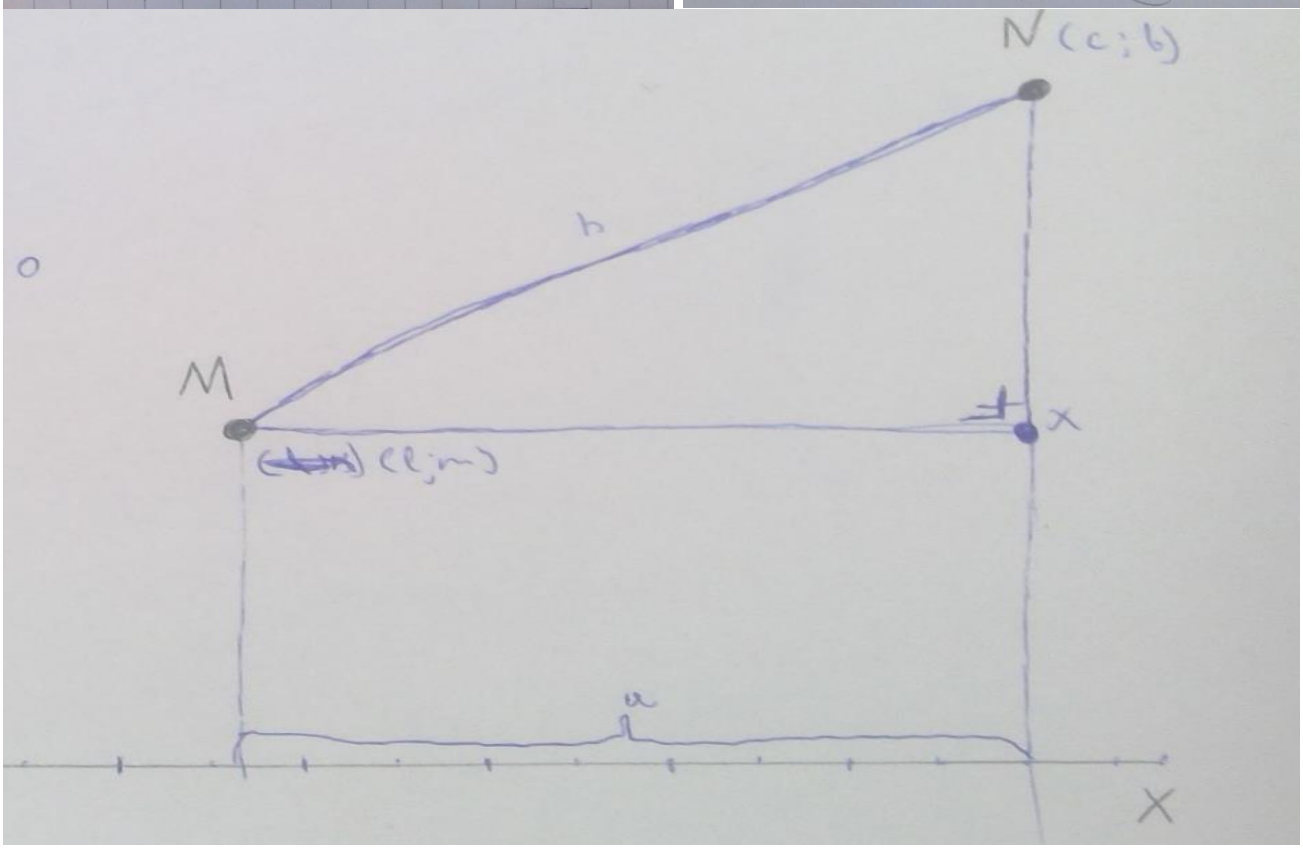


Overall in our opinion the students enjoyed the lesson study approach and would like to use this process again. We feel that regular lesson study classes would hugely benefit students with their confidence in Maths and with forming problem solving techniques. It gives them an opportunity to form solid Maths links across the curriculum.
One student said they didn't learn, a second student didn't have a clue and two students found it difficult especially the last one.

Student Solutions







Students answered questions 1 and 2 easily as anticipated. Question 3 proved difficult for most students. During the ‘Ceardaíocht’ this was rectified and addressed. Students’ understanding of Question 4 was linked to the ’Ceardaíocht’ of Question 3.

(b) Reflection

Hoped to observe
It was hoped that students would identify the co-ordinates of the given points, find lengths by subtracting x or y values, as necessary. We hoped students would use Pythagoras’ Theorem to derive the distance formula.

Actually observed
Initially some students answered questions one and two by measuring the distance instead of identifying the co-ordinates and subtracting x or y values. Some just counted the boxes. During the ‘Ceardaíocht’ students who hadn’t identified co-ordinates were encouraged to do so by observing peers explaining at the board. Most students didn’t know to use Pythagoras’ Theorem to solve the question. Some joined the co-ordinates together but most failed to draw a right-angled triangle. During the ‘Ceardaíocht’ students who correctly drew the triangle were probed to explain to others how they got the length of the given line segment. This created a ‘Eureka’ moment for others. Students found the final question very difficult to bring the numerical questions to algebraic form. Through ‘Ceardaíocht’ students were encouraged by the teacher to infer from question 3 the method used, using co-ordinates.

Post lesson discussion
The lesson progressed well and once told not to use rulers they understood a better way of finding the length. Question 3 proved difficult but through ‘Ceardaíocht’ the students identified the link to Pythagoras’ Theorem. Question 4 was very challenging because they didn’t fully understand the subtraction of co-ordinates. Another similar question to number 3 would have proved more valuable than going directly to number 4.

Future Study
When conducting this lesson again, we would

- not use 1cm grids
- allow more time on question 3
- have two more questions similar to question 3
- write the correct method that students verbalise for subtracting co-ordinates on the board and get students to emphasise this method of solving we believe this method will help students in Co-ordinate Geometry to understand formulae used for slope.

Benefits of participating in Lesson Study

We felt students have a deeper knowledge of their learning as they have reached this themselves through exploration. The ‘Ceardaiocht’ helps them to further push this learning experience. It opened our eyes to the way students should be learning through problem solving techniques. It gives us a deeper understanding of peer to peer teaching more so than teaching led teaching. We feel we could role this out within our own Maths Department. The tools and insights gained will help us develop as Maths teachers.