

Thomond Community College

Lesson Research Proposal for 2nd Year Algebraic Fractions

For the lesson on 31/01/2018
At Thomond Community College
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1. Title of the Lesson: Fractions meet Algebra

2. Brief description of the lesson

In this lesson student will learn to apply their knowledge of the addition and subtraction of fractions and their knowledge of the solving of algebraic equations, to the solving of equations involving algebraic fractions.

3. Research Theme

As part of DEIS planning and School Self Evaluation, Thomond Community College is in the process of developing a range of formative assessment tools and techniques for use within the school. Among other things we at Thomond Community College would like to improve our ability to share and co-create success criteria with students so they can assess their own learning through self-assessment and peer assessment, and identify areas for improvement and strategies to achieve improvement. The lesson study process will allow us as teachers develop clear and suitable success criteria for students. By getting students to explain their point of view and their method of solving the problem at hand, we will get an insight into all students' ability and can assess them. This will allow for short term and long term formative assessment of students.

4. Background & Rationale

Students in Thomond CC complete a Numeracy Competency test at the start of first year. This has consistently shown that fractions are one of the areas that students have the greatest difficulty with coming out of primary school.

As part of the numeracy strategy and DEIS/SSE planning within the school we are committed to identifying areas of weakness and putting in places interventions to alleviate these issues. Fractions have been chosen as part of our Numeracy Self Evaluation as an area of focus within Junior Cycle. This is due to student's difficulties but also due to the link between fractions other areas of weakness like Percentages and Probability and its link to other areas of the junior and senior curriculum like Algebra. As part of the school commitment to Numeracy, there is a lot of resources put into 1st and 2nd year students within the school for example with station teaching and team teaching initiatives. However it is the feeling as a department that 2nd years higher level students would benefit from on a Lesson Study class to introduce Algebraic Fraction as the issues and misconceptions with fraction are not completely solved by the time students start working with Algebraic fractions in 2nd year. This leads to further issues in the teaching and learning of Algebra. This is noticed in other subjects also for example Chemistry, Engineering and Physics.

There is also a need to standardize teaching methods in the math's departments and across the school. Fractions are also one of the focuses of this as part of the Numeracy SSE. We feel that Lesson study

process will help us achieve this goal.

-Same problem but see the development difference between 1st years and 5th years

5. Relationship of the Unit to the Syllabus

| Related prior learning Outcomes | Learning outcomes for this unit | Related later learning outcomes |
|---|--|---|
| <p>As part of the Common Introductory courses students will have,</p> <ul style="list-style-type: none"> ➤ Revisited addition and subtraction in the contexts of rational numbers. ➤ investigated models to help think about the operations of addition, subtraction, multiplication and division of rational numbers ➤ use the equivalence of fractions, decimals and percentages to compare proportions. <p>As part of algebra in 2nd year students will already have</p> | <p>To be able to add and subtract simple algebraic expressions of forms such as:</p> <div style="background-color: #e0f0ff; padding: 5px; margin-bottom: 10px;"> $\bullet \frac{ax + b}{c} \pm \frac{dx + e}{f}$ <p>where $a, b, c, d, e, f \in \mathbf{Z}$</p> $\bullet \frac{ax + b}{c} \pm \dots \pm \frac{dx + e}{f}$ </div> <div style="background-color: #e0f0ff; padding: 5px; margin-bottom: 10px;"> $\bullet \frac{a}{bx + c} \pm \frac{p}{qx + r}$ <p>where $a, b, c, p, q, r \in \mathbf{Z}$</p> </div> <p>Student should also be able to solve equations of the form</p> <div style="background-color: #e0f0ff; padding: 5px; margin-bottom: 10px;"> $\frac{ax + b}{c} \pm \frac{dx + e}{f} = \frac{g}{h}$ <p>where $a, b, c, d, e, f, g, h \in \mathbf{Z}$</p> </div> <p>At Higher level they should also be able to solve first degree equations in one or two variables with coefficient elements of \mathbf{Q} and solutions also in \mathbf{Q} of the form</p> | <p>At Leaving cert level students at ordinary level must be able to add and subtract expressions of the form.</p> <div style="background-color: #ffe0e0; padding: 5px; margin-bottom: 10px;"> $\bullet \frac{a}{bx + c} \pm \frac{p}{qx + r}$ <p>where $a, b, c, p, q, r \in \mathbf{Z}$</p> </div> <p>In addition at Higher level student must be able perform addition and subtraction on rational algebraic expressions that included brackets and surds.</p> <p>Student at Ordinary level must be able find solutions to equations of the form.</p> <div style="background-color: #ffe0e0; padding: 5px; margin-bottom: 10px;"> $\bullet f(x) = g(x)$ <p>with $f(x) = \frac{a}{bx+c} \pm \frac{p}{qx+r}$;</p> $g(x) = \frac{e}{f}$ <p>where $a, b, c, e, f, p, q, r \in \mathbf{Z}$</p> </div> <p>In addition students at Higher level must be able to find solutions to equations of the form</p> |

6. Goals of the Unit

- Working with algebraic Expressions up to and including algebraic fractions
- Simplifying Algebraic expressions using factors
- Solving equations up to and including equations involving algebraic fractions.

- Recognizing and being comfortable using fractional equivalent of 1 as a tool in working with algebraic fraction.

7. Unit Plan

| Lesson | Learning goal(s) and tasks |
|----------------------|---|
| 1 | Lesson Study revision lesson on the addition and subtraction of Fractions. This lesson will be for revision but also to highlight the role of fractional equivalent of 1 as a tool in working with algebraic fractions. |
| Research Lesson 2 | Addition and Subtraction of Algebraic Fractions leading to the solving of equations involving Algebraic fractions. |
| 3 | Adding and subtracting Algebraic Fractions consolidation. |
| 4 | Simplifying algebraic fractions using factors. |
| 5 | Solving equations involving Algebraic fractions. |

8. Goals of the Research Lesson:

- Student will be able to add and subtract simple algebraic rational expressions with different integers as denominators.
- Students will be able to solve algebraic equations which involve algebraic rational expressions.
- Students will recognize relevance of 1 in different fractional form forms as a tool in working with Algebraic fractions

Key Skills and Statements of learning

In the planning and development of this lesson the Junior Cert Key Skills and Statements of Learning have been considered. This lesson will implement and promote the follow JC key Skills.

1. Being Literate: Student will be given an opportunity to express their ideas clearly and accurately
2. Being Numerate: It will develop a positive attitude toward problem solving.
3. Managing myself: Students will have an opportunity to reflect on their own learning.
4. Staying Well: This lesson will promote students confidence and positive disposition to learning.
5. Communicating: Student will present and explain their own work.
6. Being Creative: Student will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with Others: Student will be given an opportunity to work with other student in pair or groups.
8. Managing Information and thinking: Student will be encouraged to think creatively and critically.

This lesson is also designed to meet the following JC statements of Learning:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning.

16. The student describes, illustrates, interprets, predicts and explains patterns and relationships.
 17. The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge reasoning and skills

9. Flow of the Research Lesson:

| Steps, Learning Activities Teacher's Questions and Expected Student Reactions | Teacher Support | Assessment |
|---|---|---|
| <p>Introduction Revision of prior knowledge on addition of fractions</p> | <p>Prompting and discussing with students the addition of Fractions</p> | <p>Teacher questioning of students and student feedback.</p> |
| <p>Posing the Task Introduction of problem and posing of Task 1</p> <p>Mary went shopping with a certain amount of money. She spent half her money on shoes. Then she spent one seventh of her money on make-up. Mary spent 90Euro. She bought the rest of her money home.</p> <p>Task 1: Write the amount of money Mary spent as a mathematical sentence.</p> | <p>Ensure students understand the requirements of Task 1</p> | |
| <p>Student individual work on Task 1</p> <p>Possible students responses</p> <p>Cost of Shoes + Cost of Shirt= 90 $\frac{1}{2}$ Total money + $\frac{1}{7}$ Total money = 90 $\frac{1}{2} T + \frac{1}{7} T = 90$ $T/2 + T/7 = 90$ $(\frac{1}{2} + \frac{1}{7}) T = 90$</p> <ul style="list-style-type: none"> • Did students understand the task? • What solution was most common among students • How many students could only come up with projected Outcome 1 • What misconceptions did students have? • What question or comments did students have? • Could students see the equivalence of all 4 sentences? | <p>Answer questions as appropriate</p> | <p>Teacher observing of student work and individual questioning</p> |
| <p>Ceardaíocht /Comparing and Discussing Presentation of student work and discussion of outcomes of Task 1</p> | <p>Invite selected student to the board to discuss their solution Prompting and probing of students work.</p> <p>Encourage students to discuss and compare their solution to</p> | <p>Teacher observing of student work and individual questioning</p> |

| | | |
|---|---|--|
| | the others presented. | |
| <p>Introduction of problem and posing of Task 2</p> <p>Task 2: How much money did Mary many bring to town in total?</p> | | |
| <p>Student individual work on Task 2</p> <p>Expected students responses are that they will use one of the following methods to solve the problem.</p> <ul style="list-style-type: none"> Using only fractions $\frac{1}{2} T + \frac{1}{7} T = 90$ $\frac{T}{2} + \frac{T}{7} = 90$ $(\frac{1}{2} + \frac{1}{7}) T = 90$ <ul style="list-style-type: none"> Do students understand the task? What solution was most commonly used by students What misconceptions did students have? What question or comments did students have? Were students able to use multiple methods to find an answer? | <p>Answer questions as appropriate</p> | <p>Teacher observing of student work and individual questioning</p> |
| <p>Ceardaíocht /Comparing and Discussing</p> <p>Presentation of student work and discussion of outcomes of Task 2</p> | <p>Invite selected student to the board to discuss their solution</p> <p>Prompting and probing of students work.</p> <p>Encourage students to discuss and compare their solution to the others presented.</p> | <p>Teacher observing of student work and individual questioning</p> |
| <p>Summing up & Reflection</p> <p><u>Summing up</u> Review the steps for adding algebraic fractions</p> <p><u>Extension Task</u> Solve the following: $\frac{x+1}{2} + \frac{x+4}{4} = 6$</p> | <p>Teacher questions and probes students understanding of the addition and subtraction of algebraic fraction.</p> <p>Teacher poses extension task</p> | <p>Observation and questioning of students ability to apply the knowledge gained during the class to the extension class</p> |

10. Board Plan

Board 1

| | | |
|---|--|--------------------------------------|
| Revision addition and subtraction of Fractions | Student Response | Student Response |
| Mary went shopping with a certain amount of money. She spent half her money on shoes. Then she spent one seventh of her money on make-up. Mary spent 90Euro. She bought the rest of her money home. | $\frac{1}{2} \text{ Total money} + \frac{1}{7} \text{ Total money} = 90$ | $T/2 + T/7 = 90$ |
| Task 1: Write the amount of money Mary spent as a mathematical sentence. | Student Response | Student Response |
| Student Response | $\frac{1}{2} T + \frac{1}{7} T = 90$ | $(\frac{1}{2} + \frac{1}{7}) T = 90$ |
| Cost of Shoes + Cost of Shirt = 90 | | |

Board 2

| | | |
|--|--------------------------------------|--|
| Task 2: How much money did many bring to town in total? | Student Response using | Student Response using |
| $\frac{1}{2} T + \frac{1}{7} T = 90$ | $\frac{1}{2} T + \frac{1}{7} T = 90$ | $(\frac{1}{2} + \frac{1}{7}) T = 90$ |
| $T/2 + T/7 = 90$ | | |
| $(\frac{1}{2} + \frac{1}{7}) T = 90$ | | |
| Student Response using Fractions only | Student Response using | Student Response using Alternative methods |
| | $T/2 + T/7 = 90$ | |

Boards 3

Review and Discussion
Steps to adding and subtracting
Algebraic equations

Extension Activity
Solve the following:
$$\frac{x+1}{2} + \frac{x+4}{4} = 6$$

Student Responses

11. Evaluation

There will be 5 observers in the room during the lesson

- 2 observers will circulate the room recording student responses and gauging understanding
- 2 observers will stay at the back of the room observing the overall flow of the lesson and the interactions between the teacher and students
- 1 teacher will record the significant sections of the lesson on LessonNote using an Ipad. They will also take pictures of significant student work as directed by the other observers.

Observers will consider the following questions during the lesson

- Were students willing to discuss their outcomes and compare methods with other students?
- Did students' understanding develop during the course of the lesson?
- Could students use their new knowledge on the more difficult question?
- How much teacher exposition was required to clarify students' understanding?
- Was the flow of the lesson coherent?
- Were students positively engaged with the class?
- Were the goals of the lesson and research theme met?
- What do you feel needs to be changed with the lesson?

12. Reflection

- Students engaged positively with the lesson and were willing to discuss and present their work when asked. The student reflections on the task were positive and indicated that they had learnt significantly during the class.
- Lesson didn't follow the flow or structure we expected. Students came up with some unexpected solutions.
- Even though the lesson flow wasn't as expected, giving the students time and scope to develop their own ideas still led to a very positive lesson where the lesson objectives were generally met.
- One common student misconception $\frac{1}{2} + \frac{1}{7} = 90$. Although students were able to come up with the mathematical equation $\frac{1}{2}T + \frac{1}{7}T = 90$, the majority of them were not able to use this to solve for T. Most students relied on using only fractions for the problem.
- We felt that some students may have benefited from pair work or group work after the initial individual work.
- We felt the board planning benefitted the flow of the lesson and gave students a clear path through the lesson.
- Students used a/a to help deal with fractional algebra.
- The extension task showed that a majority of the students were able to manipulate algebraic fraction by finding a common denominator by the end of the lesson. This was one of the main goals of the lesson.
- One aspect of the lesson we felt could be improved was the presentation of the tasks. Task 2 should have been presented on grid paper to encourage student to attempt a graphical method of solving the problem.



Appendix 1: Student worksheet

Mary went to town shopping with a certain amount of money.
She spent half of her money on shoes.
Then she spent one seventh of her money on make-up.
Mary spent €90.
She brought the rest of her money home.



Task 1:

Write the amount of money Mary spent as a mathematical sentence.



Task 2:

In as many ways as possible

How much money did Mary bring to town in total?



Appendix 2: Teacher check sheets

Task 1: Mathematical sentence

| Expected Responses | Student | Notes |
|---|---------|-------|
| Cost of Shoes + Cost of Shirt= 90 | | |
| $\frac{1}{2}$ Total money+ $\frac{1}{7}$ Total money=90 | | |
| $\frac{1}{2} T + \frac{1}{7}T = 90$ | | |
| $T/2 + T/7 = 90$ | | |
| $(\frac{1}{2} + \frac{1}{7}) T = 90$ | | |
| | | |
| | | |
| <p>Misconceptions</p> | | |

Task 2 : Solving for T

| Expected Response | Student | Notes |
|--------------------------------------|---------|-------|
| $\frac{1}{2} T + \frac{1}{7} T = 90$ | | |
| $T/2 + T/7 = 90$ | | |
| $T/2 + T/7 = 90/1$ | | |
| $(\frac{1}{2} + \frac{1}{7}) T = 90$ | | |
| | | |
| | | |
| | | |
| Misconceptions | | |

Appendix 3

Observation: Task 1 (sheet 1)

| Questions | Notes | Students |
|---|-------|----------|
| Do students understand the task? | | |
| | | |
| What solution was most common among students | | |
| | | |
| How many students could only come up with Outcome 1 | | |
| | | |

Observation: Task 1 (sheet 2)

| Questions | Notes | Students |
|--|-------|----------|
| What misconceptions did students have ? | | |
| | | |
| What question or comments did students have? | | |
| | | |
| Could students see the equivalence of all 4 sentences? | | |
| | | |

Observation: Task 2(sheet 1)

| Questions | Notes | Students |
|--|-------|----------|
| Do students understand the task? | | |
| | | |
| What solution was most commonly used by students | | |
| | | |
| What misconceptions did students have? | | |
| | | |

Observation: Task 2(sheet 2)

| Questions | Notes | Students |
|---|-------|----------|
| What question or comments did students have? | | |
| | | |
| Were students able to use multiple methods to find a answer | | |
| | | |
| | | |
| | | |

Observation: Class discussion

| Questions | Notes | Students |
|--|-------|----------|
| Were student willing to discuss their outcomes and compare methods with other students. | | |
| | | |
| Could students use the their new knowledge on the more difficult question? | | |
| | | |
| How much teacher exposition was required to clarify students understanding. Grade 1-5 1 being extensive. 5 being not at all | | |
| | | |

Observation: General 1

| Questions | Notes | Students |
|---|-------|----------|
| Was the flow of the lesson coherent | | |
| | | |
| Were students positively engaged with the class | | |
| | | |
| Did students understanding develop during the course of the lesson? | | |
| | | |

Observation: General 2

| Questions | Notes | Students |
|--|-------|----------|
| Where the goals of the lesson and re-search theme met? | | |
| | | |
| What do you feel needs to be changed with the lesson? | | |
| | | |
| | | |
| | | |