Lesson Research Proposal: 2nd Year Quadratic patterns

For the lesson on 7/02/2018
At St Mary's Secondary School Ballina Co Mayo, Colm Leonard's class
Instructor: Colm Loenard
Lesson plan developed by: Patricia Gordon, Roisin Mc Manamy, Colm Leonard and Michael Walsh

1. Title of the Lesson: Going underground on a rollercoaster

2. Brief description of the lesson

This lesson incorporates students using given heights of a rollercoaster above ground to identify that the pattern of heights over time is not linear and is in fact quadratic in nature. Students will draw a table and plot the quadratic relationship.

3. Research Theme

At St Mary's Secondary School, we want students who:

- Enjoy their learning, are motivated to learn, and expect to achieve as learners
- And teachers who value and engage in professional development and professional collaboration

As mathematics teachers, we will actively support the achievement of these goals by paying attention to the following entry points in my every day classes:

- Ensure students are motivated to learn through having a clear sense of attainable and challenging learning outcomes
- Teachers use formal meeting time and planning to reflect on their work.

4. Background & Rationale

This lesson is aimed at second year Higher Level students. The teaching of quadratic relationships through patterns brings together previously learned materials and extends students understanding to a deeper level.

It is commonly recognized that the students learn the process of solving the quadratics but do not relate them to the real world. We tend to teach these topics in isolation, thus not allowing students to make a connection of what they are doing to real life examples. Both patterns and quadratic relationships are an integral part of the Junior and Senior Syllabi.

Through discussions we have found that teachers use a varied amount of methods to teach quadratic equations. Guide numbers, factorising, box method. Students rarely gain a deep understanding of quadratic relationships from methods of solutions.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
In **first through to fourth** class students learn to: ▪ *Explore, recognize and record patterns in numbers.* ▪ *Explore extend and describe sequences* ▪ *Make patterns on the hundred square* By fifth class, students are capable of identifying relationships and record verbal and simple rules for number patterns: ▪ *81, 27, 9, …..Dividing by 3* ▪ *1, 4, 9, 16, 25, 36,…..* **In sixth class**, students deduce and record rules for given number patterns ▪ *2, 6, 12, 20, 30, ….* Students also explore the concept of a variable in the context of simple patterns, tables and simple formulae and substitute values for variables ▪ *substitute values into formulae and into symbolic rules developed from number patterns.*	In second year, students are introduced to more formal algebra and learn to rewrite algebraic expressions in equivalent forms. They also revisit patterns and generate arithmetic expressions from repeating patterns. Students learn to: • *Represent situations with tables, diagrams and graphs* • *use tables, diagrams and graphs as tools for representing and analyzing linear and quadratic patterns and relations* • *find the underlying formula written in words from which the data are derived (linear relations)* • *find the underlying formula algebraically from which the data are derived (linear, quadratic relations)* Students will recognise features of a relationship and how these features appear in the different representations. They must recognise both	Later in second year and into third year students will • utilize their understanding and show that relations have features that can be represented in a variety of ways • distinguish those features that are especially useful to identify and point out how those features appear in different representation in tables, graphs, physical models, and formulas expressed in words, and algebraically • discuss rate of change and the *y*-intercept; consider how these relate to

In **first** year, students revisit patterns and learn the rules that govern them. Students learn to: • *use tables to represent a repeating-pattern situation* • *generalise and explain patterns and relationships in words and numbers* • *write arithmetic expressions for terms in a sequence* Students derive relations from some kind of context, familiar everyday situations or arrangements of tiles blocks peg boards, the use of tables, diagrams and graphs to make predictions of what comes next. They also use patterns to explain relationships in words and numbers i.e. finding the general term. They also write arithmetic expressions for particular terms in a sequence	constant rate of change (linear relationships) and non constant rate of change (quadratic relationships). They should also recognise that a distinguishing feature of quadratic relations is the way change varies These lessons will involve the students in investigating and understanding how to use tables, diagrams, graphs and formulae as tools for representing and analysing quadratic patterns and relations in order to recognise that a distinguishing feature of quadratic relations is the way change varies i.e. the rate of change of the rate of change is constant. Historically, quadratic expressions and patterns were taught in isolation, not allowing students to make real life connections. We hope by amalgamating the two topics, that students will develop a deeper understanding of quadratics and their role in every-day life.	the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula From that they will recognize problems involving direct proportion. And from that the concept of a function as a relationship of a set of inputs and a set of outputs. Students should also use tables and diagrams to analyze quadratic patterns and relations.

6. Goals of the Unit

1. Students will understand what a repeating pattern is, what characterizes linear patterns and the many ways in which a repeating pattern may be represented with a story, a table, a graph and a formula.
2. Students can apply this knowledge to examine non-linear patterns by examining the graph and then in the context of the story analyze first and second differences.
3. Students will know how to plot these types of patterns and relate work done on quadratic equations, factorizing and solving.
4. Student will be competent in describing quadratic patterns using a general formula.

5. This will allow students to apply this knowledge to analyze and interpret real life quadratic patterns.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Introduce a linear pattern using colored unit blocks and generate a table of values. Use a suitable problem to show a linear pattern e.g. money in piggy bank. Using resources from 'Introduction to patterns' and ‘Algebra through the lens of functions’ resources from the project math’s website.
2	Revise the SALT method of graphing on a Cartesian plane. Use the data from the previous lesson to graph the linear pattern. Analyze the graph pointing out key features e.g. rate of change, steepness, co-ordinates etc.
3	Student use the graph or otherwise to devise a formula for the linear pattern. With suitable support students should be able to derive the formula through making sense of the relationship. Verify this using the graph and inputting value.
4 Research Lesson	Students are given a real-life problem where they are required to work out the first and second difference in table form and make comments on the shape of expected graph. Student will be expected to use prior knowledge of graphs to plot this and answer questions relevant to it.
5	Students must devise a method to derive an algebraic formula for the n^{th} pattern. Students will apply their knowledge to other quadratic problems.
. . .	.Students will recognize that patterns may double, triple and that these exponential relationships will be represented graphically. A suitable problem will be given to reinforce the knowledge.

8. Goals of the Research Lesson:

Students will be able to:

- Understand that not all patterns are linear
- Understand the characteristics of a quadratic pattern
- Graph a quadratic relationship
- Apply algebraic knowledge to form an equation from roots seen in the graph.
- Make a connection between the second difference and the coefficient of x^2.

9. Flow of the Research Lesson:

In the planning and design of this lesson, the Junior Cycle Key skills and statements of learning have been considered. This lesson will implement and promote Junior Cert key skills in the following ways:

1. Being Literate: Through Ceardaiocht, students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: By engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.

3. Managing myself: Students will have the opportunity to reflect on their own learning and give verbal feedback on the lesson.
4. Staying well: By engaging in tasks which are appropriate to their abilities, students' confidence and positive disposition to learning will be promoted.
5. Communicating: During Ceardaiocht, students will present and discuss their mathematical thinking.
6. Being Creative: Students' will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with others: Students will learn with and from each other by discussing different approaches to solving the problem.
8. Managing information and thinking: Students will be encouraged to think creatively and critically.

This lesson is also designed to meet the following Junior Cert Statements of Learning:

1. Student communicates effectively using a variety of means in a range of contexts.

15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.

16. The student describes, illustrates, interprets, predicts and explains patterns and relationships.

17. The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

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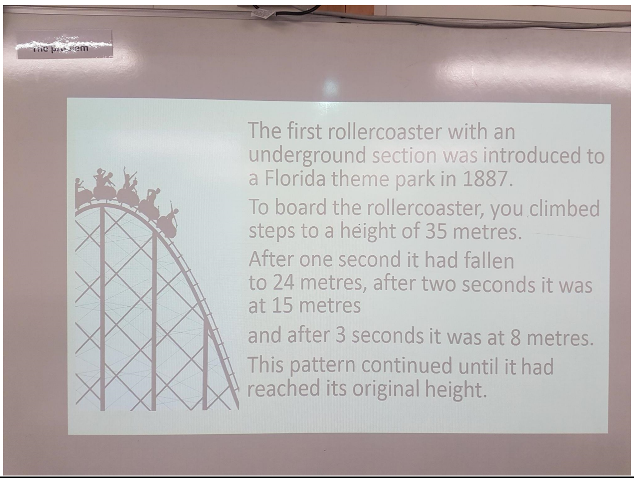
<tr><th>Steps, Learning Activities
Teacher’s Questions and Expected Student Reactions</th><th>Teacher Support</th><th>Assessment</th></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>Introduction
The teacher will introduce the class with the following problem

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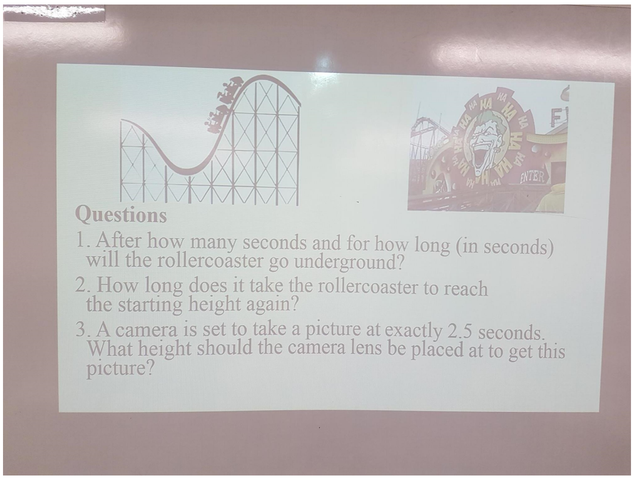
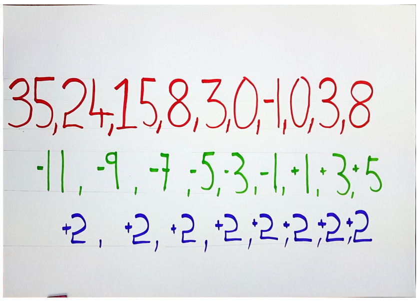
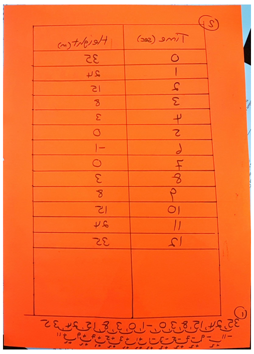
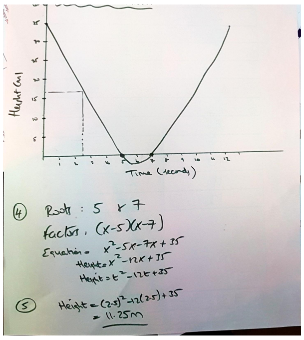
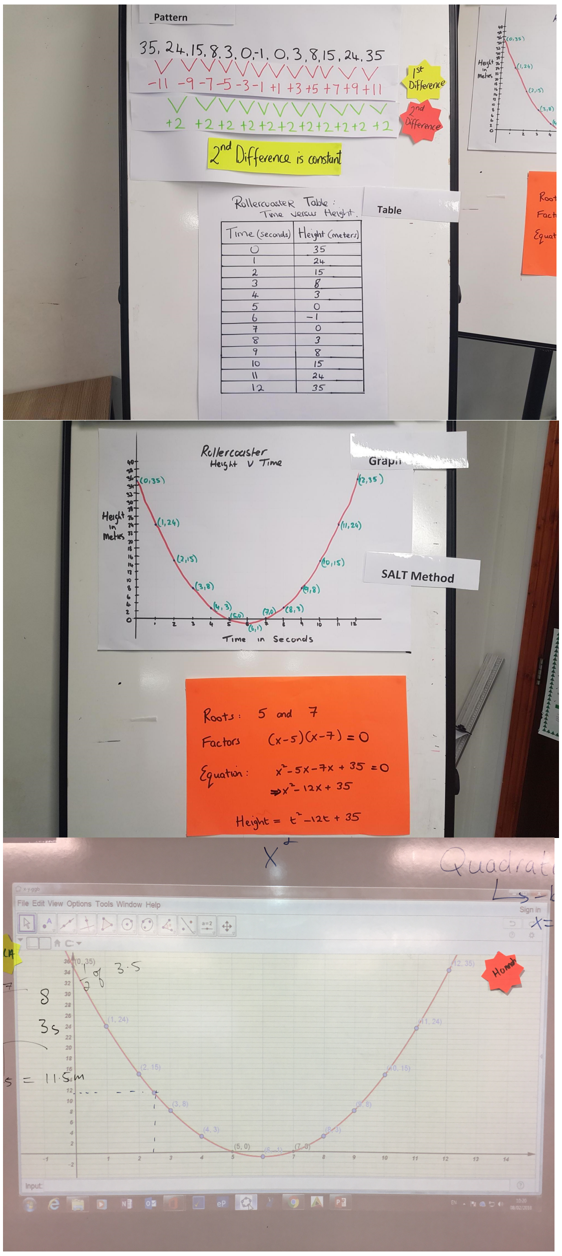
Posing the Task The following questions will be projected onto the board  3 Main Questions 1. After how many seconds will the rollercoaster be at its start height again? 2. After how many seconds will the rollercoaster be underground? 3. A Camera is due to take a picture after 2.5 seconds. What height should the camera be placed in order to get this picture?	The problem will be projected onto the overhead and a hard copy handed to each student. Ensure students know the workings of a rollercoaster? Do you recognize that the roller coaster is descending? What are we trying to find out? Where does the rollercoaster start? I want you to work on this individually and after a few minutes we will discuss this in pairs.	Do students understand the task? Do students understand that this rollercoaster will be falling and then rising? Are students aware that when the rollercoaster is underground, height will be negative? Are students eager to solve the problem?
Student Individual Work Students will guess random solutions. Students will list the heights and continue the pattern. Some will draw a table with time and height and continue the pattern. Students will draw a graph illustrating the pattern. Student may ask or discuss the change of the change.	How did you reach that solution? Teacher will note the students who listed the correct pattern and note the order in which they will call each student up to the board to display or discuss their approach. If students are stuck, we will ask appropriate questions to enable progression of problem. Did you consider finding the first difference? Did you continue the pattern?	Are students able to tackle the problem? Do students understand that the pattern will be decreasing first, and that height will be negative underground? Do they understand that this involves patterns and graphs and the link to quadratics?

	Have you drawn a sketch of what you think might happen as a roller coaster falls and then rises? Have you plotted the information given in the question?	
Ceardaíocht /Comparing and Discussing The students possible solutions will be shared with the class in the following order   		

Summing up & Reflection		
Students will be asked to reflect on today's class. They will be asked to give either positive or negative feedback on the lesson. Students will be guided by questions such as Today I learnt…. Today I understood…. Today I misunderstood ….		

10. Board Plan



11. Evaluation

Questions for the lesson Observation
Are the students

1. Enjoying their learning
2. Motivated to learn?
3. Do they expect to achieve?

As teachers have we ensured students are motivated to learn through having a clear sense of attainable and challenging learning outcomes?

12. Reflection

We as a team had hoped to observe the students continue a pattern and find that it was not like other linear patterns they were used to. As a result, we had hoped that the students would draw a graph and become aware that the pattern was in fact not linear. During the lesson we noticed students continue the pattern until it reached 0. They seemed to stumble at this point.
The post lesson discussion led to the following points, the students were a little lacking in confidence and we believe this to be down to the fact that there was four teachers in total in the room and that students are not used to this level of attention.
A major point we discovered is that in St Mary's the teaching of math's follows a plan where students solve quadratic equations before they study patterns and relationships. We agreed this should be considered the next time teachers review the Mathematics plan for second years. This point was most obvious when students could not name the shape of graph nor relate the relationship to a quadratic equation which had been taught previously. In the future teachers in St Mary's are interested in continuing lesson study and applying it to their classes at all levels. The approach taken here led to some new and enjoyable experiences that will shape our teaching for years to come.