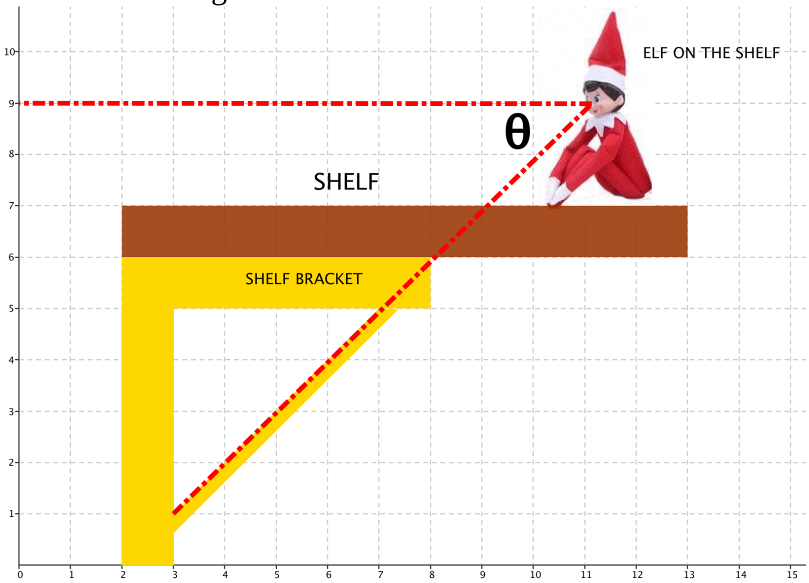
Lesson Research Proposal for 3rd Year: Trigonometry (Higher Level)

For the lesson on 23/01/2018
At Cashel Community School, Ritchie Ryan's Higher Level Junior Cycle class
Instructor: Ritchie Ryan
Lesson plan developed by: Gerard Brennan, Ritchie Ryan and Harry Langley

1. **Title of the Lesson:** Elf on the shelf : How many different methods can you use to find the angle theta in the diagram below?



2. **Brief description of the lesson**

The student must be able to use their knowledge of trigonometry, synthetic geometry and their problem solving skill to find the angle of depression from a datum point.

3. **Research Theme**

(a) Subject Knowledge: It is expected that all teachers will have a deep understanding of all subject material that they are responsible for teaching where this lacking, it is the teacher's responsibility to acquire the requisite level of understanding through personal research (eg. Textbooks, websites and research papers) and through collaboration with colleagues in the maths department (e.g through Lesson Study).

(b) Pedagogical Knowledge: As a maths department we recognize that subject knowledge is not sufficient for effective teaching and learning, but that teachers require up to date knowledge of developments in maths pedagogy. To this end, and through participation in lesson study, teachers will work together to research and choose teaching and learning strategies which are most productive.

(c) Classroom Management: It is agreed that in the coming academic year we will use structured problem solving to introduce new concepts to students. We aim that a minimum of 1/10 maths lessons will follow a structured problem solving approach over the next two school years, with this proportion rising to 1/5 thereafter. The scaled nature of introducing structured problem solving recognizes the difficulty of adopting such an approach and the need for teachers to develop their skills.

4. Background & Rationale

a) Why you chose the topic
This lesson is aimed at Third Year Higher Level Junior Cycle students. The quality of answers to questions in this topic at Junior Cycle level varies greatly between students, once they understand the fundamental concepts of the topic. They struggle to apply their problem solving skills to questions that deviate from standard textbook exams.

b) Research findings
Students at Junior Cycle level are often phased by trigonometric problems which involve real world applications. When tasked with an applied mathematics problem, students have difficulty conceptualising the problem from within the curriculum specified pure mathematics teaching and learning.

5. Relationship of the Unit to the Syllabus

For mathematics, the Primary Curriculum and Post-Primary Syllabus can be accessed at:
Primary: http://www.ncca.ie/uploadedfiles/Curriculum/Maths_Curr.pdf
Junior Certificate: http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Syllabuses_and_Assessment/JC_Maths_English_2013.pdf
3 Handbooks.
Leaving Certificate: http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Syllabuses_and_Assessment/LC_Maths_English_2013.pdf

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Primary: Shape & Space **Infant Class:** students learn to identify different geometric shapes and patterns. Students name the and describe the shapes based on distinguishing features. **First & Second Class:** 2-D Shapes- the student must identify form features and symmetries associated with 2D planar constructs. **Third & Fourth Class:** 2-D Shapes: properties (sides, angles, parallel and non-parallel lines) of 2-D shapes • construct and draw 2-D shapes 2-D shapes. Symmetry- understand the concept and applications• recognise an angle in terms of a rotation form. **Fifth & Sixth Class:** 2-D	**Third Year**: students revise geometry one, geometry two, trigonometry, algebra one and algebra two. Relevant Concepts: Axioms 1. Two points axiom. 2. Ruler axiom. The properties of the distance between points 3. Protractor Axiom. The properties of the degree measure of an angle 5. Axiom of Parallels Theorems: 2. The properties of the isosceles triangle3. The laws associated with parallel lines. 4. The angles in any triangle add to 180˚ 9. The laws regarding parallelograms. 10. The diagonals of a parallelogram bisect each other. 13. Two congruent triangles 14. Pythagoras theorem Corollaries: 1. A diagonal	**Fifth & Sixth Year** **2.1 Synthetic geometry** Students must gain proficient knowledge of: Theorem, proof, axiom, corollary, converse. Students must gain sufficient knowledge of proofs including proof by induction. **2.2 Co-ordinate geometry** Students should be able to– use slopes to show that two lines are • parallel • perpendicular – solve problems involving slopes of lines– recognise that $(x\text{-}h)^2 + (y\text{-}k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle with centre (h, k) and radius r . solve problems involving • perpendicular distance • the angle between two Students should be able to – use of the theorem of Pythagoras to solve problems (2D only) –solve problems

Shapes: 2-D shapes and their properties • use angle and line properties to classify geometric contructs. • construct triangles from given sides or angles. • classify 2-D shapes according to their lines **Post Primary: Strand 2: Geometry & Trigonometry** **First Year:** students learn the use of geometry equipment for measurement. **Second Year:** Introduction to Pythagoras' Theorem Slope of a line. Theory regarding planes, spheres and parrallelogrms.	divides a parallelogram into 2 congruent triangles. Constructions: 1. Bisector of a given angle. 5. Line parallel to a given line, through a given point 9. Angle of a given number of degrees. 10. Triangle, given lengths of three sides 11. Triangle, given SAS data 12. Triangle, given ASA data 13. Right-angled triangle theory given two or more dimensions.	using the sine and cosine rules (2D) – define sin θ and cos θ for all values of θ – define tan θ – work with trigonometric ratios in surd form – use trigonometry to solve problems in 3D– use the radian measure of angles –

6. Goals of the Unit

(a) Students will understand that problems in trigonometry can be solved using a variety of alternative methods.
(b) Students will be utilizing the skills that they already know about right angled triangles.
(c) Students will learn through groupwork, peer interaction, assessment and reflective practice.
(d) Cross curricular links with Technical Graphics at Junior Cycle. This will form preparation for Mathematics, Physics, Design & Communication Graphics, Construction Studies and Engineering at Senior Cycle.

7. The Unit

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Lesson	Learning goal(s) and tasks
1	An overview of triangle types and their properties using graphical techniques with the use of the student's geometry kits. Diagrams which are not to scale should be revised here.
2	Focus only on right angled triangles. Recap of Pythagoras Theorem with the use of the formula: $c^2=a^2+b^2$. Introduce trigonometric ratios.
3	Review of ratios and labelling of appropriate sides. Selection of the appropriate angle.
4	Finding the lengths of sides using the ratio with a given angle.
5	Finding the measure of an angle using the ratio with two given sides.
6-7	Introduce problem solving for a variety of scenarios and application using lesson four and five.

8	Review of synthetic geometry from first year Common Introductory Course.
9 The Research Lesson	The application of problem solving skills to a Trigonometric problem involving determination of an angle on a 2D co-ordinated plane at Junior Cycle, Higher Level.

8. Goals of the Research Lesson:

a) Mathematical Goals

- Students will know that there are various solutions to a single problem.
- Students will understand the characteristics of an isosceles triangle.
- Students will recognize that the trigonometric tan ratio can be used to solve this problem.
- Students will gain more confidence in self exploration in solving mathematical problems.

b) Key Skills and Statements of Learning

In the planning and design of this lesson the Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will implement and promote JC Key Skills in the following ways:

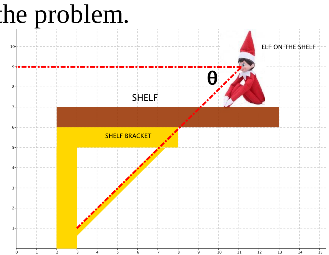
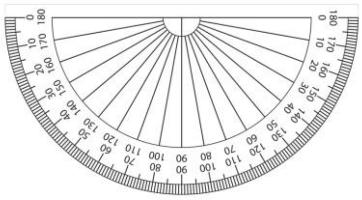
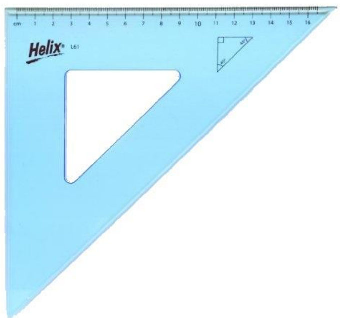
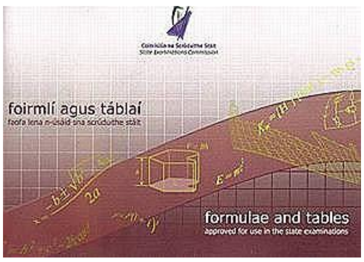
- Being Literate: Students will have the opportunity to express their ideas clearly and accurately.
- Being Numerate: Seeing patterns, trends and relationships.
- Staying Well: Student's confidence and positive disposition to learning will be promoted.
- Communicating: Students will present and discuss their mathematical thinking.
- Being Creative: Students will explore options and alternatives as they actively participate in the construction of knowledge.
- Working with Others: Students will learn with and from each other.
- Managing information and thinking: Being curious, and thinking creatively and critically. Students will be encouraged to think creatively and critically.

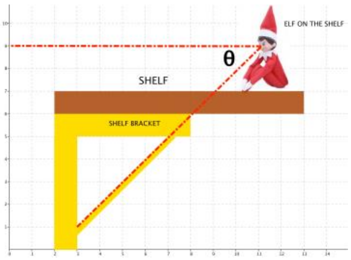
This lesson is also designed to meet the following JC Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The students describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Start (5 min.) Overview and assessment of prior learning on the topic of geometry and trigonometry. The teacher displays the first geometric problem on an auxiliary board in the classroom.	A diagram of a triangle is displayed on the pin-board. The teacher assesses the students prior learning to date on the subject of trigonometric and geometric mathematical theory.	During this lesson component (a) the level of understanding of the students with regard to junior cycle triangles and parallelograms. (b) the observer will note the students have acquired a high level of competency in trigonometry and geometry.
Introduction		

This section may set up the main task, e.g. by providing a contextualized problem out of which a pure mathematical problem will arise. 		The students analyse the problem and establish the data supplied, and the data which must be produced.
Posing the Task (5 min.) The teacher displays the primary question on the whiteboard. The question involves a bracket with a cartoon elf, supported by the literacy type question. Clarifying the problem: The teacher explains the set of data which is supplied with the questions and the variables which must be calculated using the students problem solving abilities and prior knowledge. ♦ - students are encouraged to ask questions to enable them to establish and understand the limits associated with the problem. 	The question starts as a projected image, which leads to a hard copy as the students commence the problem solving activity. What type of angle is theta? What is the value of theta?   	
Student Individual Work **(10 min.)** The teacher distributes the A3 printout of the problem to each group (groups comprising of 2 -3 students).	The teacher will reiterate the concept behind the task.	The main task forms the basis of the plenary assessment.

The teacher circulates around the room to answer questions and give guidance on the task where necessary. The students write their rough work on the A3/A4 sheet. Students are allowed to use their formula tables and calculators (as per exam conditions). Students present their final solution, with a value for theta, and the working which formed the basis of their solution. Students response: R1. Protractor R2. Angles of isosceles triangles R3. Trigonometry R4. Theory of corresponding angles R5. Theory of alternate angles R6. Utilization of set squares R7. Ascertaining congruency by the method of completing the square. R8. Folding paper templates to verify congruency.	The teacher answers some questions by referring students to their prior understanding of trigonometry and geometry. The teacher anticipates the potential questions which the students may pose, and prepares sample answers which do not bypass the learning. <u>Any misconceptions are solved during the ceardaiocht (Discussion)..</u>  	
Ceardaíocht /Comparing and Discussing (15 min.) Students pin their solution to the board prior to the discussion. Individual students are invited to explain the basis of their solution, and how they came to their final conclusion. Effective questions to ask during Ceardaíocht include: “What do you think”? (ask another student(s) other than the presenter) “Why is that”? (Looking for evidence). “Did anyone else solve it the same way? Can you explain this method” It is important that the whole class is engaged and that students understand they may be called on at any stage to reflect on what another student said. When responding to students’ efforts, try not to adopt the opinion of a particular student, as this can funnel all students down a particular path to solving the problem.	The ideas of focus are how a student interprets a problem in the initial stages. The students then learn that a problem can have multiple solutions, and that the solution that they chose is generated from their initial interpretation and the features of the question which they focus on.	The students explain the rational behind their solution. The students and teacher discuss the advantages/disadvantages and merit of the other teams working on the same “Elf on the shelf” problem. Students are encouraged to be critical of the solutions put forward by their peers. Students are encouraged the defend the logic encompassed by their solution.
Summing up & Reflection (5 min.) The teacher summarizes the overall success of the learning objectives. The teacher distributes a self assessment questionnaire.	The teacher circulates around the room whilst reading and verbally questioning the thoughts of the students concerning he learning	

The questionnaire consists of six questions. The first five questions involve the students ranking their understanding from 1-5 with regard to trigonometry theory, including key words. The sixth question requires the student to produce another real-world problem with can be solved using trigonometry. The students rank their understanding from 1-5	content of the lesson.	

10. Board Plan

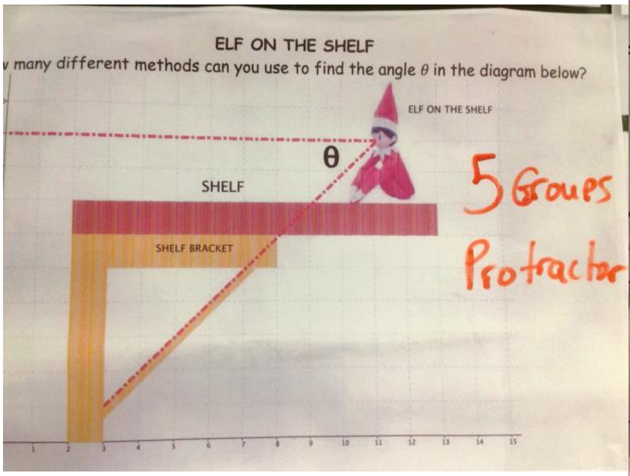
Board 1

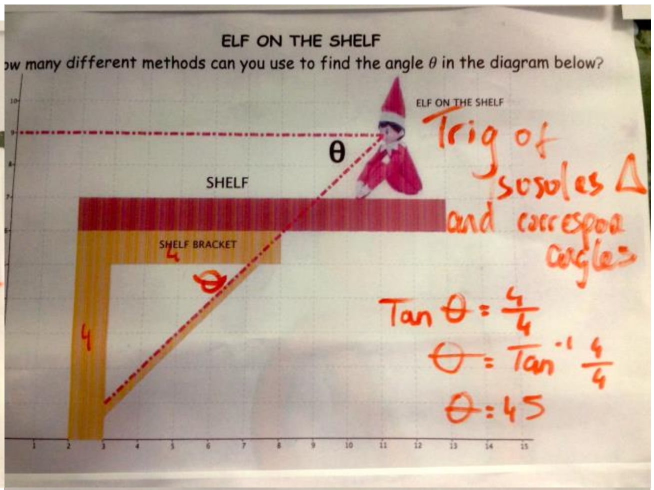
Information posters on synthetic geometry, trigonometric ratios, Pythagoras theorem.

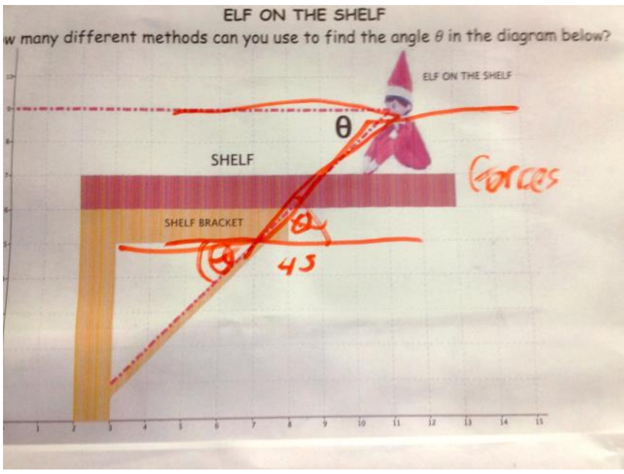
Board 2

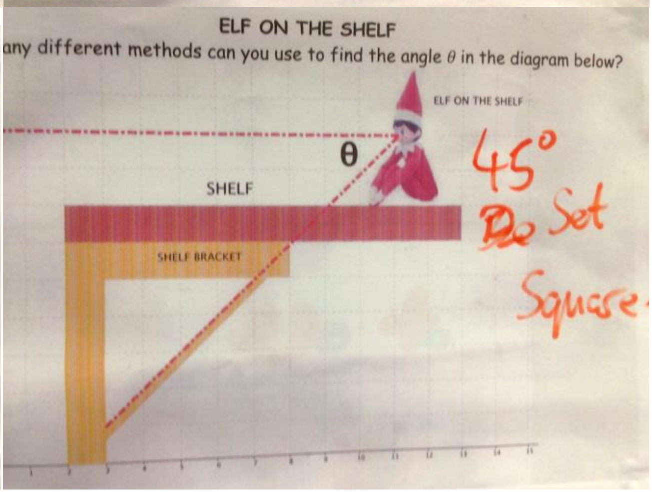
A3 Sheet size copy of the A4 handout given to each student group. Students would write on these during the Ceardaíocht,

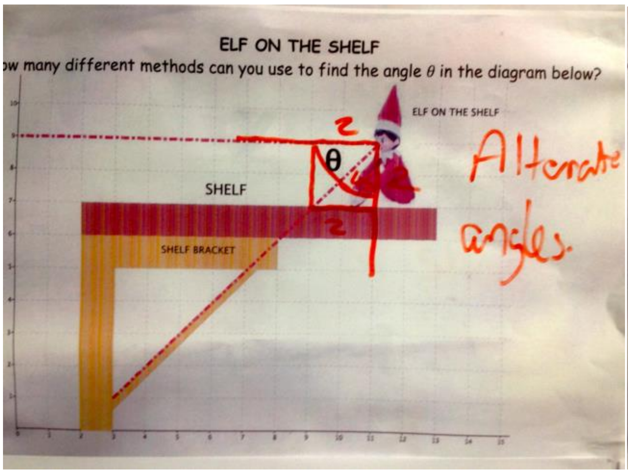
Some parts of the boardwork are shown below:

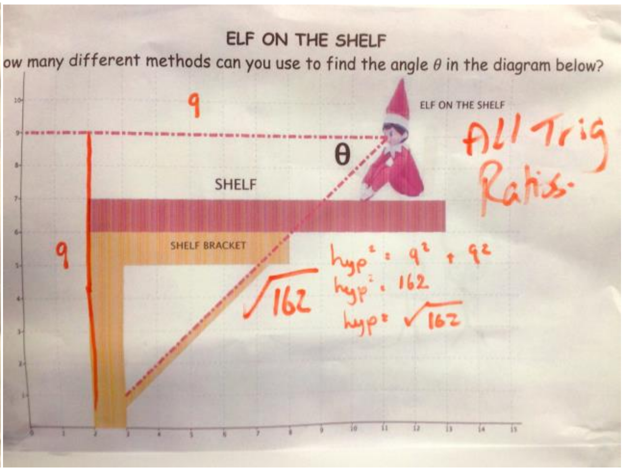












11. Evaluation

A student self-evaluation form was handed out at the end of the lesson. The students completed them in five minutes. The following is an overview of 9 student responses:

1. Name the method that you used to solve the Elf on the Shelf problem.

Protractor 9
All the trig ratios 1

2. List the other methods that you could have used to solve the problem

Protractor 1
Trigonometry tan 8 sin 7 cos 6
Corresponding 7
45 Degrees Set Square 7
Alternate Angles 5
Right angled isosceles triangle 3
Vertically Opposite 3
Pythagoras 2

3. What advantage did the method your team selected have over the other possible solutions?

"Quick and easy"X2, "It was the fastest method", "It was quicker"X4
"They were quick and easy and some didn't require a formula"
"It was quick to use, we used a protractor"

4. If the shelf bracket on which the Elf was placed was not to scale and you could not measure directly off the drawing in the question, which method could you use to find the angle?

Protractor 2
45 Degrees Set Square 2
Corresponding 3
Trig Ratios 5

5. Do you feel confident that you would be able to answer a Trigonometry question in your mock exam? Explain.

Yes 9
Reasons: "Because I find it easier to do other than other parts and we have done a lot in class to do with it"
"It has been covered well and I find it easy to understand"
"If you have all the numbers you need to solve the question you have the answer"
"I know I am able to complete more than 5 ways"
"There is more than one way of finding an angle"
"There is usually more than one way to answer it"
"I find it easy to use cos, sin, tan"

"I believe I understand trigonometry"
"There's loads of different ways of solving so if you're stuck on one, you have other options to use to solve"

12. Reflection

- Were the goals and research theme of the lesson met?
  - Yes, all students in the class realise there are multiple solutions to this problem. Students are well aware of properties of an isosceles triangle, as it was used in multiple solutions. Students are aware that Tan can be used to solve this angle of depression. Students appear to be a lot more confident in suggesting solutions to a problem.
- What methods did the students use to solve the problem? Initially Trig but 1st: Protractor method, 2nd: Use of Tan (Trig of Isosceles triangle) and corresponding angles, 3rd: Alternate angle method using vertically opposite angles, 4th: 45 degree set square method, 5th method: Assuming theta is 45, built a square and used diagonals of a square to bisect right angles, hence 45 degrees, 6th: use of Sin, 7th: Synthetic geometry: Realised a right angle in the corner and it was isosceles so with the 90, the other 90 degrees must be split in two evenly due to isosceles, hence 45 degrees.
- What comments or questions did the students have?
  - That they could also use Sin as a Trig function to calculate the angle too. The 7th solution. Use inverse Tan was suggested. 'I'm getting 45 using his protractor' was said.
- What were the common misconceptions and misunderstandings?
  - Confusion of equilateral, isosceles, scalene triangles – group labeled sides wrong while not thinking about the properties of an isosceles triangle. Some stand off-ish groups at the start – not communicating well with each other. Length of hyp = 9 (didn't understand to calculate the hyp), some unsure of how to calculate Sin/Cos/Tan, students did not see the extra theta on the 2nd slide easily which is a recurring problem in exams for students, miscounted the squares on the diagram. When the teacher said 'this diagram is to scale', we are unsure if all students understood this. Maybe prior learning of subjects may have been an issue.
- How and when did students' understanding change?
  - When the corresponding angle was highlighted, a lot of students realised the corresponding angle method. Students learned about the use of 45 degree set square, and realised the use of a protractor was very meaningful. The use of the little picture of the elf was very effective because it didn't look like an exam question. Previous knowledge and last few lessons led into this lesson very smoothly.
- Did the students' presentation and discussion promote their thinking and learning?
  - Yes, students' discussion helped the understanding of the lesson to a new level. Students' presentation helped learned using the protractor and 45 degree set square method. The text book approach to deriving a solution was found to be not always the suitable approach.
- Was the flow of the lesson coherent?
  - Lesson introduced very clearly. Students given wait time to assess their own understanding of the question posed. Towards the end of the Ceardaíocht, some students appeared to tire from turning around to watch the board and listen to the Ceardaíocht, and may have lost focus. We would look to change this if we were to re-teach this lesson. Also, once one of the 6 groups had done their presentation for the class, they kind of relaxed a little. We would look to change this if possible.
- Did the students display a positive disposition?
  - All students were totally engaged with plenty of evidence of learning through questioning, answering and written work. Some students were extremely comfortable with suggesting

possible solutions which spurred on others to try their solutions. Great energy and interest shown by all.

- Did the activities support the goals?
  - A lot of groups got 2 methods, one got 6 methods. A 7th method was realised by a student during the Ceardaíocht,

13. Key skills

Being Literate: Using terminology in relation to trigonometry and synthetic geometry during the lesson.
Being Numerate: Use of formulae and calculators during the lesson.
Staying Well: Students felt safe in their class environment and displayed enthusiasm for the lesson.
Communicating: Students took a while to start discussion when they got into their groups at first but then settled in at a later stage when the second angle was revealed. They pointed out information from their diagrams and formulae. Students who went to Board 2 were very clear in their explanations and display of angles involved.
Being Creative: Creativity was shown by most groups during the lesson but one student came up with a seventh method during the Ceardaíocht,
Working with Others: All students showed cooperation with others in the group. It was clear groupwork was practiced before.
Managing information and thinking: Some students checked the information on board 1. Others checked their notes in their copies. They drew diagrams and labeled them sharing this with their neighbours when complete.

14. Possible changes:
Use of different measurements on the horizontal and vertical parts of the shelf brackets to provide more of a challenge to students over a longer class time.
A clearer grid in the colour photocopy of the handout would have helped in clarifying the problem to students.

Appendix 1

Quality Framework for Post-Primary Schools – Teaching & Learning

Learner outcomes	Students enjoy their learning, are motivated to learn, and expect to achieve as learners	☐
	Students have the necessary knowledge and skills to understand themselves and their relationships	☐
	Students demonstrate the knowledge, skills and understanding required by the post-primary curriculum	☐
	Students attain the stated learning outcome for each subject, course and programme	☐
Learner experiences	Students engage purposefully in meaningful learning activities	☐
	Students grow as learners through respectful interactions and experiences that are challenging and supportive	☐
	Students reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning	☐
	Students experience opportunities to develop the skills and attitudes necessary for lifelong learning	☐
Teachers' individual practice	The teacher has the requisite subject knowledge, pedagogical knowledge and classroom management skills	☐
	The teacher selects and uses planning, preparation and assessment practices that progress students' learning	☐
	The teacher selects and uses teaching approaches appropriate to the learning intention and the students' learning needs	☐
	The teacher responds to individual learning needs and differentiates teaching and learning activities as necessary	☐
Teachers' collective / collaborative practice	Teachers value and engage in professional development and professional collaboration	☐
	Teachers work together to devise learning opportunities for students across and beyond the curriculum	☐
	Teachers collectively develop and implement consistent and dependable formative and summative assessment practices	☐
	Teachers contribute to building whole-staff capacity by sharing their expertise	☐

Junior Cycle Key Skills and Statements of Learning

Key Skills

KS1	Managing myself
KS2	Staying well
KS3	Monitoring information & thinking
KS4	Being numerate
KS5	Being creative
KS6	Working with others
KS7	Communicating
KS8	Being literate

Statements of Learning

	The student
SL1	communicates effectively using a variety of means in a range of contexts in L1*
SL2	listens, speaks, reads and writes in L2* and one other language at a level of proficiency that is appropriate to her or his ability
SL3	creates, appreciates and critically interprets a wide range of texts
SL4	creates and presents artistic works and appreciates the process and skills involved
SL5	has an awareness of personal values and an understanding of the process of moral decision making
SL6	appreciates and respects how diverse values, beliefs and traditions have contributed to the communities and culture in which she/he lives
SL7	values what it means to be an active citizen, with rights and responsibilities in local and wider contexts
SL8	values local, national and international heritage, understands the importance of the relationship between past and current events and the forces that drive change
SL9	understands the origins and impacts of social, economic, and environmental aspects of the world around her/him
SL10	has the awareness, knowledge, skills, values and motivation to live sustainably
SL11	takes action to safeguard and promote her/his wellbeing and that of others
SL12	is a confident and competent participant in physical activity and is motivated to be physically active
SL13	understands the importance of food and diet in making healthy lifestyle choices
SL14	makes informed financial decisions and develops good consumer skills
SL15	recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning
SL16	describes, illustrates, interprets, predicts and explains patterns and relationships
SL17	devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills
SL18	observes and evaluates empirical events and processes and draws valid deductions and conclusions
SL19	values the role and contribution of science and technology to society, and their personal, social and global importance

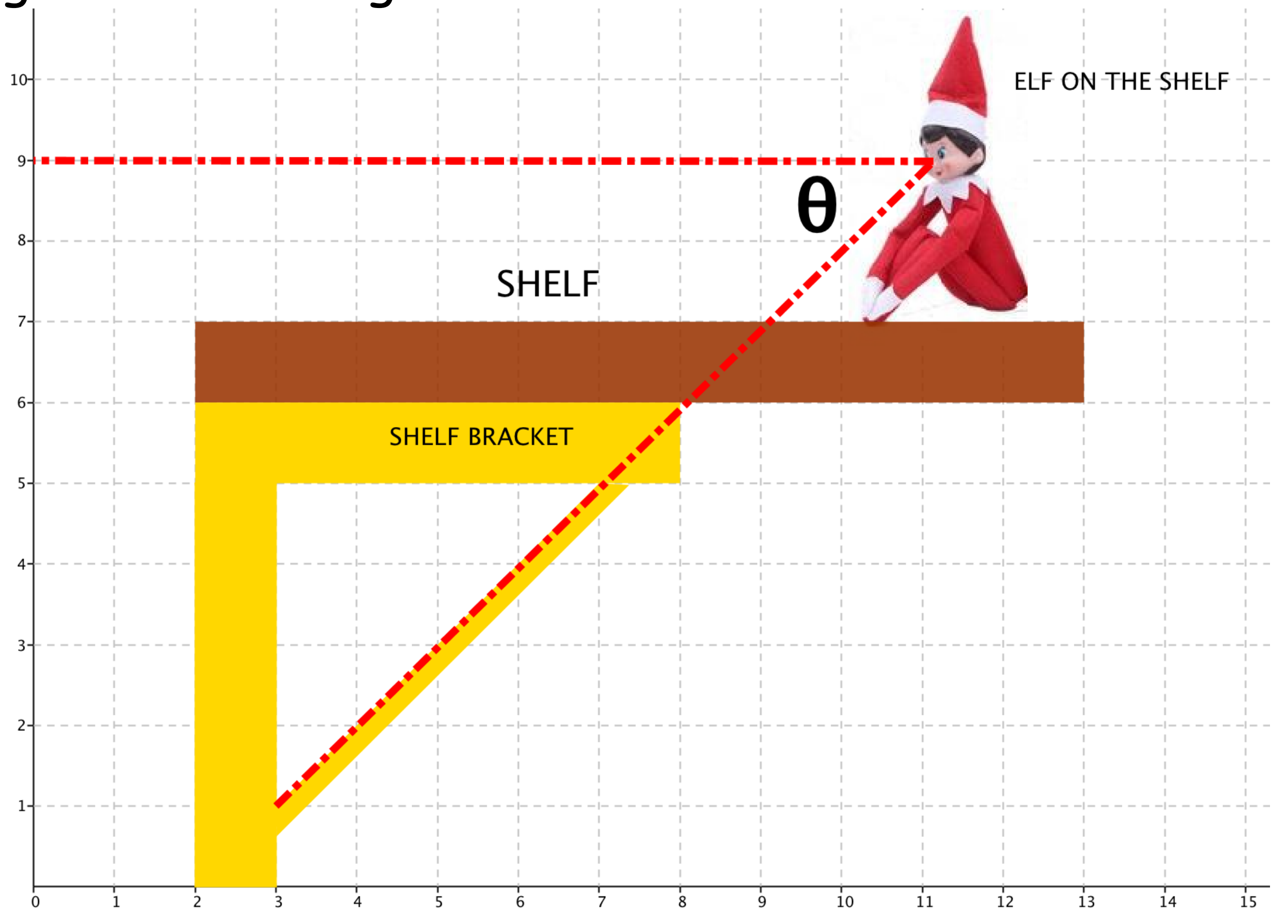
SL20	uses appropriate technologies in meeting a design challenge
SL21	applies practical skills as she/he develop models and products using a variety of materials and technologies
SL22	takes initiative, is innovative and develops entrepreneurial skills
SL23	brings an idea from conception to realisation
SL24	uses technology and digital media tools to learn, communicate, work and think collaboratively and creatively in a responsible and ethical manner

L1 is the language medium of the school (Irish in Irish-medium schools). L2 is the second language (English in Irish-medium schools).

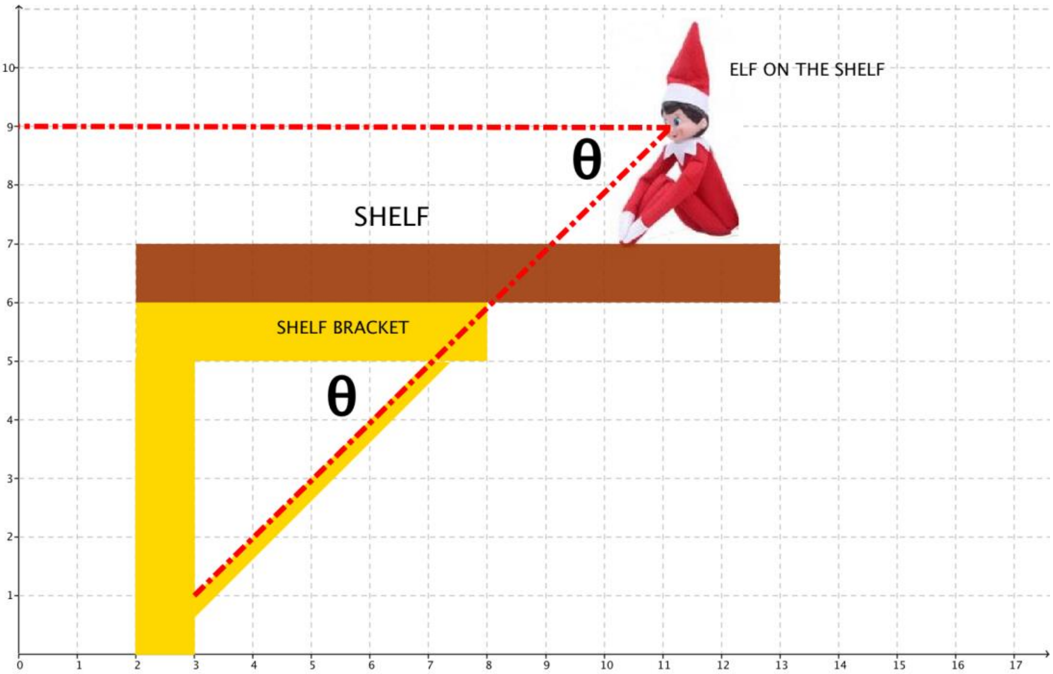
APPENDIX 2 STUDENT HANDOUT / SLIDE 1 ON POWERPOINT

ELF ON THE SHELF

How many different methods can you use to find the angle θ in the diagram below?



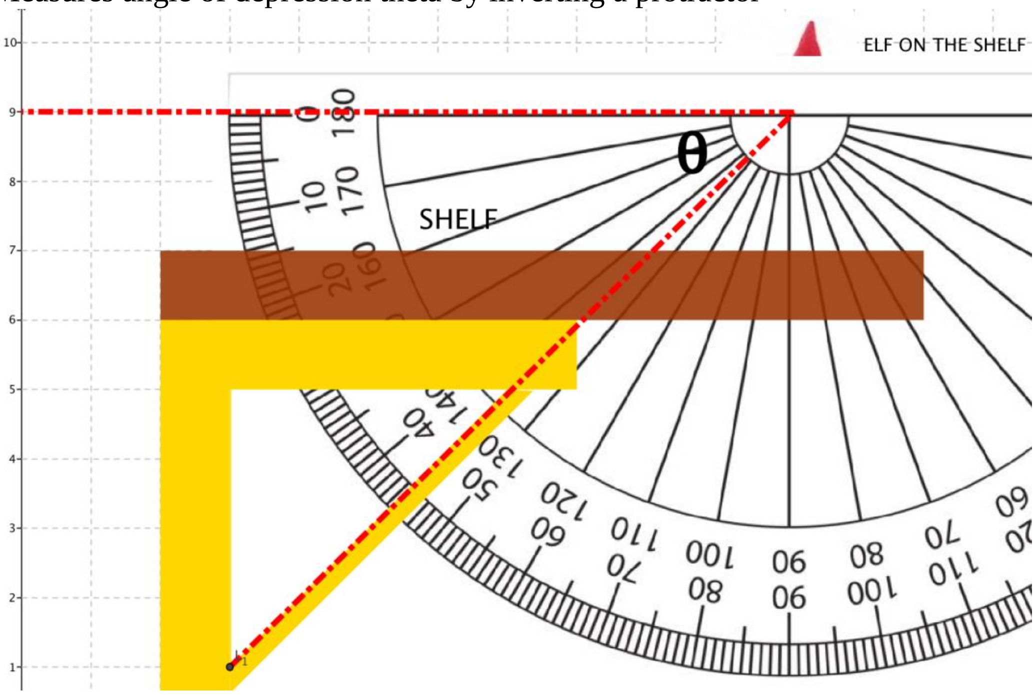
APPENDIX 3 SLIDE 2 POWERPOINT REVEAL OF CORRRESPONDING ANGLE



APPENDIX 4 PROTRACTOR METHOD

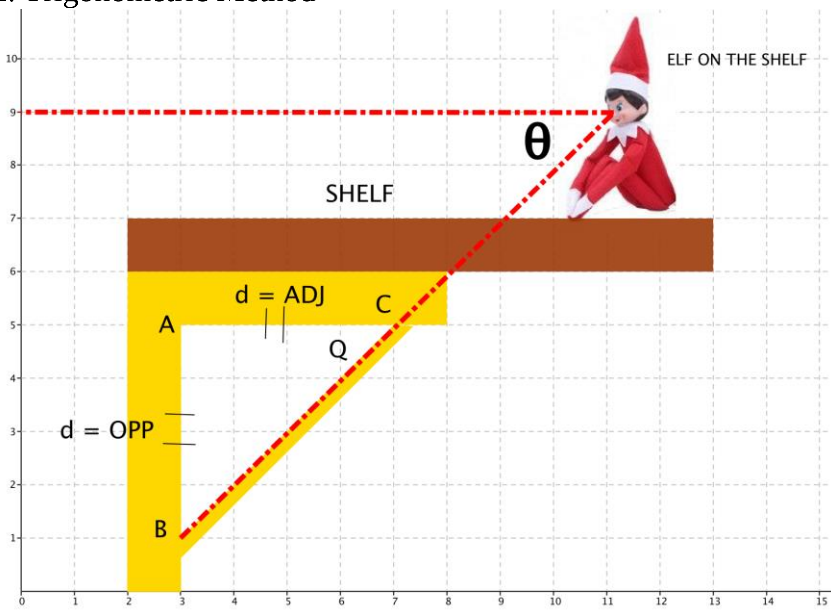
1. Protractor

Measures angle of depression theta by inverting a protractor



APPENDIX 4 TRIGONOMETRIC TAN METHOD

2. Trigonometric Method



$|<Q| = |<q|$ Corresponding angles (All horizontal lines parallel)

|AB| = |AC| = d = 4 units each in length

$$\tan Q = \frac{Opposite}{Adjacent}$$

$$\tan Q = \frac{d}{d}$$

$$\tan Q = \frac{4}{4} = 1$$

$$Q = \tan^{-1}(1)$$

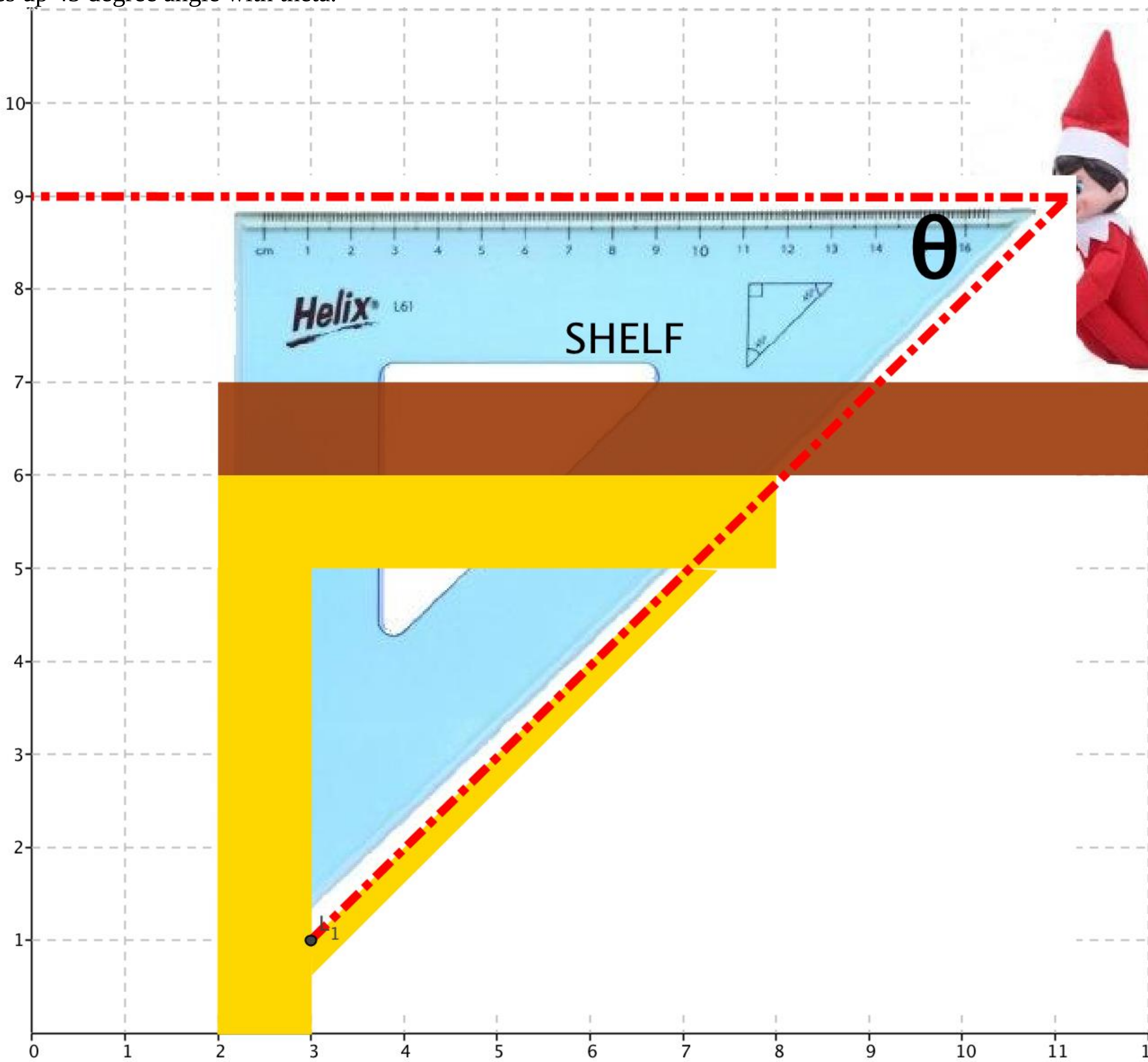
$$Q = 45^0$$

Þ $|<q| = 45^0$

APPENDIX 5 SET SQUARE METHOD

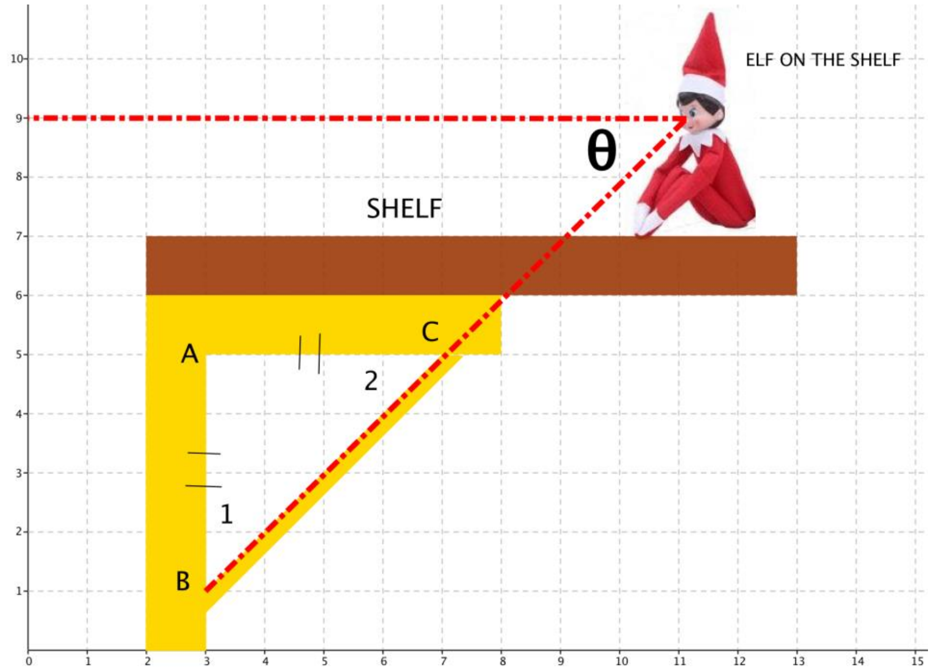
4. Set Square Method

Lines up 45 degree angle with theta.



APPENDIX 6 ISOSCELES RIGHT ANGLED TRIAGLE METHOD

7. Isosceles Triangle Method



$|< 2 | = |< q |$ Corresponding angles (All horizontal lines parallel)

|AB| = |AC| 4 units each in length

$\Rightarrow$ DABC is isosceles

$\Rightarrow$ |<1| = |<2| Theorem

|<1| + |<2| + 90^0 = 180^0 Theorem

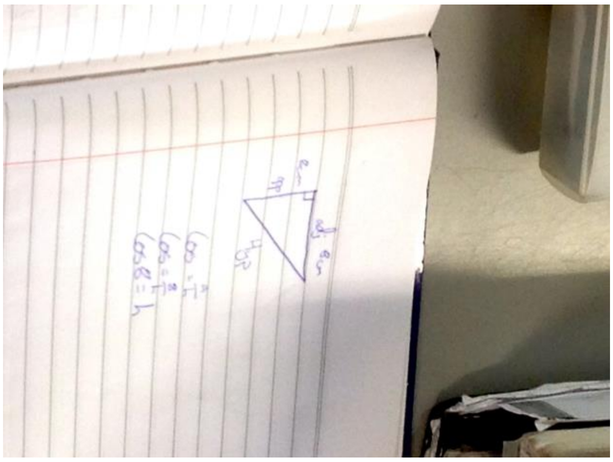
$\Rightarrow$ |<1| + |<2| = 90^0

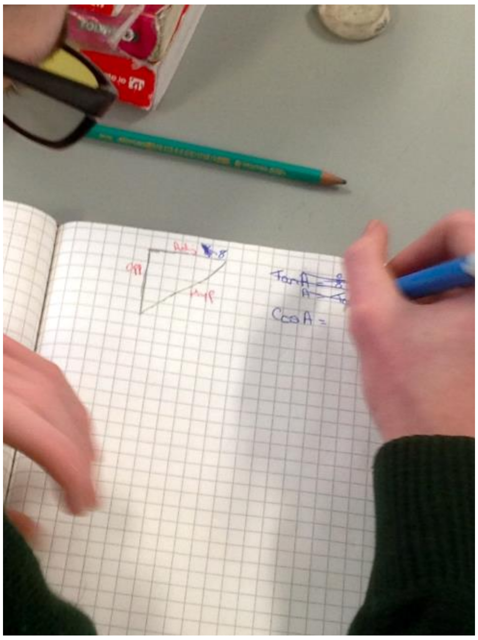
2|<2| = 90^0 |<1| = |<2|

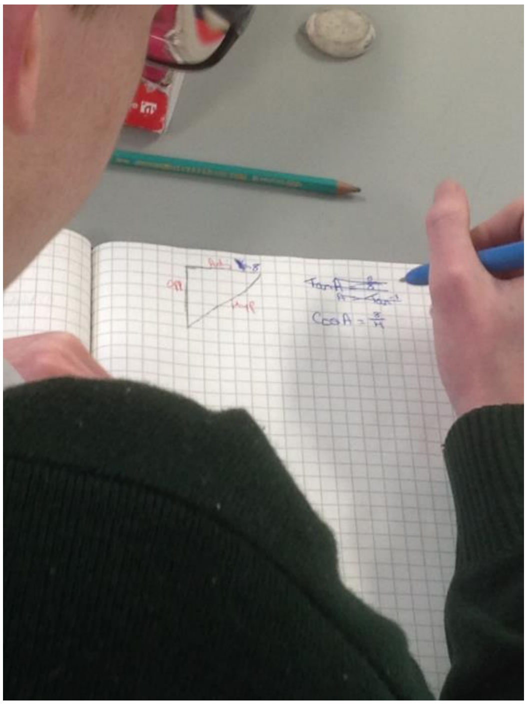
$\Rightarrow$ |<2| = 45^0

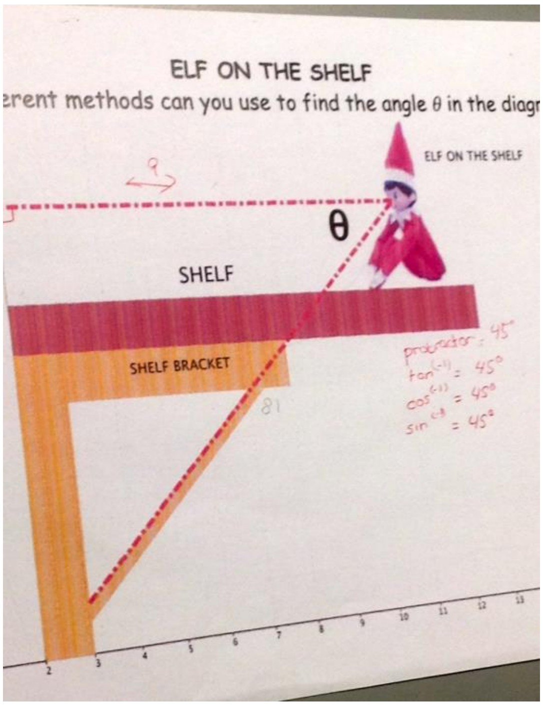
$\Rightarrow$ $|< q |$ = 45^0

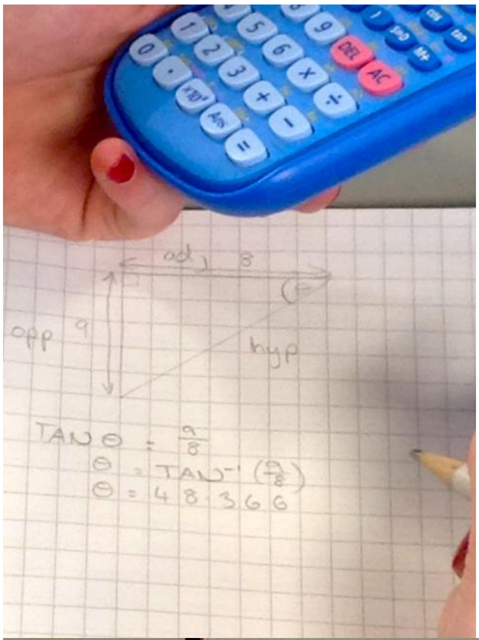
APPENDIX 6 OTHER PICTURES FROM LESSON

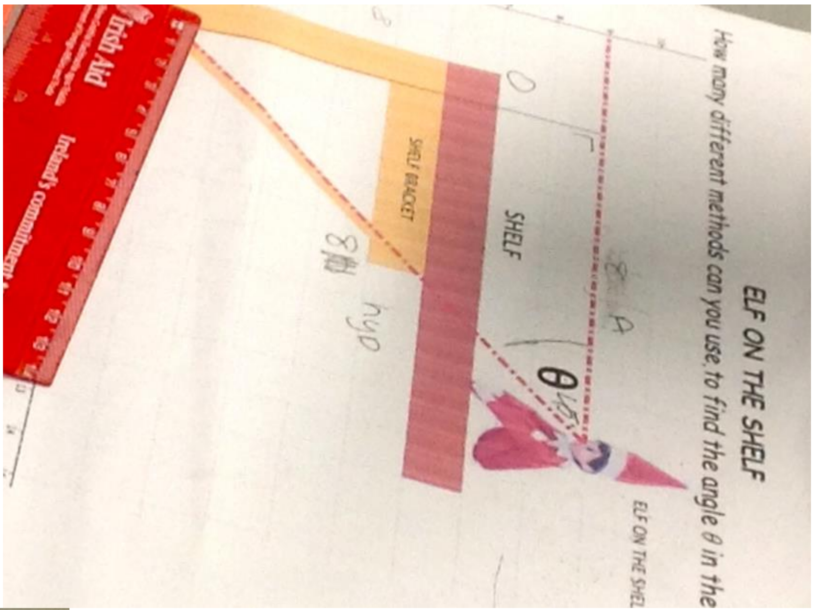


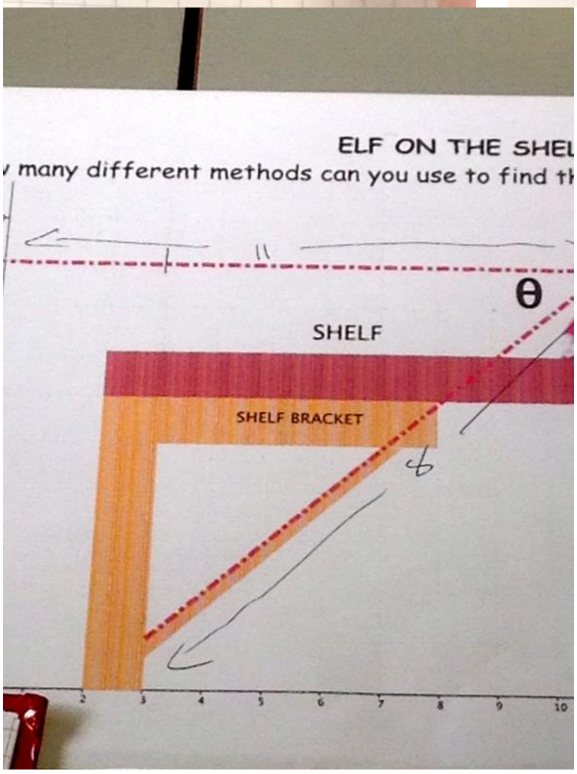




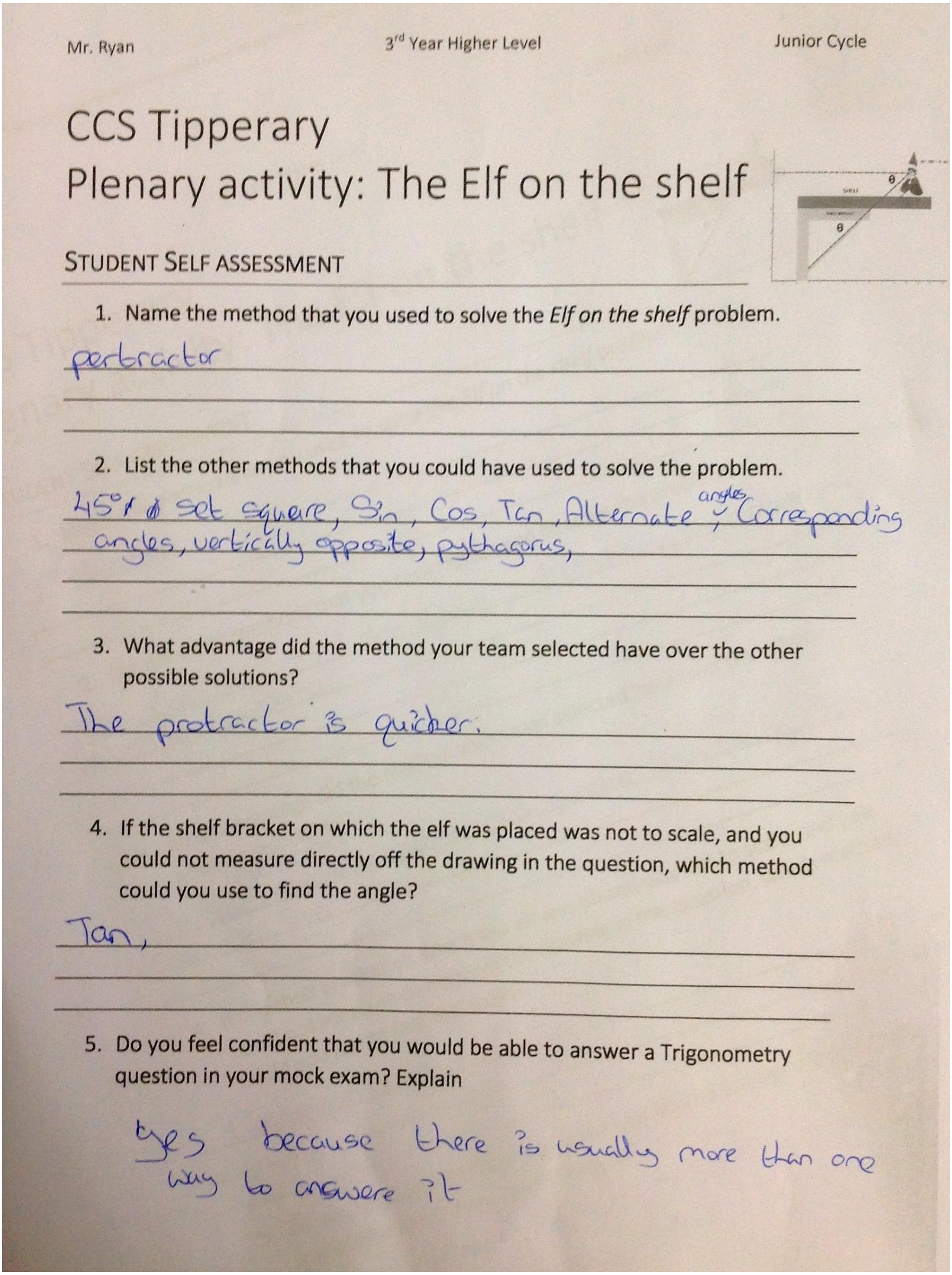



adj 8
opp 9
hyp
TAN θ = 9/8
θ = TAN⁻¹ (9/8)
θ = 48.366


ELF ON THE SHELF
SHELF
SHELF BRACKET
Irish Aid
hyp


SHELF
SHELF BRACKET
θ

APPENDIX 7 A SAMPLE STUDENT RESPONSE FORM



Mr. Ryan 3rd Year Higher Level Junior Cycle

CCS Tipperary

Plenary activity: The Elf on the shelf

STUDENT SELF ASSESSMENT

1. Name the method that you used to solve the *Elf on the shelf* problem.

pertractor

2. List the other methods that you could have used to solve the problem.

45° / set square, Sin, Cos, Tan, Alternate angles & Corresponding angles, vertically opposite, pythagorus,

3. What advantage did the method your team selected have over the other possible solutions?

The protractor is quicker.

4. If the shelf bracket on which the elf was placed was not to scale, and you could not measure directly off the drawing in the question, which method could you use to find the angle?

Tan,

5. Do you feel confident that you would be able to answer a Trigonometry question in your mock exam? Explain

yes because there is usually more than one way to answere it