Lesson Research Proposal for 1st Year- Algebraic Word Problems

For the lesson on: 29/01/2018
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Class: 1st Year
Instructor: Andrew Hough
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1. Title of the Lesson: Kevin's Cash Conundrum

2. Brief description of the lesson

The students will show in many ways how to represent a word problem which will lead to an algebraic equation. Following this they will solve an equation and explore the concept of 'balancing' an equation.

3. Research Theme

We want students who:

a) Attain the stated learning outcomes for each subject, course and programme.
b) Reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning.

As mathematics teachers, we will actively support the achievement of these goals by paying attention to the following entry points in my every day classes:

a) Clear attainable learning outcomes
Will differentiate appropriately so that all students can achieve and at times surpass the intended learning outcomes of the lesson.

b) Support for students to assess and take ownership of their learning
Encourage students to reflect on their work realistically and describe their strengths and areas for improvement. Provide the opportunity for students to have ownership of their work through facilitation of problem solving lessons and continued self-reflection and assessment.

4. Background & Rationale

1. Why we chose this topic:

The lesson is aimed at first year students. We chose algebra worded problems because it is a problematic area from 1st Year all the way up to 6th Year. We find that there is a fear or stigma around algebra with many students. Students find it difficult to formulate mathematical equations from words and lack confidence when faced with worded problems. Students find it difficult to generalize and explain patterns and relationships in words and numbers.

2. Our Research Findings

Through our discussion as maths teachers we realised that we generally bring students through a series of steps when dealing with worded problems. We would try to translate words to mathematical symbols and operations and lead them to form the equation. When students are left to approach an unfamiliar problem alone they often struggle. We find that students aren't given enough time to understand how exactly to solve the problem and maybe as teachers we jump in to quickly to help or assume students know how to tackle the problems so we move on.

Because of the above we have decided to commence the teaching of algebraic word problems using a problem-solving situation which gives rise to an algebraic equation.

In designing the research lesson, we believe it is imperative to engage students enthusiastically with the subject matter and therefore our lesson proposal will try to devise creative ways to write and solve an algebraic equation from a word problem. The approach requires allowing the students time to think, confront their difficulties and progress on their own.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
In primary school, students are introduced to variables and equations. In fifth class, students translate number sentences with a frame into word problems and vice versa. In sixth class students explored the concept of a variable in the context of simple patterns, tables and simple formulae and substitute values for variables. This is continued on in first year where students are reintroduced to variables and constants through patterns and conclude by solving given linear equations.	− consolidate their understanding of the concept of equality. − solve first degree equations in one variable, with coefficient elements of **Z** and solutions also elements of **Z.** − solve equations of the form $ax + b \pm dx + e = g$, where *a, b, c, d, e, f, g, h* $\in$ **Z.** − solve linear inequalities in one variable of the form $g(x) \leq k$ where $g(x) = ax+b$, $a \in \mathbf{N}$ and $b, k \in \mathbf{Z}$; $\boldsymbol{k \leq g(x) \leq h}$ **where** $\boldsymbol{g(x) = ax+b,}$ **and** $\boldsymbol{k, a, b, h, \in Z}$ **and** $\boldsymbol{x \in R}$	− solve first degree equations in one or two variables with coefficients elements of Q and solutions also in Q − form quadratic equations given whole number roots

6. Goals of the Unit

a. Students will predict possible solutions to equations, building confidence with the topic and resolving any misconceptions.
b. Students will approach solving equations in one or two variables by trial and error and relate simple equations to problem solving completed at Primary School Level.
c. Students will know how to solve equations using the balancing method and find a method to verify their answer either by substitution or elimination.
d. Students will write relationships as mathematical statements building on their previous knowledge of mathematical symbols and their understanding.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1 Research Lesson	Introduce linear equations through a worded problem in a problem-solving context.
2-3	Balancing single variable equations to solve and verify simple linear equations. Incorporate a worded problem to reinforce previous lesson.
4-5	Solving and verifying single variable linear equations involving the expansion of brackets. Incorporate a worded problem to reinforce topic.
6	Revision lesson of unit. Students create own worded problems, swap with others, solve and verify.
7	Assessment.

8. Goals of the Research Lesson:

a. Mathematical Goals

Students will:

- Apply their prior knowledge of expressions in formulating the equation.
- Appreciate that there is a variety of methods to solve and verify a worded problem.
- Understand how to write a mathematical equation from words and feel less intimidated by these problems.

b. Key Skills & Statements of Learning

In the planning and design of this lesson the Junior Cycle Skills and Statements of Learning have been considered. This lesson will implement and promote Junior Cycle Key Skills in the following ways:

1. **Being Literate:** Students will have the opportunity to develop their understanding the use of mathematical language.
2. **Being Numerate:** It will develop a positive and confident feeling towards problem solving.
3. **Managing Myself:** Students will have the opportunity to reflect on their own learning.

4. **Staying Well:** Students will become confident and positive about learning.
5. **Communicating:** Students will be listening, expressing, performing, presenting, discussing and debating in class.
6. **Being Creative:** Students will explore options and alternatives through problem solving and learning creatively.
7. **Working with Others:** Students will respect all other student's views and learn how to solve problems in different ways.
8. **Managing Information & Thinking:** Students will be curious about how could they make up their own mathematical problem.

This lesson is also designed to meet the following Junior Cycle Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.

15. The student recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning.

16. The student describes, illustrates, interprets, predicts and explains patterns and relationships.

17. The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge reasoning and skills.

9. Flow of the Research Lesson:

Steps, Learning Activities **Teacher's Questions and Expected Student Reactions**	**Teacher Support**	**Assessment**
Introduction: **(2 mins)** The students will have just finished patterns and expressions in the previous classes, so no prior knowledge recap will be required. Students will be brought into class and told that they are going to solve a new problem. We are going to try and solve the problem ourselves and then come together as a class to discuss what we have found.		To explain what students are expected to do in the class.
Posing the Task: **Task 1: (3mins)** Kevin has saved €20 in his bank account. He gets €10 pocket money a week and spends €3 on sweets every week. In as many ways as you can, show what Kevin's total savings amount to each week. **Clarifying the problem:**	Students will see the problem written on the board and also have a worksheet with the problem on it and room for their workings and solutions.	Do students understand the task being asked of them? Observe students and if

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- If students are struggling to make a start say "How much does Kevin have at the start, after one week, after two weeks, etc."

- If students are struggling to find many ways to solve the problem we might suggest some examples such as "Is there a way you can visually show this?"

- Responses to student's anticipated questions: ***Student questions in italic.***

"how many weeks do we go up to?"
"Do a few weeks" but then pose the question "what if he is saving for years?"

"what do you mean in how many ways?"
"Is there a way you can show this visually or explain it mathematically?"

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Students will be aided with sections of graph paper given on the worksheet.

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<td>not understood, clarify the question.

Do students understand they should use their prior knowledge of patterns to complete the task?</td>
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Student Individual Work

See Appendix 1 for worksheet.

Individual Student work (10 mins)

<u>Student Response 1</u>: Make a list (2 ways).

Students may write a list such as:

Way 1:

Week 0: 20
Week 1: 20 + 7 = 27
Week 2: 27 + 7 = 34
Week 3: 34 + 7 = 41
Week 4: 41 + 7 = 48
Week 5: 48 + 7 = 55

Way 2:

Week 0: 20
Week 1: 20 + 10 – 3 = 27
Week 2: 27 + 10 – 3 = 34
Week 3: 34 + 10 – 3 = 41
Week 4: 48 + 10 – 3 = 48
Week 5: 48 + 10 – 3 = 55

<u>Student Response 2</u>: Lists but placed in a table format.

Week 1	27
Week 2	34
Week 3	41
Week 4	48
Week 5	55

We don't expect students to show workings when doing the table.

<u>Student Response 3</u>: Line graph or a possible bar chart.

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Teachers will circulate the room while students are trying to solve the problem.

If individual students are struggling we may lead them to a certain solution or for them to get an idea of their partner.

If the majority of students have misconceptions it may be necessary to put the entire class on the right path.

If students finish early without reaching the 5 ways, we may lead them to complete their solution set.

If students finish early with all 5 ways, we may give them an extension such as "how would you write a formula if he saved €9 per week?"

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<td>Are students finding different ways to complete the task?

Do students forget the amount saved at the beginning and continue with just €7 every week?

Can students convert the worded problem into a mathematical statement and understand it?</td>
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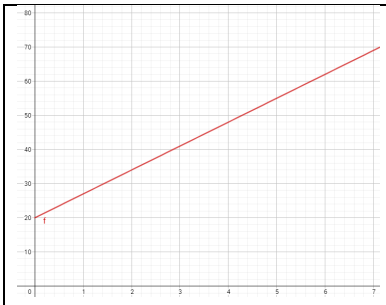
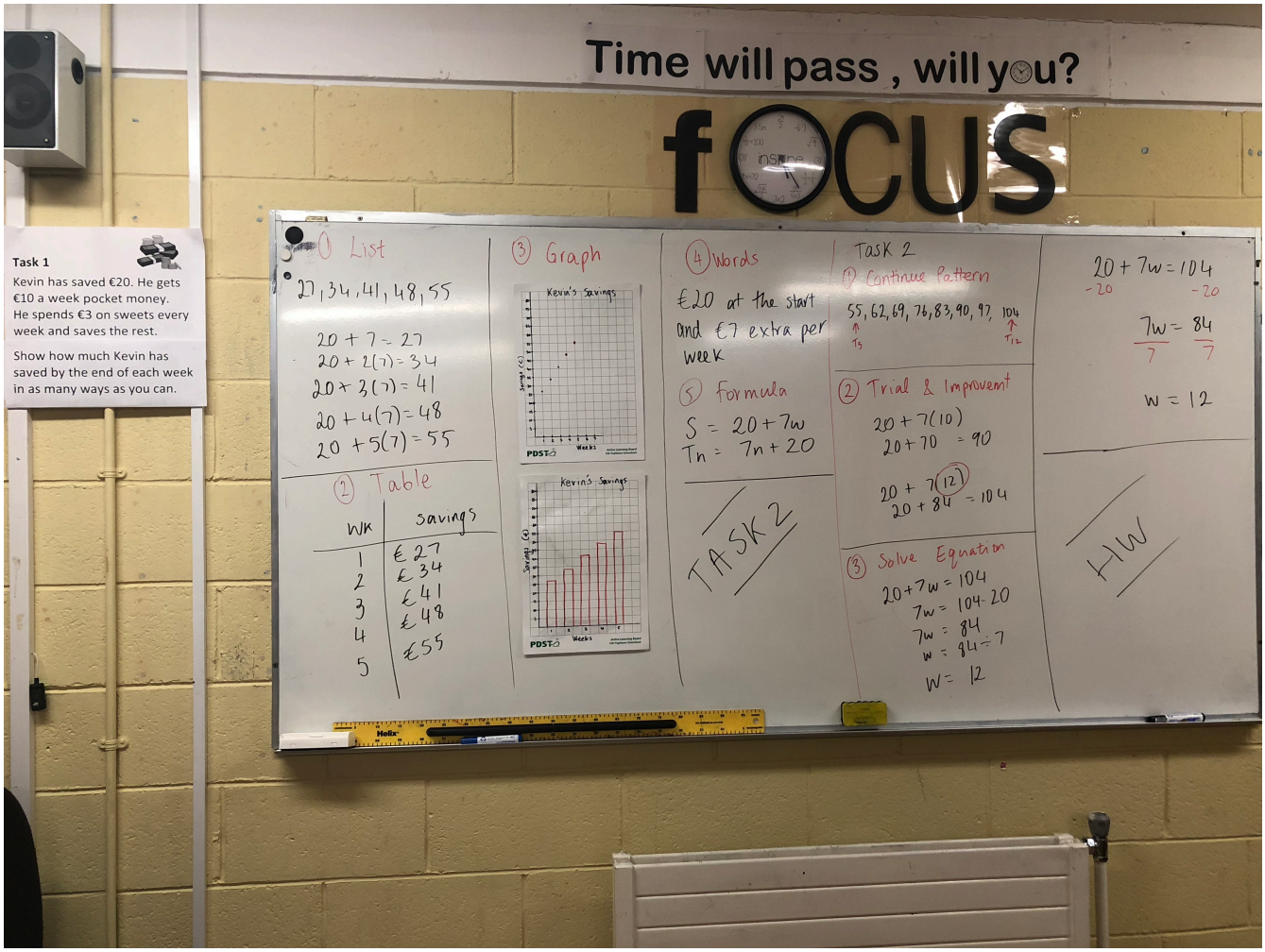
 **<u>Student Response 4</u>:** Describe the problem using words. Students will write "€20 at the beginning and then €7 each week in savings". **<u>Student Response 5</u>:** Students will develop a formula. 20 + 7n = Total.		
Ceardaíocht /Comparing and Discussing (15mins): **Response 1:** Listing, basic and table. The teacher might bring up students with a basic approach and then a slightly more advanced approach (response 1 then response 2). **Response 2:** Listing more advanced. **Response 3:** Line / Bar Chart **Response 4:** Explanation of formula in words **Response 5:** Algebraic formula	Did anyone do it the same way? Did anyone make a list in a different but similar way? Is there a connection between the two ways. Is either of these methods more useful than the other method? Questions as above. Which do you think is better and why? Did anyone word it slightly differently? Does it mean the same thing? Does anyone have the same formula with a different letter? Does this mean the same thing? Did anyone explain their letters in their formula?	Are students comparing ideas? Are students defending their methods? Observe what types of graphs students devise on their graph paper and their reasons for this. Observe what words students are using to describe the problem. Can students defend the language they are using in their description? How many students arrived at a mathematical formula. Do students understand that any letter can be used and it represents a variable?

Posing the task: **Task 2: (2 mins)** How many weeks does it take Kevin to save for a pair of football boots that cost €104? **Clarifying the problem:** Repeat the question to students.		
Student Individual Work: (5 mins) **Student Response 1:** Continue numerical pattern. 27, 34, 41, 48, 55, 62, 69 ,76, 83, 90, 97, 104 **Student Response 2:** Trial and improvement with formula 20 + 7(10) 20 + 70 = 90 20 + 7(11) 20 + 77 = 97 20 + 7(12) 20 + 84 = 104 **Student response 3: Working backwards to solve an equation** **2**0 + 7w = 104 7w = 104-20 7w=84 w=84/7 w=12	Is there a possible faster way to arrive at the answer? Students may be asked to refer back to their graph solution if they had one if they make no attempt to. Teacher may need to lead students to use the equation to solve the extension question.	Can students link the word problem to the formula previously devised? Do students attempt more than one method at solving the problem?
Ceardaíocht /Comparing and Discussing (15mins): **Response 1:** Continued numerical pattern	Did anyone do it the same way? Did anyone make a pattern in a	Are students defending their method?

Response 2: Trial and error with formula **Response 3:** Working backwards to solve an equation	different way? Is either of these methods more useful than the other method? Why did you choose to substitute in those numbers? Did anyone else substitute in any other numbers and what was the reason? Is this a more suitable method than trial and error? Teacher will at this point show how an equation should be balanced on each side to find a solution.	Are students attempting multiple methods? How did students go about this method? Do students recognize that solving and equation is the most efficient and quickest way to arrive at their solution?
Summing up & Reflection (8 mins): Today we learned that: • there are several ways to tackle a single problem. • some ways are more efficient than others while all have merit and are still valid and correct • algebra can be used to simplify a worded problem • one algebraic expression can be used to generalize an infinite amount of weeks • balancing is the most mathematically valid way to solve an equation		Do the students reflections represent the teacher's view of the lesson?

10. Board Plan



11. Reflection

We felt that our research theme accomplished the aims we had laid out. By working through various methods to complete a task and sharing these methods with other students at the board, the students were taking ownership of their work. All students reflected at the end of the lesson about what they had learnt and any areas they felt they had difficulty understanding. The majority of students attained the clear stated outcomes for this lesson which was a success.

Students were able to apply their prior knowledge of expressions in formulating the equation, appreciate that there is a variety of methods to solve and verify a worded problem and understand how to write a mathematical equation from words and feel less intimidated by these problems.

As observers, we hoped to observe the students actively engaging with the problem, this was definitely the case. The students were confident in sharing their ideas and methods of completion on the board. The students were also engaged with new learning of balancing an equation.

The main point raised about the actual problem was that a large number of students thought that each "way" meant each week and a lot of students just did week 1 for "way 1". This was noticed by the teacher immediately and the issue was clarified. It may need clarification going forward if this lesson is used.

In studying the students collected work, all students had attempted several ways to approach the problem. Their reflections showed that they really enjoyed the lesson and learnt other methods from other students at the board work. Some students also stated that they wanted more practice on ‘balancing equations’ which was expected as this was the initial introduction to balancing an equation.

Appendix 1

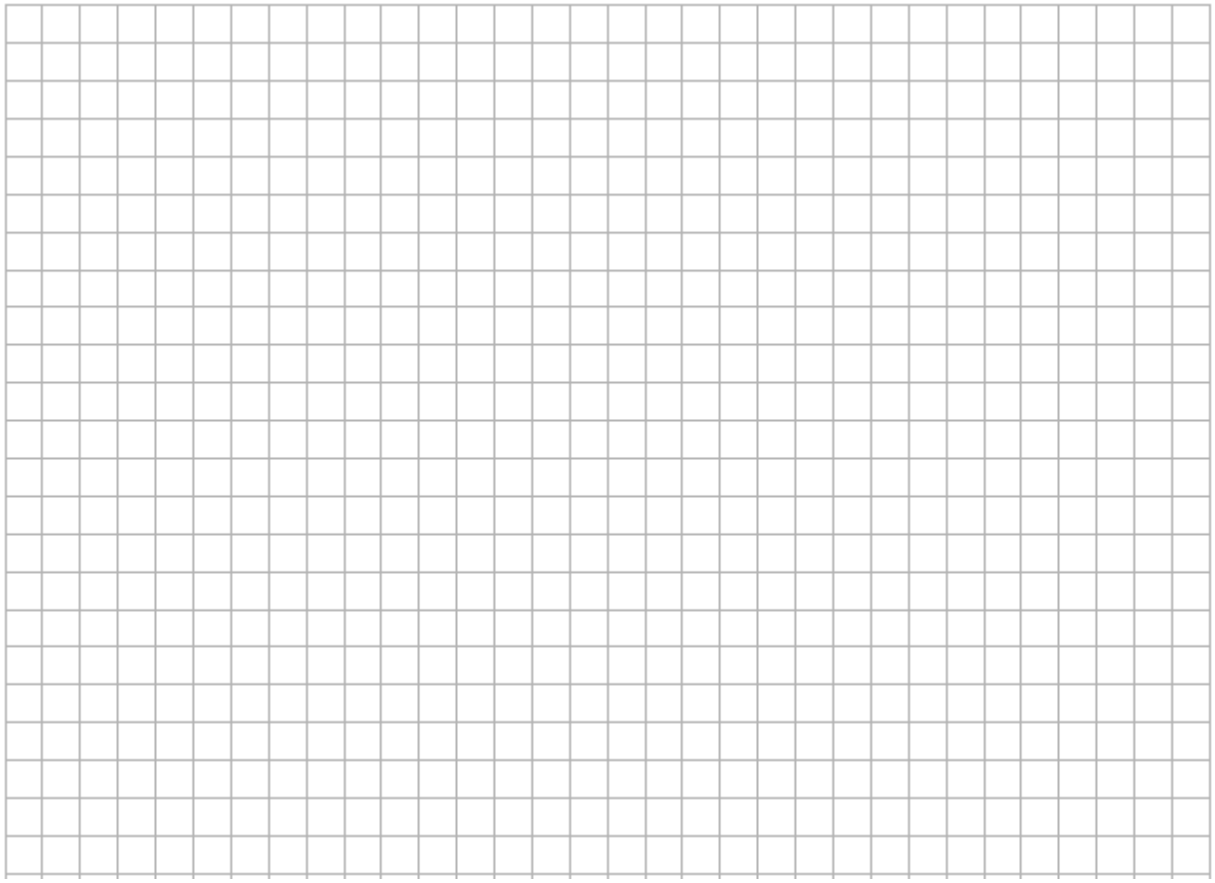
1st Year Maths Algebra Lesson Name: _____

Task 1
Kevin has saved €20. He gets €10 a week pocket money.
He spends €3 on sweets every week and saves the rest.
Show how much Kevin has saved by the end of each week in as many ways as you can

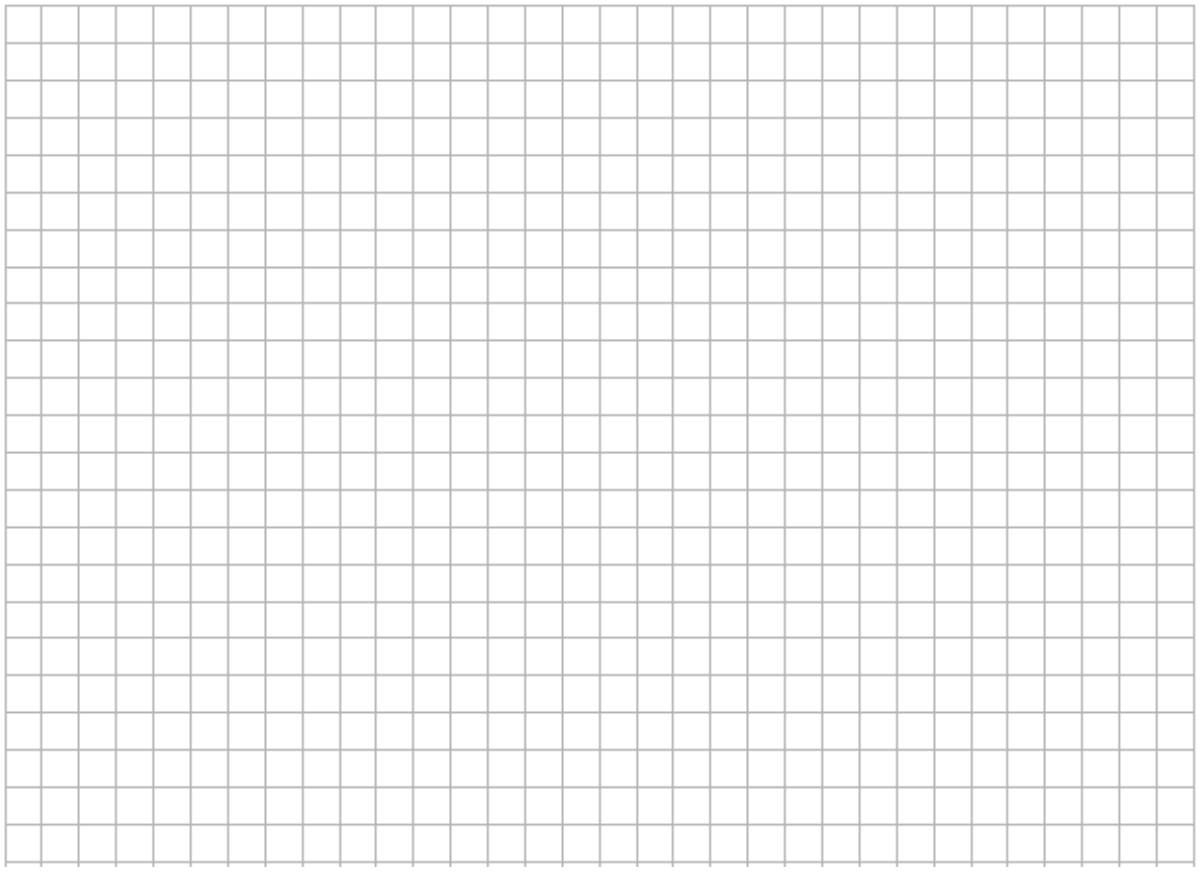
Way 1

Way 2

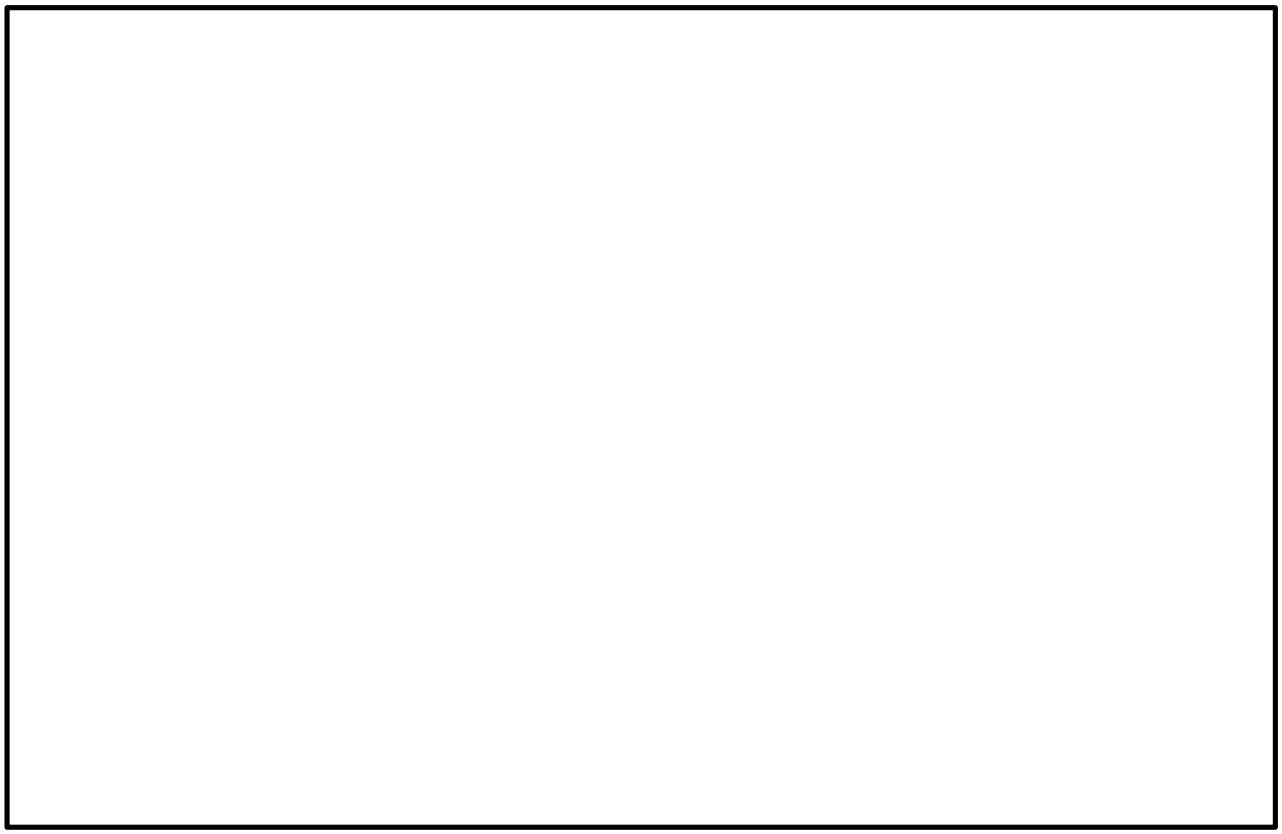
Way 3



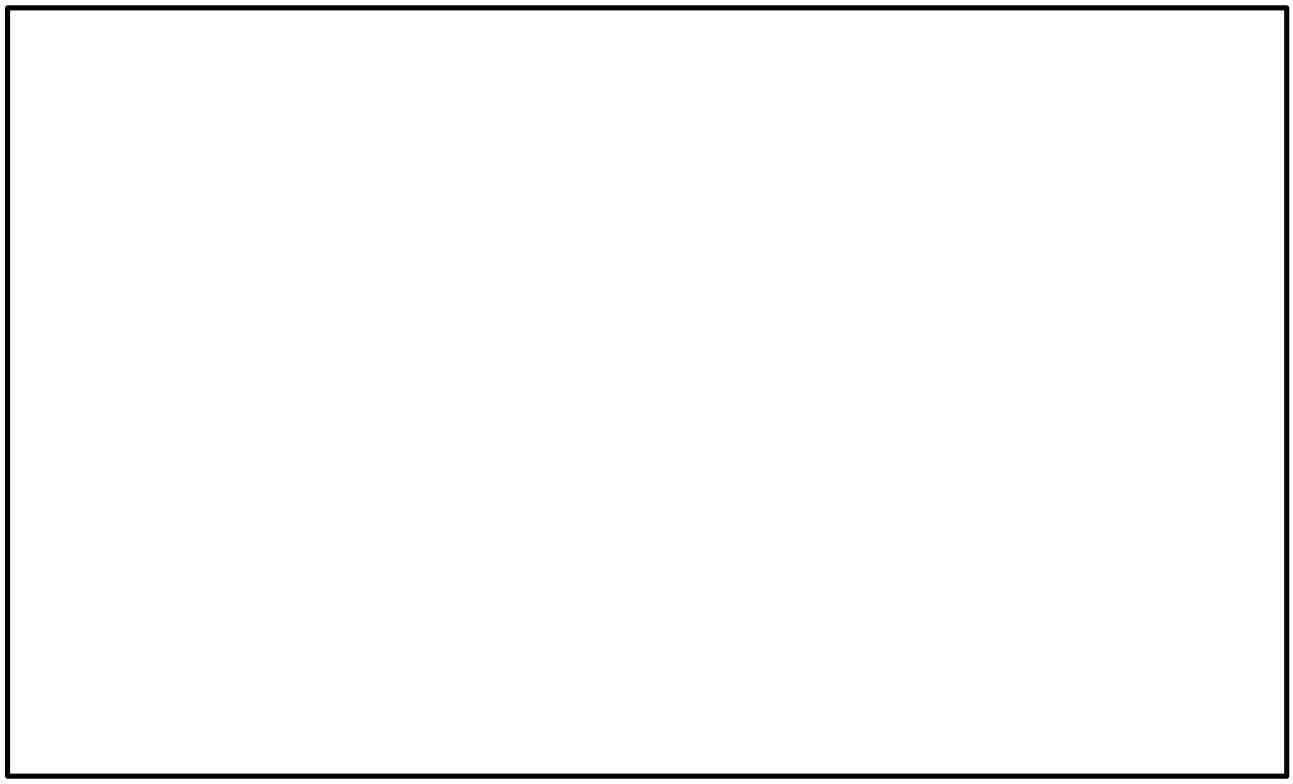
Way 4



Way 5



Way 6



Way 7

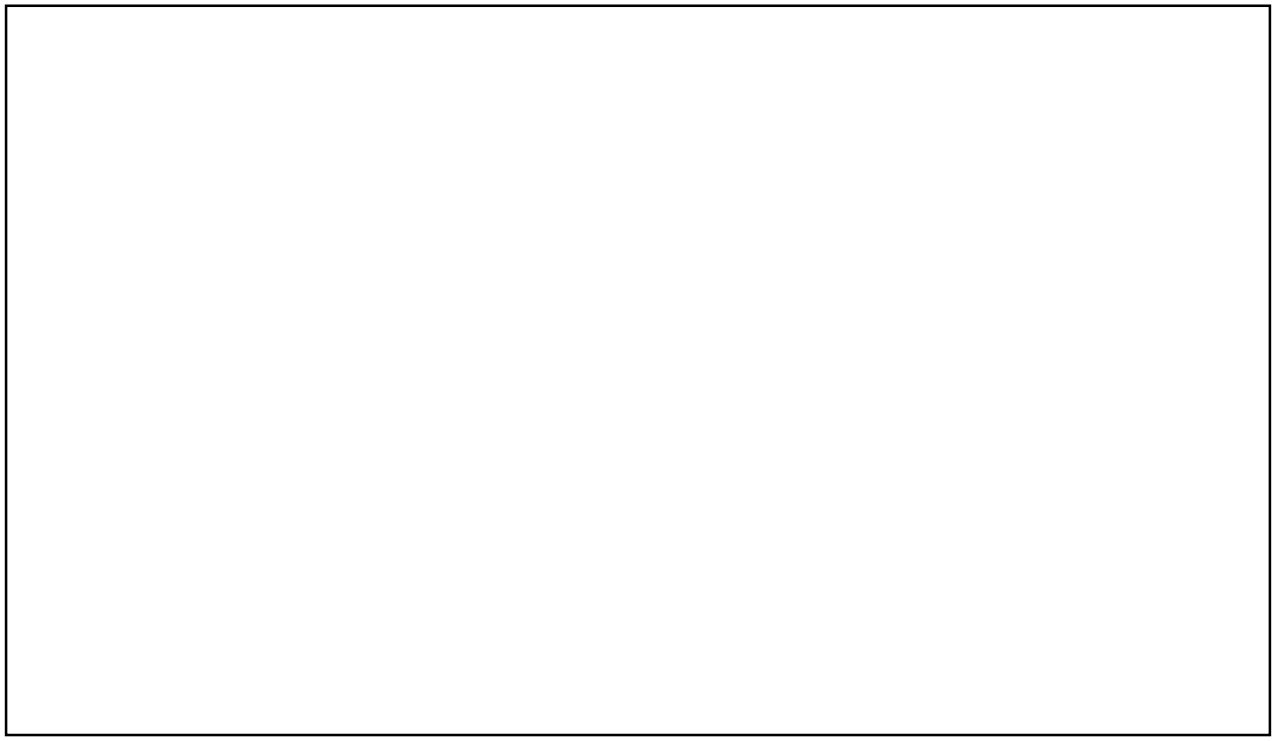
Task 2

How many weeks does it take Kevin to save for a pair of football boots which cost €104?

Show as many methods as you can.

Way 1

Way 2



Way 3