Lesson Research Proposal for 5th Year Higher Level

For the lesson on 24th January 2018
At Woodbrook College, Brian Connaughton, 5th Year Higher Level
Instructor: Mr Connaughton
Lesson plan developed by: Angela Dwane, Brian Connaughton

1. Title of the Lesson: It's a Matter of Principles

2. Brief description of the lesson

The students will investigate a word problem to find the speed of a rocket travelling on a path that can be mapped as a quadratic function.

3. Research Theme

At Woodbrook College our teachers are encouraged to:
"value and engage in professional development and professional collaboration"[1]
"responds to individual learning needs and differentiates teaching and learning activities as necessary"[2]

We as the Maths Department support these goals by:

- Using a shared drive to which all Maths teachers contribute resources.
- All teachers share access to online classrooms (Edmodo & Build Up).
- We engage in peer assessment and team teaching including participation in Lesson Study.
- Individual attainable targets are set by teachers for students based on their learning needs.
- These targets are communicated to parents and monitored over the course of the year. Targets are adjusted as necessary during the course of the year.

The Maths Department adheres to the target setting guidelines as part of the whole school policy. In addition we review numeracy competency tests, track patterns of uptake and attainment generated through the State Exam Analysis Tool provided by the PDST and informally monitor attainment through following a common syllabus and standardising all class tests and exams across each year group.

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1 Looking-at-Our-School-2016-A-Quality-Framework-for-Post-Primary-schools.pdf pg12

2 ibid

4. Background & Rationale

This lesson is aimed at Leaving Certificate Higher Level students. We have observed over the past few years that students fixate on attempting to study calculus through learning off particular notation without having a grounded understanding of what a derivative is and why the derivative rules are applicable.
This can be due to the way calculus is introduced to students by teachers emphasising the rules in a particular format and referring to specific notation.
Leaving Certificate papers are moving towards deeper conceptual problems that have real world applications and challenge students to demonstrate their understanding of a topic rather than simply testing their competency of known procedures.

Traditionally students did well in the State Exams on questions involving calculus where procedure was tested, however the applications proved to be an area of weakness.

The chief examiners report on Leaving Cert Higher Level Maths in 2005 stated:

"Answering indicated that candidates can competently execute the technique of differentiating by rule, (as evidenced by success in part (i)), but are not able to apply their knowledge with any degree of understanding, (as evidenced by their failure to engage meaningfully with part (ii)).In part (iii), although candidates did appear to know how to find the equation of a line, the vast majority failed to recognise any connection between the derivative of the function and the slope of the tangent. "[3]

Throughout the correcting process over the past few years it has been evident that there are still some difficulties surrounding the topic of functions and in particular with calculus applications. Areas of weakness include the appropriate time to use a derivative and the correct application of derivatives.

The Chief Examiners Report 2015 specifically identified a difficulty that several students encountered:

"*many candidates did not know that solving* $f'(x) = 0$ *would yield the x-coordinates of the turning point of the function. In many instances, candidates sought to solve* $f''(x) = 0$ *instead, which illustrated a lack of comprehension of the concept of a turning point and its relationship to the derivative.* "[4]

3 Chief Examiners Report Maths 2005 pg32
4 Chief Examiners Report Maths 2015 pg25

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Junior Cycle: Functions were introduced to students through graphs and algebra. Real life problems are used. Appropriate graphing technologies have been used. Students will have learnt to use graphical methods to find solutions to problems including finding a maximum or minimum. Students will be familiar with function notation. "*use graphical methods to find approximate solutions where f(x) = g(x) and interpret the results… find maximum and minimum values of quadratic functions from a graph*"[5]	Students will expand their understanding of a slope. Introduction to derivatives. Investigate rates of change. "*find the derivatives of sums, differences, products, quotients and compositions of functions of the above form – apply the differentiation of above functions to solve problems*"[6]	Co-ordinate Geometry "*Equivalence of the slope of the graph and the rate of change of the relationship.* *Comparing linear relationships in real-life contexts, paying particular attention to the significance of the start value and the rate of change*"[7] Geometry "*use trigonometric ratios to solve real world problems involving angles*"[8] Number (arithmetic, area/volume) "*calculate, interpret and apply units of measure and time* *solve problems that involve calculating average speed, distance and time* *Modelling real-world situations and solving a variety of problems (including multi-step problems) involving surface areas, and volumes of cylinders and rectangular solids*"[9] Algebra "*explore graphs of motion… describe both quantity and change of quantity on a graph*"[10]

5 NCCA Junior Cert Syllabus pg31
6 NCCA Leaving Cert Syllabus pg43
7 NCCA Leaving Cert Syllabus pg24
8 NCCA Leaving Cert Syllabus pg24
9 NCCA Leaving Cert Syllabus pg32
10 NCCA Leaving Cert Syllabus pg36

6. Goals of the Unit

a. Students will present quadratic functions in a graphical context
b. Students will find the derivative of a quadratic function from first principles
c. Students can associate a derivative with slope and tangent lines
d. Students will understand that position changes over time
e. Students can apply the differentiation to solve problems
f. Students will apply derivatives to rates of change

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Investigating slopes of a quadratic function through problem solving
2 The Research Lesson	Investigating rates of change through problem solving
3	Formalising a platform for finding slope of a quadratic
4	Discover the general rule through testing
5	Develop the product rule through problem solving
6	Formalise the rule through testing
7	Investigate the quotient rule through problem solving
8	Formalise the quotient through testing
9	Problem solving challenge

8. Goals of the Research Lesson:

The Goals of the lesson should refer to:

a) Mathematical Goals

- Students will understand displacement of an object over time
- Students will apply their knowledge of slopes to a quadratic context

a) Key Skills and Statements of Learning

In the planning and design of this lesson we considered the 5 key skills of the learner as identified in the Leaving Certificate Mathematics Syllabus. This lesson will promote these skills in the following ways

Critical and creative thinking

- Engaging in investigations
- Critically evaluating information
- Solving problems in a variety of ways

Communicating

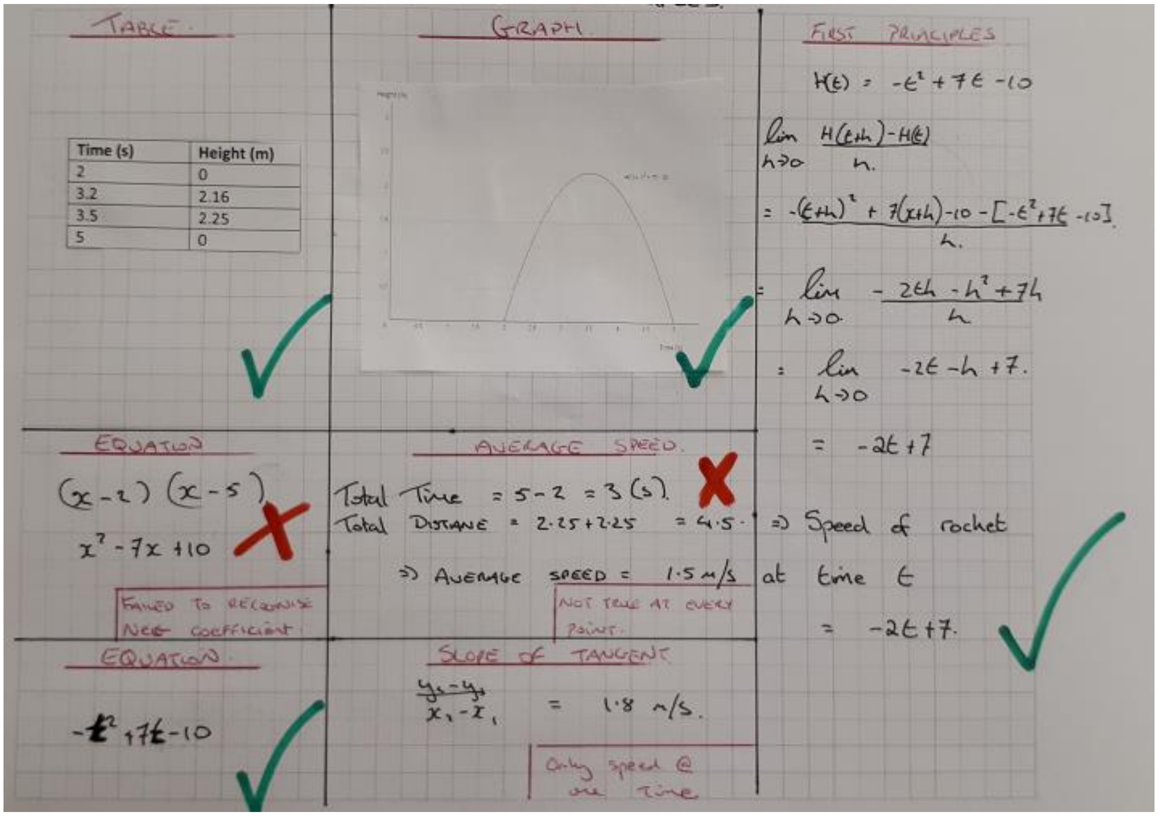
- Discuss approaches to solutions
- Consider and listen to other viewpoints
- Communicate findings to an audience

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	**Teacher Support**	**Assessment**
Introduction (5 mins) Video of student rocket launch (5th Year Chemistry project run this term) shown to students.	Show video on projector. Describe the process.	Real-life application and very relevant to students.
Posing the Task (5 mins) Distribute the problem.	Hand out printed copy of problem. Project on whiteboard. Distribute A3 work sheets (squared) **Proposed Problem** One day a fantastic Chemistry class was given a task of designing and launching a mini  Rocket. Team A were the most successful team. A timer was started when the rocket was placed on the launch pad. It took two seconds for the chemical reaction in the rocket to complete. At 3.2 seconds the rocket was at a height of 2.16 metres. At 3.5 seconds the rocket was 2.25 metres. At 5 seconds the rocket hit the ground. Some of the teachers in the staffroom had a debate about the speed at which the rocket was travelling. Mr Bergin Claimed that speed of the rocket was 1.5 m/s Mr Fitzpatrick disagreed claiming that the speed is 0.3m/s Mr O'hAnnaidh objected stating that both were incorrect. Investigate each of these statements, state what you think the speed of the rocket is. (Justify your answer)	Observe facial reactions of students to ensure they understand the task at hand.
Student Individual Work (25 mins) • Recognise quadratic • Tabulate answers • Graph the time path • Calculate average speed • Calculate first principles • Choose A • Choose B • Choose C	• Early finishers – encourage to investigate how many ways the student could approach the problem. • Stuck students – Can you use an alternative strategy to find a solution? Refer to identifying key words, applying prior knowledgeable, remind them of the video. • Incorrect solutions – allow them time to work it through. Encourage them to think through fully the solution step by step explaining each step of the way. Remind them to attempt multiple approaches.	• Check that students are actively engaged with the activities. • Ensure students are verbalising (writing) explanations justifying their work. • Circulate the room checking in on students. • Take note of student work, designing board plan.

Ceardaíocht **Comparing and Discussing (20 mins)** 1. Table – Points put into a table, recording the locations against time. 2. Equation – Recognised roots formulates equation 3. Pattern – from either a list of points or the table, identifying a quadratic pattern. 4. Graph – student has graphed a quadratic function. 5. Discusses Teacher A 6. Discuss Teacher B 7. Calculates the slope using first principles.	Questions 1. Why did you use a table? Can you spot anything from the information listed? 2. How did you know it was quadratic? Was there any other indication from the table that it was quadratic? (Leads to 3) 3. What could you use the pattern now to do? Is there another way to represent the data? 4. Why did you draw a graph? 5. Where did the figure come from? Does anyone agree with this? Does anyone think anything different? 6. Where did the figure come from? Does anyone agree with this? Does anyone think anything different? 7. Why did you use first principles? What have you calculated? How does this help you make your decision? What can you conclude?	Questions distributed among the remaining students, not targeted at the presenter. 1. Possible misconception starts at (0,0) 2. Possible misconception U shaped graph. H(x) instead of h(t) 3. Draw through discussion that the response that it is a correct calculation but it applies to a linear function, not quadratic. Looking for the keywords average speed. 4. Possible misconception U shaped graph from the equation. 5. Draw through the discussion the response that it is a correct calculation but it applies to a particular location of time. Look for an alternative calculation to dispute the solution. 6. Draw a discussion concluding with the slope changing at various points, therefore while both Teacher A and B had valid points the situation changes depending on time.
Summing up & Reflection (5 mins) Refer to original video, the problem. Summarise the argument from beginning through. Give instruction to students to write a reflection.	Look at the board work and use it to describe the argument. Encourage students to reflect on their experience. Consider what they learnt, how they found the experience.	

10. Board Plan



11. Evaluation

a. Students will present quadratic functions in a graphical context
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1. Did the problem produce the expected responses? (Graph, Equation, Average Speed, First Principles)
2. Did the lesson promote active discussion among students?
3. Did the lesson comprehensively address expected misconceptions?
4. Do students understand that the slope of a quadratic function can be calculated through first principles?
5. Were students comfortable with the notation?
6. What were the most common approaches?
7. Was there anything particularly interesting that a student said or any insight a student identified?

12. Reflection

The lesson went well. The video engaged students from the start and contributed to a relaxed environment. The problem was suitably challenging and catered to multiple levels of ability. Several students recognized that the function was quadratic and most made a decent attempt at drawing the graph. Almost all students highlighted either verbally or through calculation that the speed varied over time. Only a couple of students got as far as using first principles to find the speed. Most students did make reference to the slope and by the end of the discussion all students had a good understanding of the relationship between the slope and first principles. The students did not struggle with the alternative notation. Some misconceptions were highlighted such as the n-shaped quadratic requires a negative coefficient of x.

There was a variety of approaches to the problem, some students went straight to graphing, and some relied on calculations testing the figures first. Many attempted to use the Distance Speed Time formulae. The discussion dealt well with the misconceptions and students appeared to have good understanding of the concept of change and in particular were comfortable with the alternative notation.

In the post lesson discussion it was flagged that students should have been encouraged to direct the discussion a little more and to summarise the concepts instead of relying on the teacher. The expected board plan was largely achieved and the approaches of the students were anticipated.

One interesting approach was a student attempted to plot the given speed figures and work from the graph to interpret them.

The reflection sheet from the students largely showed enjoyment of the lesson and a strong learning experience. Most highlighted the correction of their misconception about linear speed, average speed and negative quadratic functions which was encouraging to read.

Appendix 1

Proposed Problem

One day a fantastic Chemistry class was given a task of designing and launching a mini

Rocket. Team A were the most successful team.
A timer was started when the rocket was placed on the launch pad.
It took two seconds for the chemical reaction in the rocket to complete.
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Some of the teachers in the staffroom had a debate about the speed at which the rocket was travelling.

Mr Bergin Claimed that speed of the rocket was 1.5 m/s
Mr Fitzpatrick disagreed claiming that the speed is 0.3m/s
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Investigate each of these statements, state what you think the speed of the rocket is. (Justify your answer)

Appendix 2
Student Reflective Sheet

Reflecting on my Learning

Student:

What did you learn in today's lesson?

Is there anything that you don't fully understand?

Did you enjoy todays lesson? Why or why not?

Lesson Study