Lesson Research Proposal for 2nd Years - Indices

For the lesson on 17/01/2018
At Loreto Community School, Milford - Ms. Morrow's Class
Instructor: Margaret Bonner
Lesson plan developed by: M. Bonner, A. Morrow & H. Ferry

1. Title of the Lesson: Partying with Indices

2. Brief description of the lesson
Developing an understanding of Law 1 of indices and emphasise that this is a more efficient method to finding solutions to problems (involving repeated multiplication).

3. Research Theme
At Loreto Community School we want students who will:
a) Reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning.
b) Demonstrate the knowledge, skills and understanding required by post primary curriculum.
As mathematics teachers we will actively support the achievement of these goals by paying attention to:
a) Selecting and using teaching approaches appropriate to the learning intention and the students' learning needs.
b) Working together to devise learning opportunities for students across and beyond the curriculum.

4. Background & Rationale

Why we chose our topic:
This lesson is aimed at 2nd year Higher Level students but we hope that it would be also be useful for 4th year Ordinary Level students. The teaching of indices is an important subject topic. That said we feel that there is an over use of calculators and students fail to develop their understanding of the laws of indices when applied in algebra.

Our Research Findings:
When teaching indices within our Mathematics department to first year students, the students are often encouraged to check their answers expressed in index form using a calculator. However as examiners of SEC exams, mock papers and class tests, it is commonly recognized that when tackling problems algebraically, students experience difficulty in applying the laws of indices. We feel that many students substitute values for the variable which allows them to use a calculator but not necessarily solve the problem posed. At this point, they find the use of the calculator is insufficient in aiding problem solving within algebra.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Primary Level of Understanding: Identify and explore square numbers, ie.16 = 4 x 4 = 4^2 -explore and identify simple square roots -record and relate to square numbers -identify common factors and multiples -explore and record factors and multiples to identify common factors and multiples -write whole numbers in exponential form 1000 = 10 x 10 x 10 = 10^3 8 = 2 x 2 x 2 = 2^3 *Common Introductory Course* -Investigate models such as decomposition to make sense of the operations of addition, subtraction, multiplication, and division in **N** where the answer is in **N**, including the inverse operations − Investigate the properties of arithmetic: commutative, associative and distributive laws and the relationships between them − Consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value − Analyse solution strategies to problems − begin to look at the idea of mathematical proof −Consolidate their understanding and their learning of factors, multiples and prime numbers in **N** −Check a result by considering whether it is of the right order of magnitude − Check a result by working the problem backwards	*Junior Certificate Level of Understanding :* − **use and apply rules for indices (where *a*, *b* □ R, *a*, *b* ≠ *0*;** ***p*, *q* □ Q; a ^{pp}, a^q, □ R; complex numbers not included):** • $a^p a^q = a^{p+q}$ • $a^p/a^q = a^{p-q}$ • $a^0 = 1$ • $(a^p)^q = a^{pq}$	− use and apply the rules for indices (where a ∈ Z, a ≠ 0; p, q ∈ N): **− use and apply rules for indices (where a, b □ R, a, b ≠ 0; p, q □ Q; a^p, a^q, □ R; complex numbers not included):** • $a^{1/q} = \sqrt[q]{a}$, , q □ Z, q ≠ O, a > O • $a^{p/q} = \sqrt[q]{(a^p)} = (\sqrt[q]{a})^p$ p, q □ Z, q ≠ O, a > O • $a^{-p} = 1/(a^p)$ • $(ab)^p = a^p \times b^p$ • $(a/b)^p = a^p/b^p$ − **operate on the set of irrational numbers R \ Q** − use the notation a½, a ∈ **N** − express rational numbers ≥1 in the approximate form a x10n, where a is in decimal form correct to a specified number of places and where n = 0 or n ∈ N − express non-zero positive rational numbers in the approximate form a x10n, where n ∈ Z and 1 ≤ a < 10 − compute reciprocals -multiply expressions of the form: (ax+b)(cx+d) **$(ax+b)(cx^2+dx+e)$ where a,b,c,d,e ∈ Z**

− Justify approximations and estimates of calculations *Junior Certificate Level of Understanding :* -Appreciate the order of operations, including the use of brackets		**-**divide expressions of the form: $(ax^2+bx+c) / (dx+e)$ **$(ax^3+bx^2+cx+d) / (ex+f)$ where a,b,c,d,e, f∈ Z** **Leaving Cert syllabus** – solve contextual problems involving numbers represented in the following ways: $\sqrt{a}$, $a^{1/2}$, a^2, a^3, $1/a$

6. Goals of the Unit

- Students will understand that Natural numbers can be written in various forms: factors, fractions, decimals and product of primes.
- Students will recognise that the operation of repeated addition can be replaced with multiplication. In the same manner students will discover that that the solutions to repeated multiplication can be written in index form.
- Students can apply their prior knowledge of square numbers with the intention to extend to higher exponential values.
- Students will understand that situations involving very large/small numbers expressed in index form, e.g. powers of 10.
- Students will be able to apply the skills developed in the Research lesson in areas such as scientific notation, multiplication and division in algebra, and solving equations in index form.

7. The Unit Plan

Lesson	Learning goal(s) and tasks
1 The Research Lesson	Introduce indices in a problem solving context: ● Utilizing a pattern solving approach to gain a solution to the problem. This solution can be written in index forms which will emphasise the usefulness of indices when predicting solutions for larger outcomes.
2 & 3	● Formalising Law 1 ($a^p \times a^q = a^{p+q}$) ● Investigating: Law 2 ($a^p / a^q = a^{p-q}$) Law 3 $(a^p)^q = a^{pq}$ Law 4 $(a^o) = 1$ ● Using numbers and formalizing these laws using algebra as the base.
4	● Investigating negative indices using the division law (Law 2). ● Investigating a quotient to a power and product to a power using the multiplication law (Law 1) and the division law (Law 2).
5	● Using all of the above laws in developing the fractional law (Law 6 & 7).
6 & 7	● Solving equations involving indices
8	● Introduction to surds and how to add, subtract, multiply and divide. ● Writing numbers in surd form. ● Numbers in standard form/Scientific notation

9	• Use the above laws to solve problems in scientific notation.

8. Goals of the Research Lesson:

a) Mathematical Goals

Students will:

- Understand that any number can be expressed in various forms. In particular writing the answer in index form, the students can use this information to aid their problem solving process.
- Identify that finding patterns within the problem can save time and lead to a quicker/multiple solution(s).
- Understand that indices is repeated multiplication and the significance of the laws of indices.

b) Key Skills & Statements of Learning

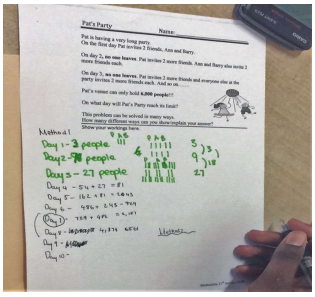
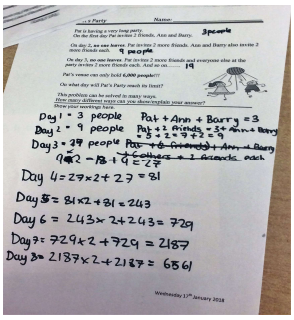
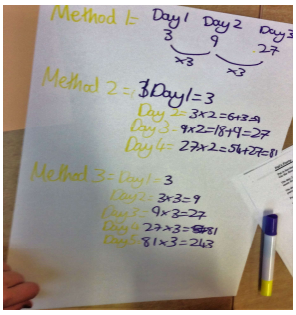
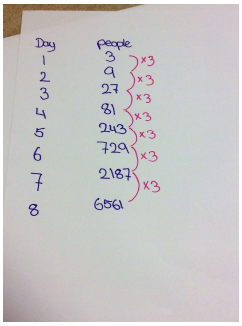
Throughout this lesson students will be encouraged to:

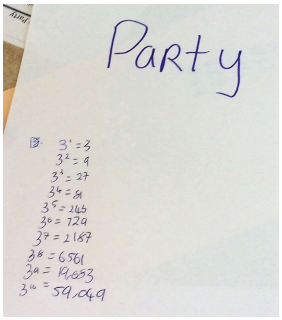
- Communicate effectively using a variety of means in a range of contexts in L1 (SL1)
- Recognize the potential uses of mathematical knowledge, skills and understanding in all areas of learning (SL15)
- Describe, illustrate, interpret, predict and explain patterns and relationships (SL16)
- Devise and evaluate strategies for investigating and solving problems using mathematical knowledge, reasoning and skills (SL17)
- Observe and evaluate empirical events and processes and draw valid deductions and conclusions (SL18)
- Take initiative, is innovative and develops entrepreneurial skills (SL 22)
- Express ideas clearly and accurately (KS 8)
- Make considered decisions (KS 1)
- Be confident and positive about learning (KS 2)
- Be curious and gather, record, organize and evaluate information and data (KS 3)
- Express ideas mathematically. (KS4)
- Estimate, predict and calculate problems (KS4).
- Develop a positive disposition towards investigation, reasoning and problem- solving (KS4).
- See patterns, trends and relationships (KS4).
- Gather interpret and represent data (KS4).
- Implement ideas and take action whilst learning creatively (KS5).
- Co-operate, respect difference and learn with others (KS6).
- Use language and numbers to express themselves whilst discussing and debating problems. (KS7)

9. Flow of the Research Lesson:

Time	Task	Resources	Teacher input	Expectations
10.30 **2 mins**	(Individually) How many ways can you write the number 36 using factors?	Markers A4 page	Set out expectations for behaviour, use of resources, timing and end of lesson. Pose question Time activity Observe work	1x36 2x2x9 2x18 2x2x3x3 3x12 4x9 6x6 and various sub factors of each.
10.32 3 mins	Sharing of students' answers to compile a comprehen - sive list.	Whiteboard Marker	Select students' answers and tease out prime factors $2x2 = 2^2$	
10.35 3mins 2mins	Pat's Party Problem (Individually) Establish the number of people at the party for the first 3 days.	Worksheet Marker	Read problem Ask for feedback from various students to make sure they take account of the number of people <u>still</u> at the party. Reinforce this to all. By way of comparing answers.	Day 1: 3 3^1 Day 2: 9 3^2 Day 3: 27 3^3 ---------- Day 1: 3 Day 2: 6 Day 3: 12 *doesn't include those already at the party*

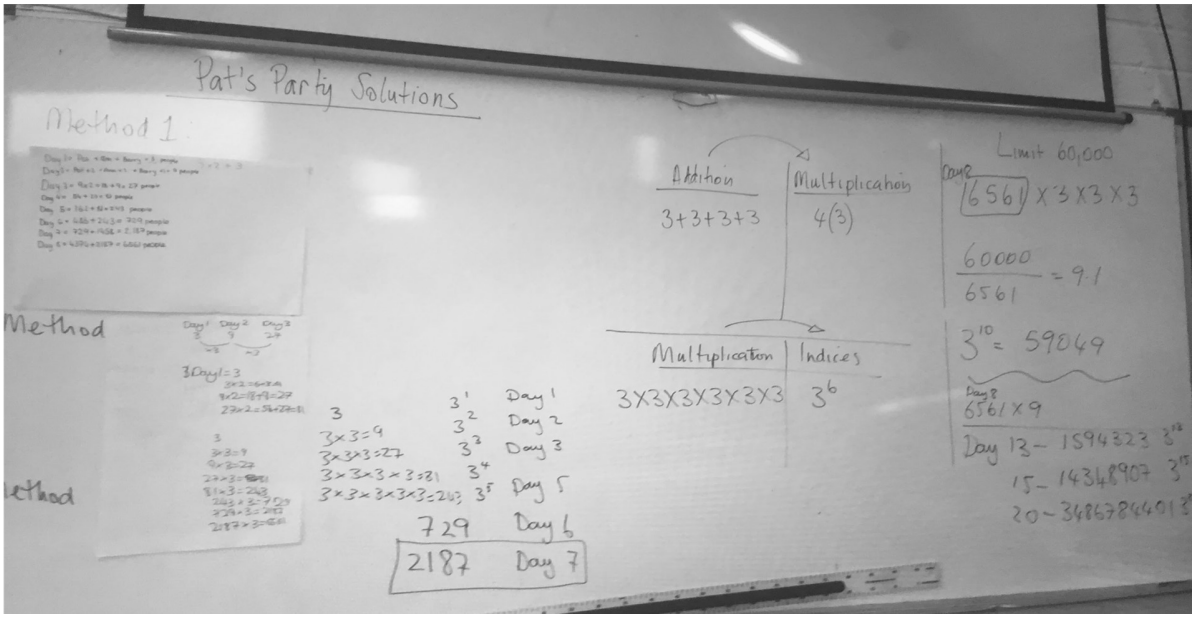
10.40 10mins	Pat's Party Problem (Groups of 3) Complete the problem. How many days will the party last if the limit is 6,000?	Worksheet Marker A3 Calculators	Observation Reinforce the requirement to <u>show your work in various ways</u>. Be able to explain your answers / procedures on A3 paper	

10.50 7mins	**<u>Ceardaíocht /Comparing and Discussing</u>** Feedback from students	A3 pages with student work Blutack Whiteboard	Select various methodologies Display on whiteboard	*Methods used* Addition   Multiplication  

				Indices  Day 4 – 81 Day 5 - 243 Day 6 – 729 Day 7 – 2187 (Day 8 – 6561) Exceeds the limit
10.57 3mins	Extension task How many more days can the party last if the limit is now 60,000?	Problem 2 handout Question only	Using <u>two</u> of the above methods answer problem 2.	Multiplication Indices $3^{10} = 59049$ ð Party will last another 3 days. ($3^{11} = 177147$)
11.00 5mins	**<u>Ceardaíocht /Comparing and Discussing</u>** Feedback from students	Write on the whiteboard.	Tease out that: Multiplication is repeated addition. Indices is repeated multiplication. (See Board Plan)	

11.05 5mins	**<u>Summing Up & Reflection</u>** How many will be at the party on Day 13 Day 15 Day 20		Highlight the following $3^{10} \times 3^3 = 3^{13}$ 3x3x3x3x3x3x3 … $=3^{13}$ $3^{15} = 3^{10} \times 3^5$ $= 3^{13} \times 3^2$ Algebra; $a^4 x a^3 = a^7$ $a^p \times na^q = a^{p+q}$ LAW 1	$3^{12} =$ 531,441 $3^{13} =$ 1,594,323 $3^{14} =$ 4,782,969 $3^{15} =$ 14,348,907 $3^{16} =$ 43,046,721 $3^{17} =$ 129,140,163 $3^{18} =$ 387,420,489 $3^{19} =$1,162,261,467 $3^{20} =$3,486,784,401 Or 59049x3x3x3 1594323x3x3 14348907x3x3x3x3x3
11.10 2mins	Student reflection Today I learned that ……..…. Today I learned from my peers that ……. Today I'm unsure about ……. The questions I still have are ……..	Original Worksheet (evaluation space included)	Collect in worksheet for evaluation of the lesson and for feedback from students.	

10. Board Plan



11. Evaluation

Goals

- Students have an appreciation for the use of indices as a useful way to represent number in some circumstances.
- They are aware that a problem can be solved using various approaches.
- Students understand that multiplication is repeated addition, indices is repeated multiplication.

The above is evidenced by way of the students' rough work and students' reflections.

- Formalising of Law 1 did not occur but the foundations were laid for easy development of Law 1 and Law 2 in the next lesson, this was due to excess time being used to tease out the writing of answers in index form.

Timing of lesson changes

- Prior Knowledge and Posing the Task: 10 mins → 10 mins
- Students working on the Problem: 10 mins → 15 mins
- Presentation of Solutions and Ceardaíocht: 7 mins → 15 mins
- Extension task: 3 mins → 5 mins
- Presentation of Solutions and Ceardaíocht: 5 mins → 10 mins
- Summing up and Reflection: 7 mins → 5 mins

Misconceptions/Misunderstanding

- Some students forgot to add the people who were already at the party. This however was anticipated and strategy adopted to deal with it.
- Some students attempted the solution by way of getting the nth term. Identifying the pattern as quadratic using the first three terms. Not understanding that they would require more terms to draw such a conclusion.
- The wording of the problem asks students to find more than one way to find the solution. However it is the answer itself that required multiple formats rather then the procedure.

Student Thinking

- All students engaged in individual think time before sharing their ideas with their peers.
- Some students engaged in meaningful analysis of their work and that of their peers through peer collaboration, asking questions from each other and sharing ideas.
- Students had to explain their answers and were encouraged to think of another way to write their answers.

Student Motivation

- All students were keen and able to attempt the problem.
- The use of the individual time to come up with solutions to day 1,2 & 3 helped motivate the students to pursue the problem further as they had confirmation that they had understood the problem. This also allowed other students (those who had made errors or who has misconceived the problem) to get back on track to continue to investigate the problem.

Key Skills

The following key skills were used;

- Express ideas clearly and accurately (KS 8)
- Make considered decisions (KS1)
- Be confident and positive about learning (KS2)
- Be curious and gather, record, organize and evaluate information and data (KS3)
- Express ideas mathematically. (KS4)
- Estimate, predict and calculate problems (KS4).
- Develop a positive disposition towards investigation, reasoning and problem- solving (KS4).
- See patterns, trends and relationships (KS4).
- Gather interpret and represent data (KS4).
- Implement ideas and take action whilst learning creatively (KS5).
- Co-operate, respect difference and learn with others (KS6).
- Use language and numbers to express themselves whilst discussing and debating problems. (KS7)

12. Reflection

During the research lesson, it was envisaged that students, through problem-solving of a patterns question, would formalise Law 1 of indices. Due to the unforeseen amount of time spent on teasing out answers so that they could be written in index form, the aim was not fully realised. However it was observed that the stages of student learning was somewhat richer than anticipated, due to the individual work and thinking, collaboration and their full engagement in the process. The journey of learning saw students realise that:

- repeated addition can be replaced by multiplication
- solutions to repeated multiplication can be written in index notation
- "see patterns, trends and relationships (KS4)" within the problem where answers can be found more efficiently if using index notation
- "gather, interpret and represent data (KS4)" in index notation

It is felt that the research lesson has led students to solving this problem in index notation without relying on the safety net of the calculator.

The research lesson has laid the foundations for the following:

- Laws of Indices
- Solving equations in index form
- Multiplying in algebra
- Long division in algebra

……