Heavenly Slopes

Research Lesson Proposal

Coordinate Geometry: Parallel and Perpendicular Slopes
5th Year Ordinary Level

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Brief Description of the Lesson

Students will engage in structured problem solving to develop their understanding of the "slope" of a line, and of the relationships between the slopes of parallel and perpendicular lines.

Research Theme

At Coláiste Éamann Rís, as a focus for our school improvement and School Self-Evaluation, we want students to:

- Enjoy their learning, be motivated to learn and expect to achieve as learners.
- Engage purposefully in meaningful learning activities

As mathematics teachers, we will actively support the achievement of these goals by using:

- Group work in the classroom.
- Developing rich learning tasks that are relevant to the students' lives.
- Supportive comments, constructive advice and positive reinforcement.
- A range of teaching techniques to promote student motivation and participation such as higher order questioning, discovery learning, use of visual media and by teaching mathematics through problem solving.

Background and Rationale

We chose this topic because we wanted students to have a deeper understanding of slope, and the relationship between their slopes parallel and slopes perpendicular lines. We would like our students to develop their mathematical confidence by giving them opportunities to: think about and deepen their mathematical conceptual understanding of topics; use their initiative in the development and consideration multiple approaches to problems; express, present and discuss their mathematical ideas in a receptive learning environment.

Through discussions of members of the Mathematics Department and with this Lesson Study Group we realise that that teaching of the topic "slopes of parallel and perpendicular lines" there is an imbalance towards procedure and language. It is rare that our students are given time to question, discuss and discover the meaning of slope and why it is that for example perpendicular slopes are "negative reciprocals" of each other.

Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes (Leaving Certificate Ordinary Level)
5th class students should be able to recognise, classify and describe angles and relate angles to shape and the environment Then in 6th class they can relate angles to shapes. Junior Certificate OL students learn about the parallel and perpendicular lines and the relationships between the slopes. They also work with parallel and perpendicular lines in constructions	Students should be able to: • use slopes to show that two lines are parallel or perpendicular • prove that two lines are parallel or perpendicular • solve problems involving slopes of lines	Strand 2: Geometry: Constructions of: a ray, parallel and perpendicular lines. Coordinate Geometry of the circle: Students will use their knowledge of parallel and perpendicular slopes to deal with tangents and to solve problems involving a line and a circle with centre (0, 0). Strand 5: Rate of change, Differential Calculus, increasing and decreasing functions.

Goals of the Unit

On completion of this unit students should be able to:

- Plot points on a coordinate plane.
- Calculate the distance, slope, midpoint and equation of a line using various formula.
- Understand the difference between parallel and perpendicular lines.
- Understand that parallel have slopes that are the same.
- Understand why it is that perpendicular slopes are "negative reciprocals" of each other, and that the product of two perpendicular slopes is -1.
- Calculate the point of intersection between two lines.
- Calculate the area of a triangle when one of the points is (0, 0) and to be able to perform transformations in order to use this formula.
- Graph lines using a pair of points or by using slope-intercept.

The Unit Plan

1 The Research Lesson	Slopes of parallel and perpendicular lines
2	Plotting points and finding the distance between two points.
3	Using formula to calculate the midpoint and slope of a line.
4	Calculating the equation of a line
5	Finding the equation of parallel and perpendicular lines
6	Graphing lines using the x and y axis
7	Finding the point of intersection between two lines
8	Calculating the area of a triangle which has (0,0) as one of its points
9	Performing transformations
10	Revision lesson and exam question practice

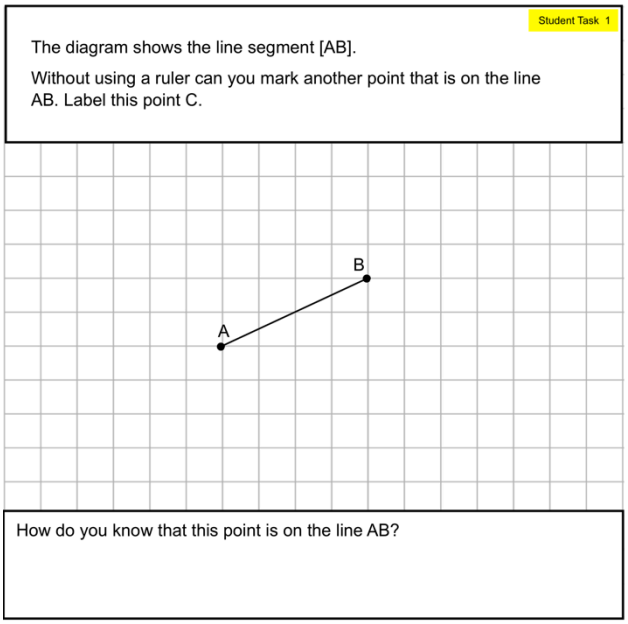
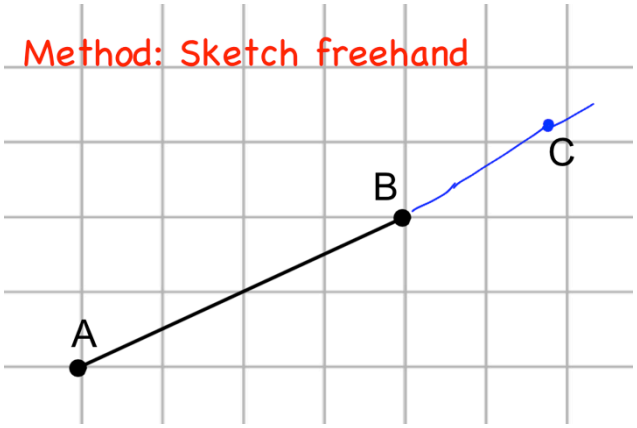
Goals of the Research Lesson

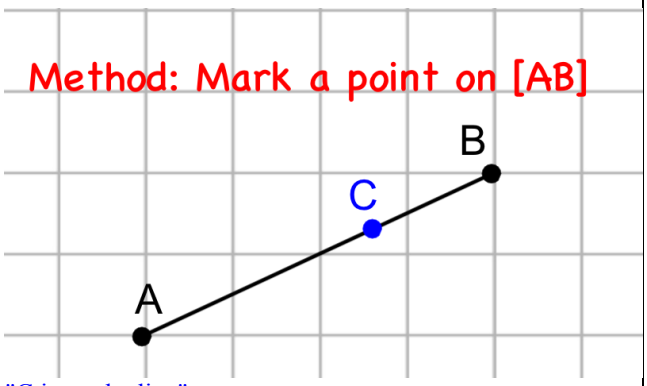
Having completed this lesson students will:

1. Have a conceptual understanding of slope.
2. Be able to find the slope of a line using different methods (slope formula, "rise over run", slope triangles) and understand the usefulness, efficiency and possible misconceptions associated with each.
3. Be able to recognise positive and negative slopes and understand that positive slopes "rise" or "increase in height" as "x increases" and *vise-versa* or describe slopes in terms of "positive" or "negative" in "x" and "y" in moving from one point to another along a straight line.

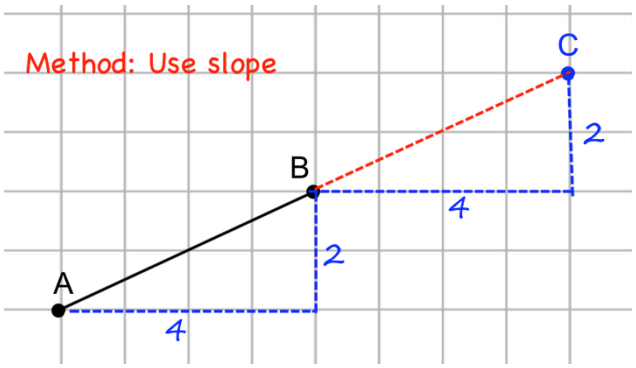
4. Be able to extend a ray from a point using its slope.
5. Understand that parallel lines have equal slopes and be able to explain and show why this is.
6. Understand that, and why, perpendicular lines have slopes that are negative reciprocals of each other.
7. Discover that the product of perpendicular slopes is -1.
8. Be able to construct parallel and perpendicular lines using different methods.

Flow of the Research Lesson

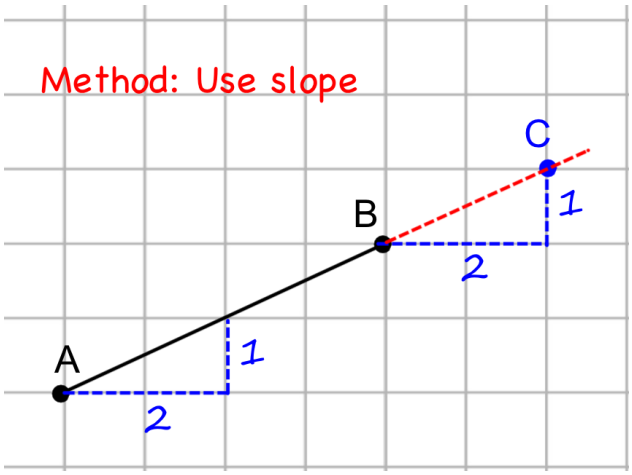
Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Roll call. Check students have materials for the first task ready along with: pens, pencil, ruler, maths set. Outline expectations for the class for students re individual work, presenting at the board, respectful participation...	(2 minutes) Materials ready on desks	Any questions?
Posing the Task 1  Teacher will check that students understand that a line segment is part of a line.	Clarifying the Problem: Students have the problem on a Worksheet (see Appendix) "What is the difference between a line segment and a line?" "In this question what is the difference between [AB] and AB?" (2 minutes)	Checking literacy. Students starting to explain their ideas. Are students engaged? Working to task? Understanding the problem?
Student Individual Work Anticipated student solutions:  "It looks about right."	Individual work (4 minutes) Presentation of work at the board by students. "Why did you pick the position you did for your point?" "Can we pick a 2nd point and label it point D" "Why did you pick a point on the gridlines"	Checking understanding of notation and literacy. Students starting to explain their ideas.



"C is on the line"



"AB goes up 1 square for every 2 squares to goes across."



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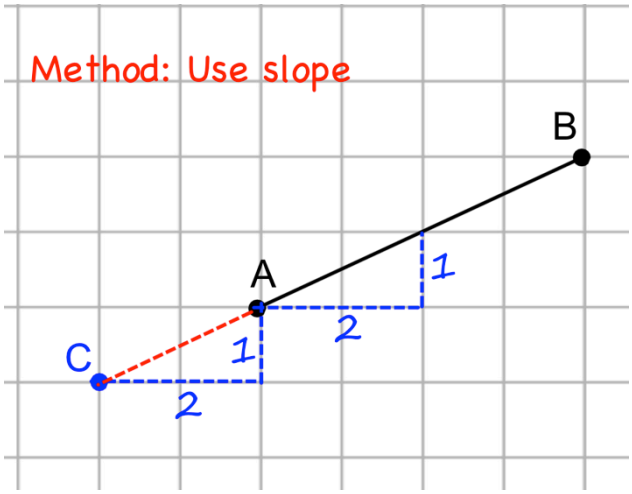

"AB goes down one square and left 2 squares."

(5 minutes)

Comment on the midpoint

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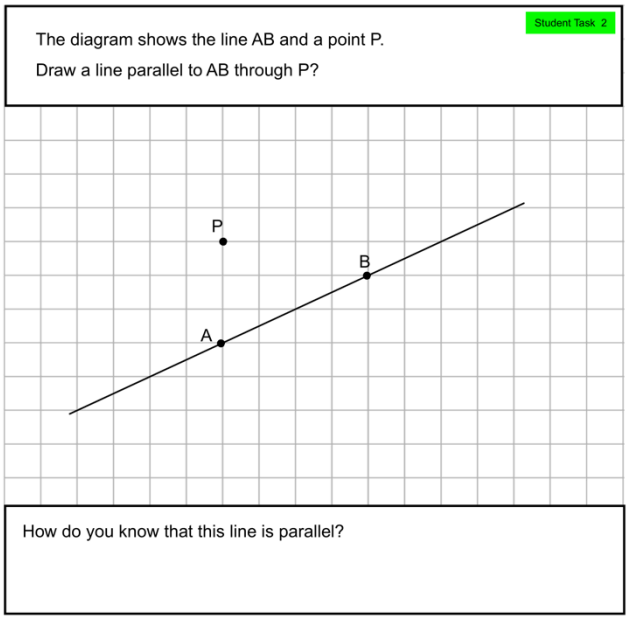
Student Individual Work
Posing Task 2



How many different ways can you do this?

</td>
<td>Clarifying the Problem

Will ask students to explain the terms “Parallel”

Can students give examples in the room, in real life?

“When we want to draw a straight line what should we use?”

“Is it ok to draw a straight line freehand?”

“How did you draw a parallel line from point P”

(2 minutes)</td>
<td>Checking literacy. Students starting to explain their ideas.</td>
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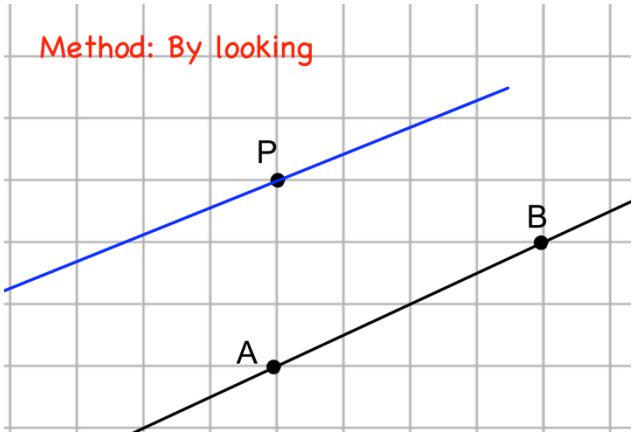
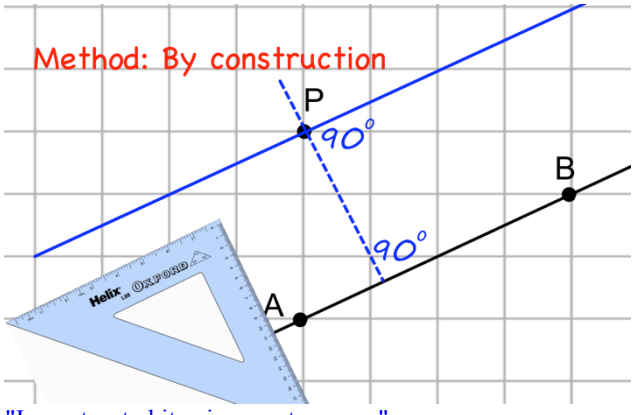
Anticipated Student Responses



"They look parallel." Roughly parallel



"I constructed it using a set square."

</td>
<td>Individual work

“Can you think of another way of doing this?”

(If nobody uses the gridlines to make a parallel line) “Can you think of a way of using the gridlines to help draw a parallel line”

(5 minutes)
Presentation

(7 minutes)

(If nobody uses the gridlines)</td>
<td>“Is this parallel?”
“Who agrees?”
“Does anyone disagree? Why?”</td>
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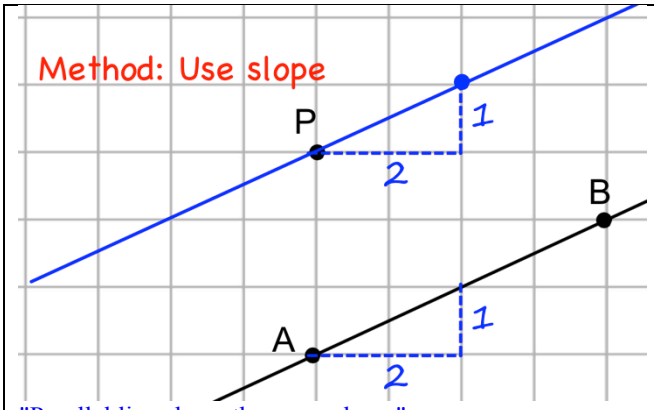


"Parallel lines have the same slope."

"The slopes of both of these lines is ½ "

"Slope = rise/run = (4/2) or ½"

"How would you travel from A to B using the gridlines?"

"What horizontal and vertical movement did you use?"

"What type of triangle does the movement create?"

"Can we relate this to our JC knowledge of rise and the run?"

(If nobody refers to slope)
"What does it mean to say two lines are parallel?" Probe until a student says something like:
"They have the same slope."

"What are the slopes of these lines?" (Assuming grid units are unitary) Give time for students to calculate and ask a student to present.

Developing an understanding of slope from Task 2

Individual Student Work

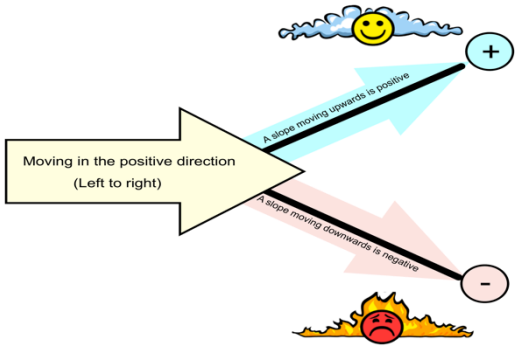
Students are handed out a sheet with the same line segment [AB] "What would the slope be if the page is rotated 90 degrees?"
This is demonstrated at the board with a 90-degree anti-clockwise rotation. Students are given a few minutes to get an answer.

Presenting Solutions

A student who uses a "slope triangle" approach with an incorrect positive solution will present at the board.

The student will use arrows to show the "change in x" and "change in y" these changes will be compared with movement along the "x" and "Y" axes.

The "Heavenly slopes" image (below) is shown to students and students are asked if they can explain the reason one slope is positive and the other is negative.



The purpose of this task is to consider negative slopes. It is expected that students using the "Rise over the run" method will get an incorrect "positive" answer to this problem.

"Did anyone else get that answer?

"Did anyone get a different answer?"

"Which is correct?"

"Is that positive?"

"Why is this a positive slope?"
"Why is this a negative slope?"

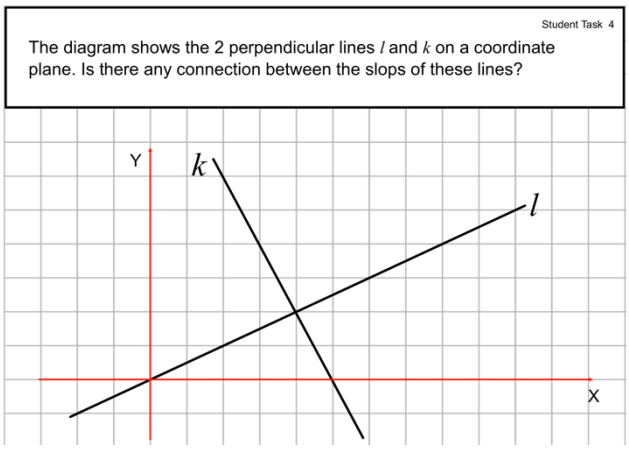
"Is that correct?"

"Can you check your answer using another method?"

Do students have any misconceptions?

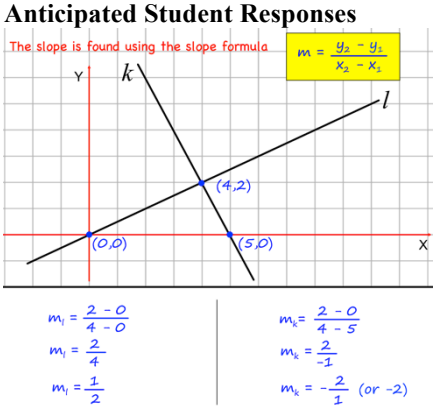
Is their understanding changing? Can they correctly explain the reason that a slope is either positive or negative?

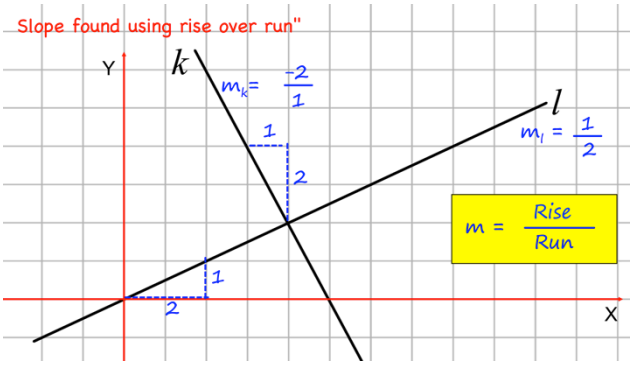
Student Individual Work
Posing Task 3



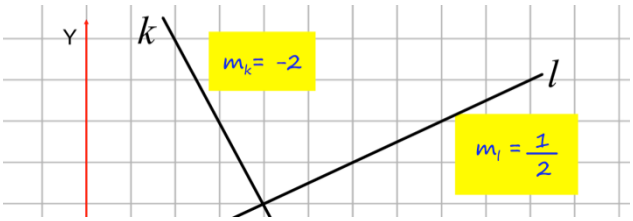
Clarifying the Problem
(2 minutes)

Anticipated Student Responses





Having correctly found/written the slopes students may notice:



"One is positive and the other is negative."

"If you invert the slope and change the sign you get the perpendicular slope." Negative Reciprocal

Individual work
(6 minutes)
Presentation
(7 minutes)

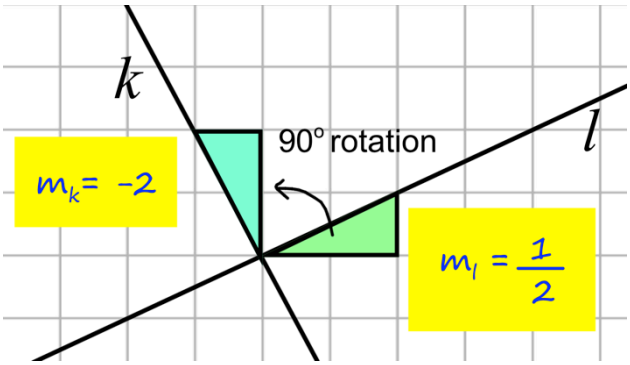
What can you say about the relationship between the two lines?
What is the slope of the line AB?
What is the slope of the line CD?

If we are rising by vertical movement in the triangle formed by the line AB, what are we doing in the vertically movement in the triangle formed by the line CD?
Is our movement or rise part going to be positive or negative as we drop from C?
Is there a relationship between 4/8 and -8/4/?
Can you tell me what happens if we multiply 4/8 and -8/4 on our calculator?
Can we make up a rule based on the result of 4/8 x -8/4 = -1?

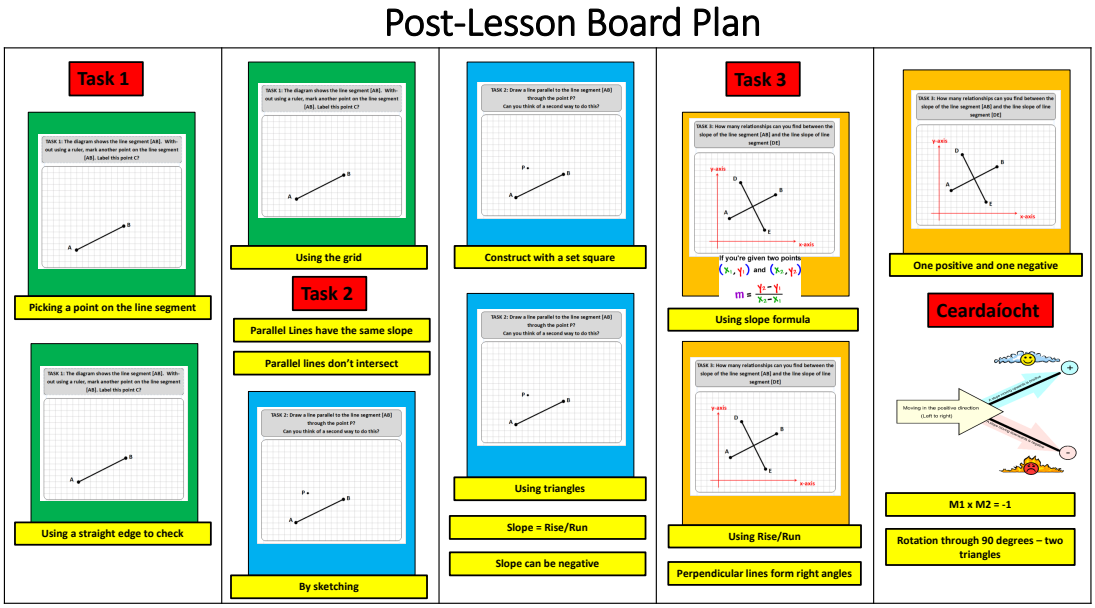
How do you tell if two lines are parallel?
How do we tell if two lines are perpendicular?

Can students explain, discuss key ideas in a way that demonstrates understanding?

$-2 \times \frac{1}{2} = -1$ "If you multiply perpendicular slopes the answer is -1."		

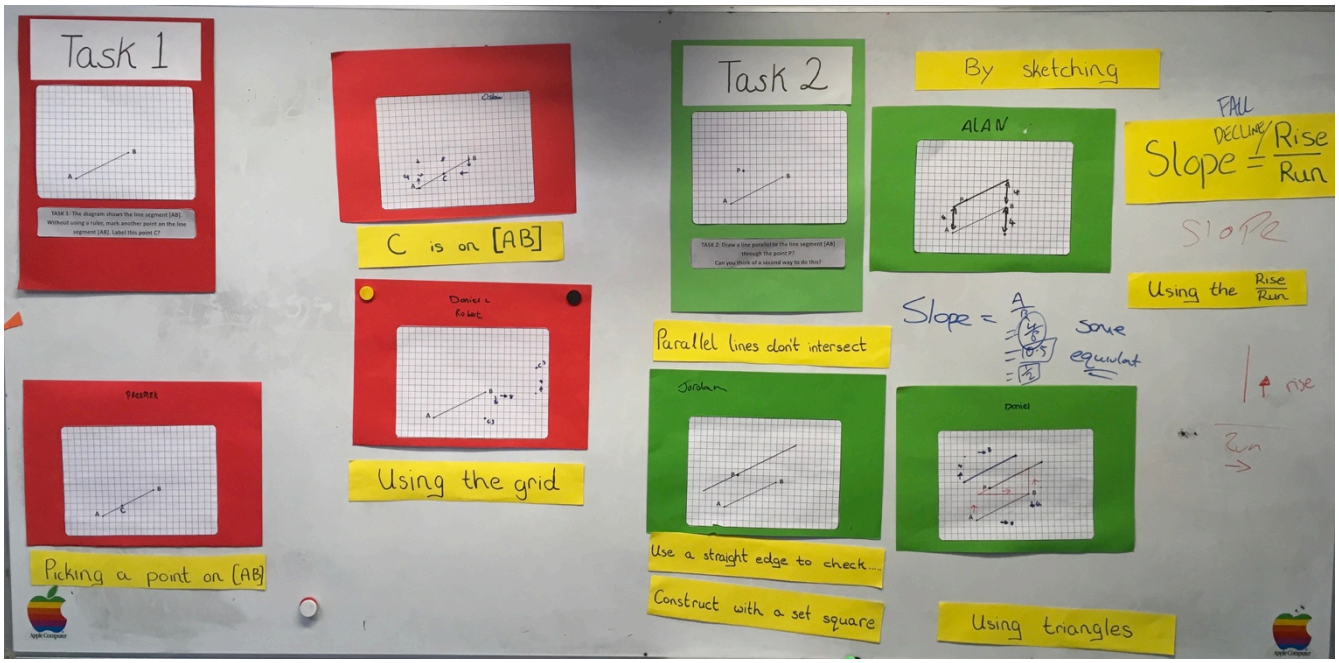
Ceardaíocht /Comparing and Discussing  Can students understand why perpendicular lines have "negative reciprocal" relationship?	Ceardaíocht (6 minutes) Ask probing questions to recognise the negative negative reciprocal relationship between perpendicular slopes. Ask the students to (use their calculator) to investigate what product of the perpendicular slopes is? Once we established it is -1 Discuss: Do you think this is always true? Demonstrate rotation of perpendicular slopes	Do students understand that "perpendicular" lines are mapped onto each other by rotating 90 degrees? Can students explain the reason for the "negative reciprocal relationship?
Summing up & Reflection What did you learn today? What do you think we wanted you to learn from this class? Who's idea did you find most helpful in understanding…?" Students complete "Reflection Sheet" (see appendix). Homework worksheet (see appendix).	Reflection (5 minutes) This will be written on a Reflection sheet to be collected at the end of the class	Do students articulate an understanding and appreciation for the learning goals of the lesson?

The Board Plan

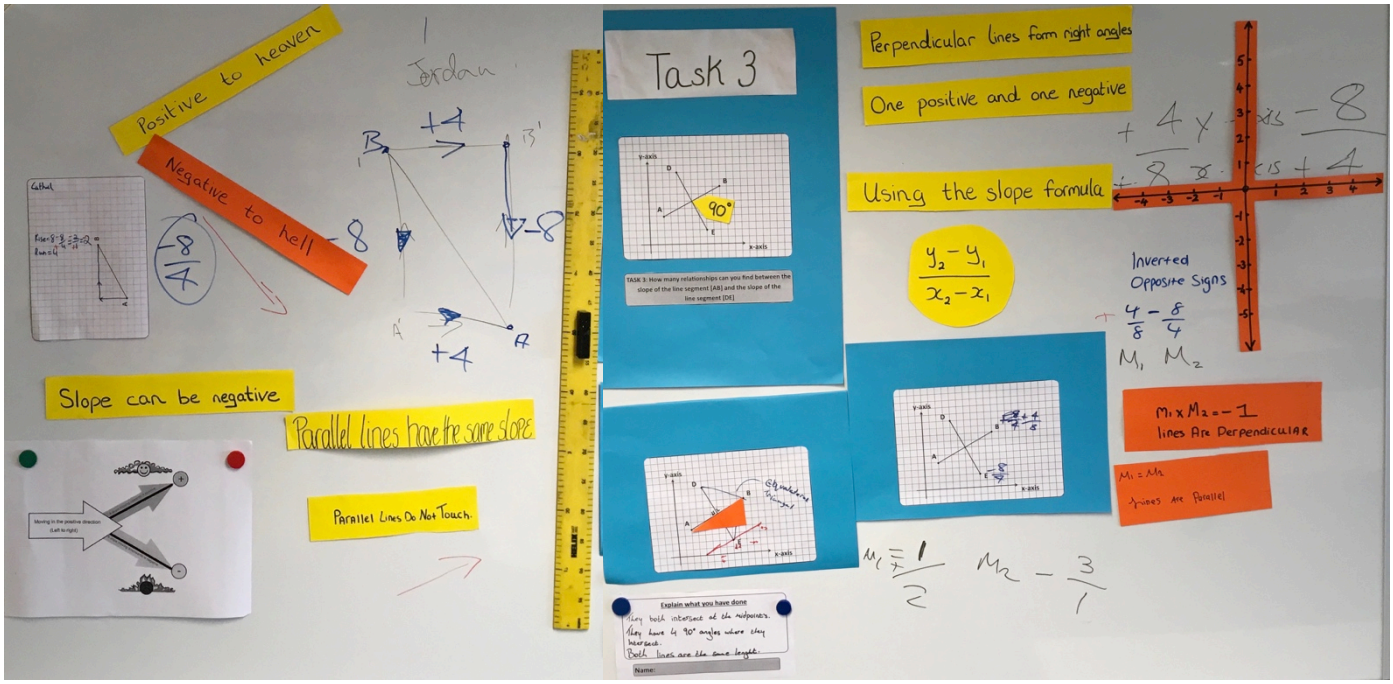


The Lesson Boardwork

Board 1



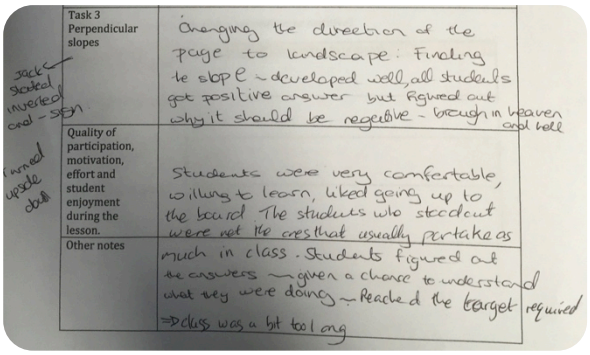
Board 2



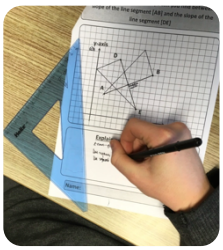
Reflecting on the Lesson

The research lesson was taught over a double class (120 minutes) on 7th February 2018. The lesson did achieve the 8 stated goals. The attention and engagement of the students was held throughout the lesson, but it was generally agreed by the teachers that the lesson was "a bit too long". It was discussed how tasks might be rephrased to achieve this. It was observed that: "Students were very comfortable, willing to learn, liked going up to the board. The students who stood out were not the ones that usually partake as much in class.

At different stages the students didn't immediately come up with the required solutions/methods to move the learning in the desired direction. On two occasions the teacher skillfully presented incomplete answers and used them to affect students thinking and then some more individual time was given after which time students had arrived at a more "productive" approach. For example during Task 1 at the first effort no student had picked a point outside of the line segment [AB], and in calculating the negative slope in the extension to Task 2 every student arrived at the correct dimension but the incorrect "sign". Being able to present and challenge this mistake was central to the success of the lesson. It was clear that "Students figured out the answer when given a chance to understand what they were doing."

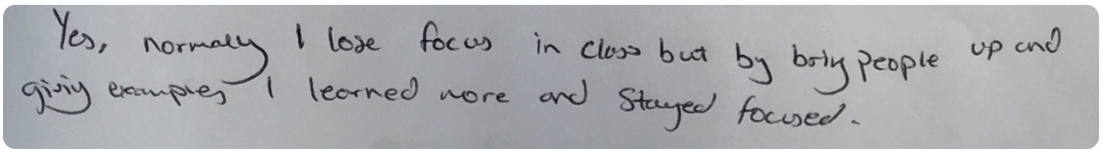


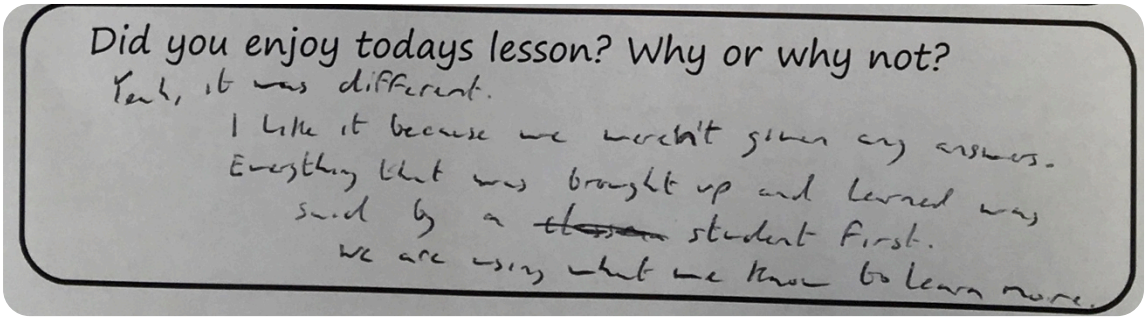
It was noted that students brought many and varied mathematical ideas into their discussion including Pythagoras, equilateral and isosceles triangles, midpoints, in this regard the richness of this approach to teaching as a means of making greater connections between topics and strands of the course was noted.



A high point of the class was when a student correctly was able to say: "invert and change the sign" in relation to comparing perpendicular slopes. There was also a rich discussion and understanding developed regarding positive and negative directions in the coordinate plane. During the class students became more confident at drawing triangles to understand the slope relationships. During the Task 1 no student drew a triangle, by Task 3 all students were drawing triangles.

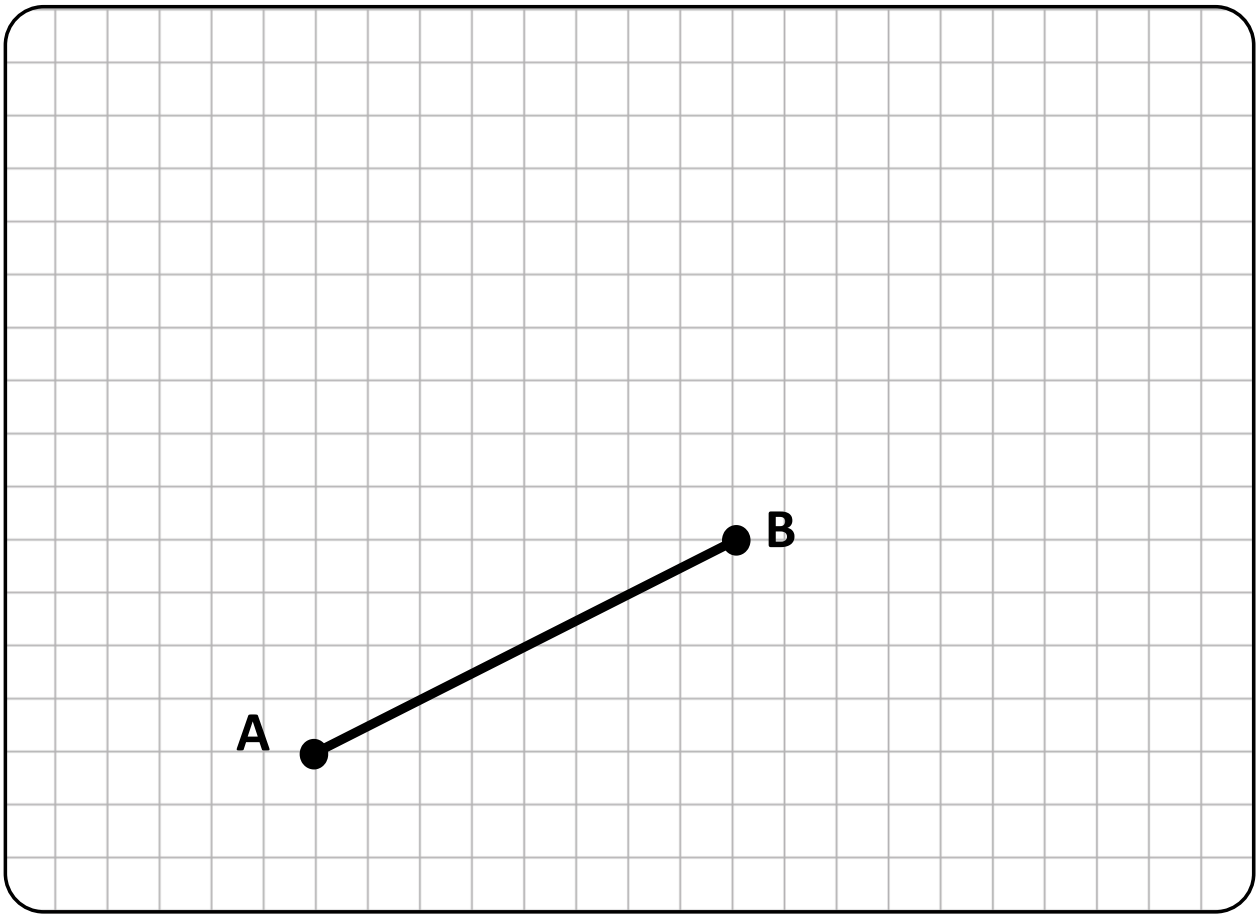
The students' "Reflections" were very positive although one student expressed "I prefer to be working all the time, writing out and trying sums instead of doing one and then looking at the board for a few minutes." Student's enjoyed: "Being included in the class", sharing each other's ideas, "by bringing people up and giving examples I learned more and stayed focused." "It was different. I liked it because we weren't given any answers. Everything that was brought up and learned was said by a student first. We were using what we know to learn more."





Appendices

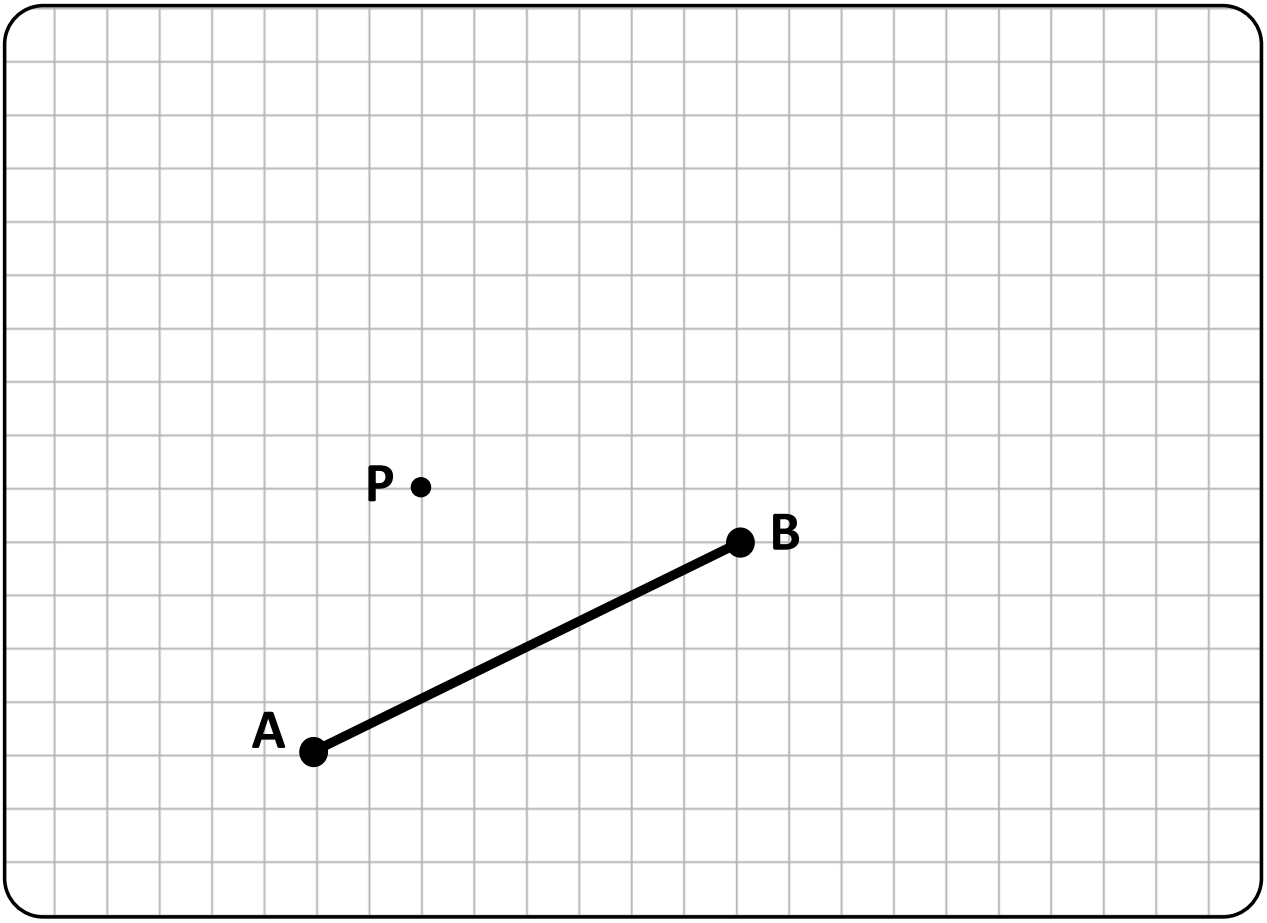
TASK 1: The diagram shows the line segment [AB]. Without using a ruler, mark another point on the line segment [AB]. Label this point C?



Explain what you have done

Name:

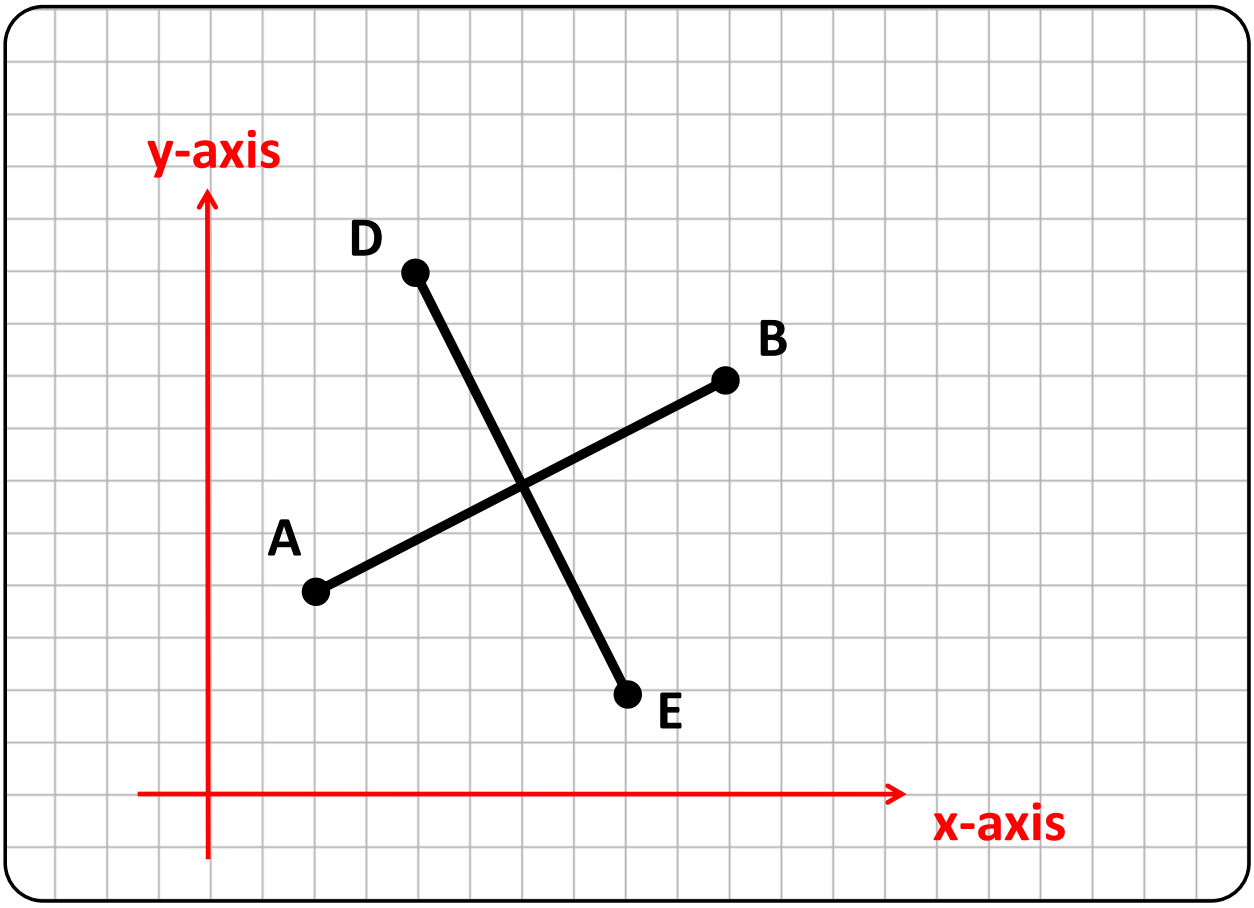
TASK 2: Draw a line parallel to the line segment [AB] through the point P?
Can you think of a second way to do this?



Explain what you have done

Name:

TASK 3: How many relationships can you find between the slope of the line segment [AB] and the line slope of line segment [DE]

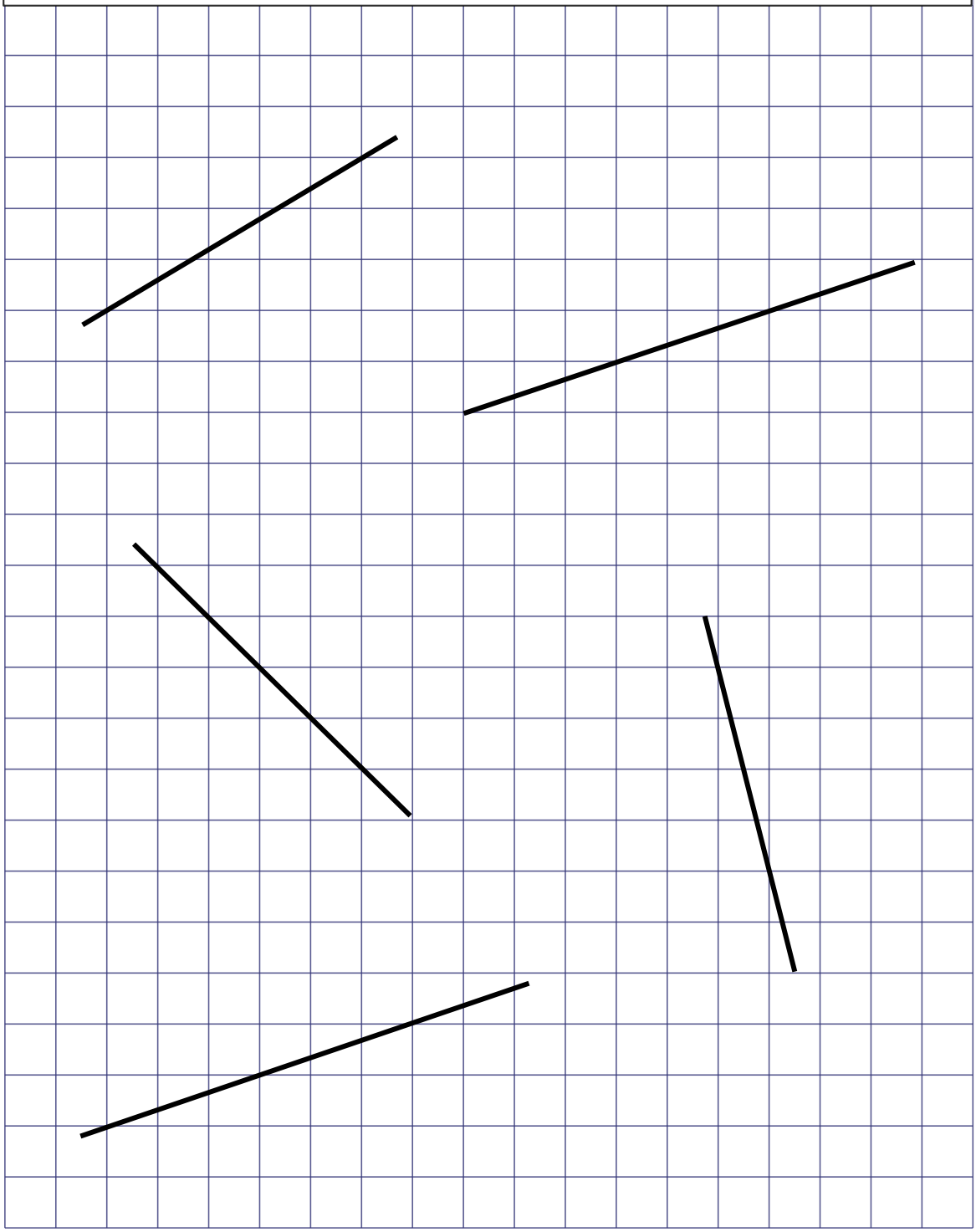


Explain what you have done

Name:

Homework Task

Without using a set square or a protractor draw lines that are exactly perpendicular to each of the line segmets shown below, and state the slope of each line that you draw.



Student:

Heavenly Slopes Observation Sheet

Aspect of Lesson	Observation, solutions, misconceptions, evidence, data, comments…
Task 1 Point on a line	
Task 2 Parallel line	
Understanding of slope	
Task 3 Perpendicular slopes	
Quality of participation, motivation, effort and student enjoyment during the lesson.	
Other notes	

Heavenly Slopes - Research Lesson Observation

Task 1 –Response	Student(s) (Tick if pre-asked to present)
Point in [AB]	
A slightly incorrect point	
Correct point outside of [AB]- using vertical & horizontal movement of the line	
Misconception needing to be addressed	

Task 2 –Response	Student(s) (Tick if pre-asked to present)
Construct correctly with a set square	
Use triangles – or vertical and horizontal gridlines	
Use slope And can explain	
Misconception needing to be addressed	

Can students explain why slopes are positive or negative?

Task 3 –Response	Student(s) (Tick if pre-asked to present)
Work out slope of CD correctly	
Identify the negative reciprocal relationship	
Identify that the product of perpendicular slopes is negative one	
Misconception needing to be addressed	

Notes:

Heavenly Slopes Image