Lessons Research Proposal for 2nd Year Higher Level Maths

Year Group: 2nd Year

Level: Higher Level

Topic: Geometry & Patterns

For the lesson on: 31/01/'18 at Coláiste Chill Mhantáin,

Instructor: Mary Kilgallen

Lesson plan developed by: Mary Kilgallen, Mariosa O' Callaghan & Natasha Smyth

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1. Title of Lesson: Patterns in Polygons

2. Brief Description: Enable students to use their prior knowledge of polygons & linear patterns to solve problems.

3. Research Theme:

At Coláiste Chill Mhantáin, we want our students to:

- Enjoy their learning, to be motivated to learn, to make progress and achieve as learners.
- Reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning.
- Attain the stated learning outcomes as set out in the Junior Certificate curriculum.

As mathematics teachers, we will actively support the achievement of these goals in the following ways:

- Engage students through selecting problem solving questions which are relevant to their lives.
- Linking in prior knowledge as a means to progress with the question and get a sense of achievement.
- Use approaches to match the learning intentions of the lesson and to match the learning needs of the student.
- Reflect and evaluate their learning by asking questions, giving supportive comments and advice and by giving students time to think about the work they have completed and the work other students have completed in class.

4. Background & Rationale

a) Why we chose the topic:

We identified a problem school wide with students failing to identify patterns in complex or wider/lateral/practical contexts (questions) in exams. Students may feel that patterns only exist in specific numerical questions e.g., [1, 5, 9, 13,] and not be evident in geometrical shapes, coordinate geometry or profit and loss questions. Students don't take the time to reflect, describe and analyse their attempts. We are attempting to build on their prior knowledge from primary school and the common introductory course to extend students' learning to a deeper level. To equip students with the necessary knowledge and skills to identify, analyse a pattern leading to formulating into new knowledge e.g. Tn. Following discussion we decided that there were wide differences in ability in spotting a pattern. This requires higher order thinking where many students struggle.

b) Your research findings:

- Text & Tests 2 and Connect with Maths 2.
- Exam Questions from past papers.
- Board work.
- Recent findings from PISA.
- Syllabus.
- Worksheet.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes:	Learning outcomes for this unit:	Related later learning outcomes:
Primary School Curriculum *Junior & Senior Infants* Algebra Extending Patterns – students should be able to identify, copy and extend patterns in terms of shape and size. Shape & Space 3D Shapes – students should be able to sort 3D shapes, regular and irregular. Students should also be able to sort, describe and name 3D shapes: cube,	Students will be able to: • Understand the characteristics of triangles and use them to solve problems. • Use patterns to continue the linear sequence • Generalise and explain patterns and relationships in algebraic form. • Recognise that properties of geometrical shapes	Students will be able to: • Generalise and explain patterns and relationships in words and numbers. • Investigate and understand the properties of different quadrilaterals.

cuboid, sphere and cylinder. 2D Shapes – students should be able to sort and name 2D shapes: square, circle, triangle and rectangle. Students should be able to solve problems involving shape. *1st & 2nd Class* Algebra The students should be able to recognize pattern and predict subsequent numbers. Shape & Space The students should be able to explore, discuss, develop and use the vocabulary of spatial relations. The students should be able to construct and draw 2D shapes. The students should be able to explore and recognize angles in the environment. *3rd & 4th Class* Algebra The students should be able to recognize and record patterns in number. The students should be able to explore, extend and describe patterns. Shape & Space	can produce a pattern.	

The students should be able to identify, describe and classify 2D shapes: square, rectangle, triangle, hexagon, parallelograms, pentagon, octagon, rhombus, circle, semicircle and irregular shapes. The students should be able to explore, describe and compare the properties (sides, angles, parallel and non-parallel lines) of 2D shapes. The students should be able to identify line symmetry in the environment. The student should be able to identify, describe and classify vertical, horizontal and parallel lines. The students should be able to draw, discuss and describe intersecting lines and their angles. The students should be able to solve problems involving lines and angles. *5th & 6th Class* Algebra The students should be able to explore the concept of a variable in the context of simple patterns, tables and simple formulae. The students should be able to use angle and line properties to classify and describe triangles and quadrilaterals.		

Shape & Space The students should be able to recognise, classify and describe angles and relate angles to shape and the environment. The students should be able to estimate, measure and construct angles in degrees. The students should be able to explore the sum of the angles in a triangle. *1st Year Introductory Course* Introduction to geometry The students should be able to estimate angles in degrees, naming angles. The students should be able to investigate the measurement of angles using a protractor. The students should be able to investigate and understand a straight angle has 180^{o} and vertically opposite angles are equal in measure. Patterns & Algebra The students should be able to use tables, graphs, diagrams and manipulatives to represent and analyse patterns and understand the concepts of variables and constants. The students should be able to generalise and explain patterns and relationships in words and numbers.		

The students should be able to write arithmetic expressions for particular terms in a sequence, linear only.		

6. Goals of the Unit

1. Students will apply their prior knowledge of angles and patterns to solve problems and tasks.
2. Students will be able to generalise and explain patterns and relationships in words and numbers.
3. Students will understand the procedures involved and the reasons for the procedures for each operation.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	The Plane - Points , Lines, Angles, Straight Angle, Vertically Opposite Angles (lines forming Angles)
2	Triangles- Types, Sum of Angles and Exterior Angle.(Lines forming Shapes)
3	Enclosed spaces -What is a polygon? - Types of polygon.
4	Concept- What is a line? Can it represent anything else apart from shapes? Leading to the idea of sequences. Shape & Number Sequences-forming patterns by tabulating and graphing-leading to a linear pattern.
5	Linking linear to the constant rate of change and slope of a line.
6	Overview of Lessons 1 – 5
7.	Research Lesson

8. Goals of the Research Lesson:

Mathematical Goals:
Students will:

- Understand the characteristics of triangles and use them to solve problems.
- Use patterns to continue the linear sequence
- Generalise and explain patterns and relationships in algebraic form.
- Recognise that properties of geometrical shapes can produce a pattern

Key skills and statements of learning:

- Being Numerate: By engaging in suitable tasks, students will have the opportunity to develop a positive attitude towards investigating, reasoning and problem solving.
- Being Literate: Expressing ideas clearly and accurately.
- Managing myself: Student will be given a chance to reflect on their own learning by being given time to fill in a reflection sheet at the end of the lesson.
- Communicating: Through going through the solutions students will be given the chance to discuss and debate their answers
- Staying well: By engaging in tasks which are appropriate to their ability, students confidence and positive disposition to learning will be promoted.
- Working with others: Through discussing the different approaches and listening to other students work students will be able to learn from each other.
- Being Creative: Students will be given the opportunity to explore options and alternative approaches to their work
- Managing information and thinking: Students will be encouraged to think creatively and critically.

Statements of learning:
The lesson is also designed to meet the following Junior Certificate Statements of learning in particular:

- Statement 1: The student communicates effectively using a variety of means in a range of contexts.
- Statement 15: The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
- Statement 16: The student describes, illustrates, interprets, predicts and explains patterns are relationships.
- Statement 17: The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
1. Introduction (Recap Prior Knowledge - 10 mins) Using all the information that we have learned to date what can you tell me about each of the following? • Straight angle • Full angle • Sum of the interior angles of a 3 sided regular polygon. • Sum of the exterior angles of a 3 sided regular polygon. • 2 other ways of finding the sum of the interior angles in a 3 sided regular polygon. • Linear Pattern.	- Teacher displays images to assist with the prior knowledge.	Questioning students' knowledge. Correcting misconceptions . Teasing out correct facts.
Posing the Task (10 mins) The task today will involve understanding and making use of all information that we have learned to date. All students are given the worksheet below with the problem. The table below shows some information about regular polygons. These are shapes where all of the angles are the same size. • Teacher poses the task and hands out the worksheet (below) to all the students. • When worksheet is handed out the teacher will read the problem aloud and will clarify any misconceptions. <u>Worksheet</u>	- The question being posed will be on a worksheet for the students to complete.	Teacher walks around the classroom and ensures the students are kept on task.

The table below shows some information about regular polygons.

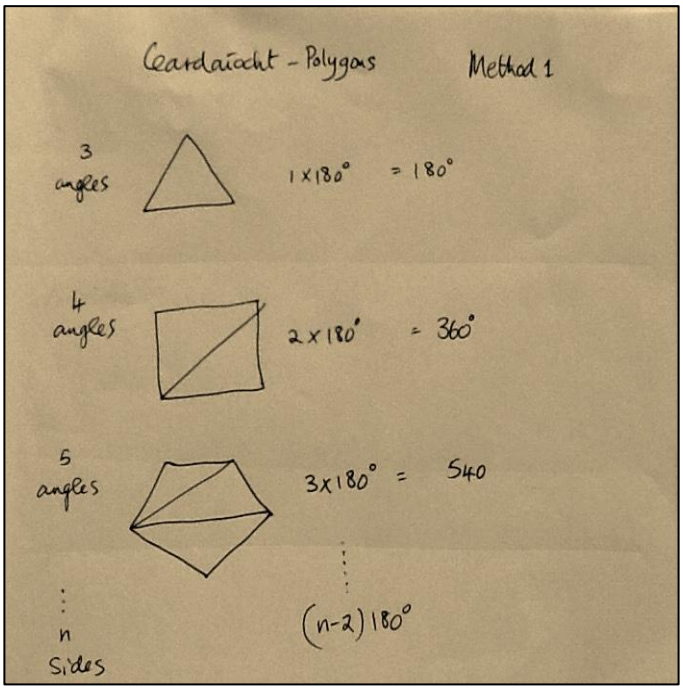
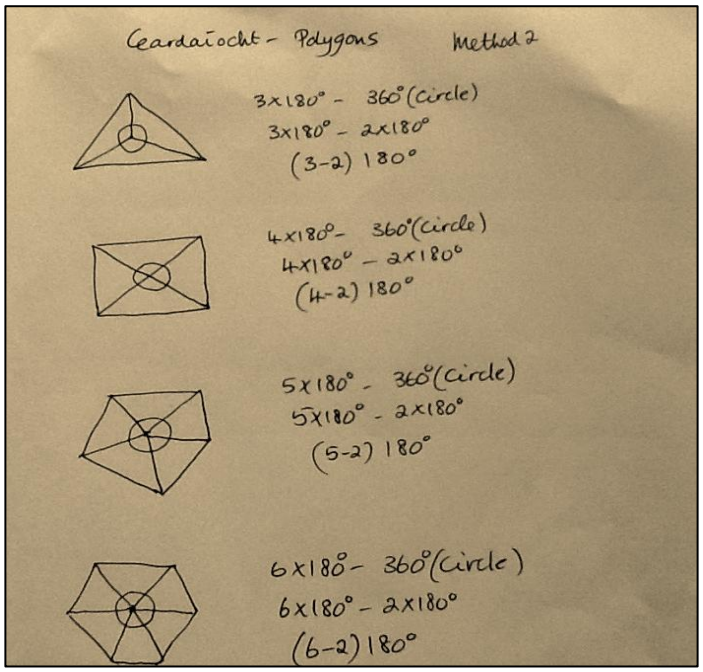
These are shapes where all of the angles are the same size.

Number of angles in the polygon	Shape (diagram)	Sum of the interior angles	Pattern	Pattern
3		180°		
4		360°		
5				
6				
7				
8				
. . . n				

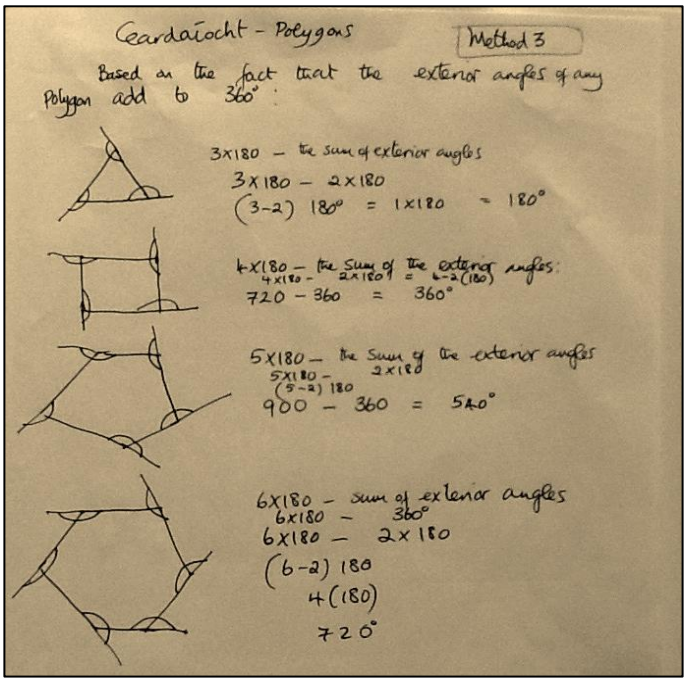
(a) The sum of the angles increases in a linear pattern.
Complete the columns in the table above showing the sum of the interior angles in each of these shapes.

(b) In as many ways as you can, find a formula for the sum of the angles in a regular polygon with **n** angles.

(Remember that these values follow a linear pattern.)

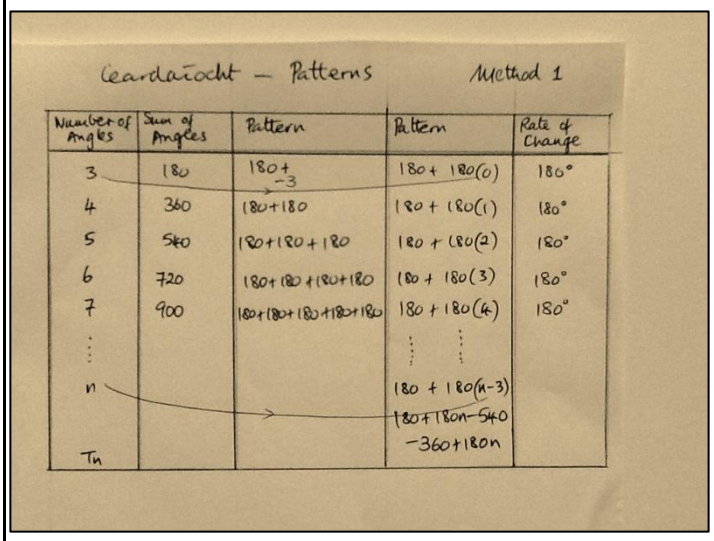
Anticipated Responses: a) <u>Method 1</u>  Method 2  	Teacher will walk around the room and take note of the different methods being completed by the students. Teacher will invite students to the board to explain the different solutions they each discovered. Teacher will invite students up if there is a common misconception.	Teacher will question the students on the solution they each came up with.

Method 3

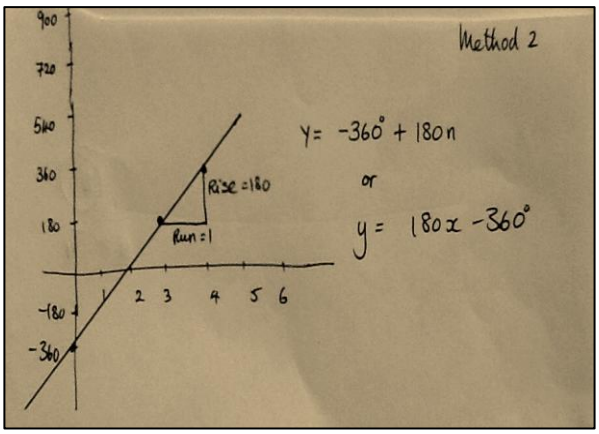


b)

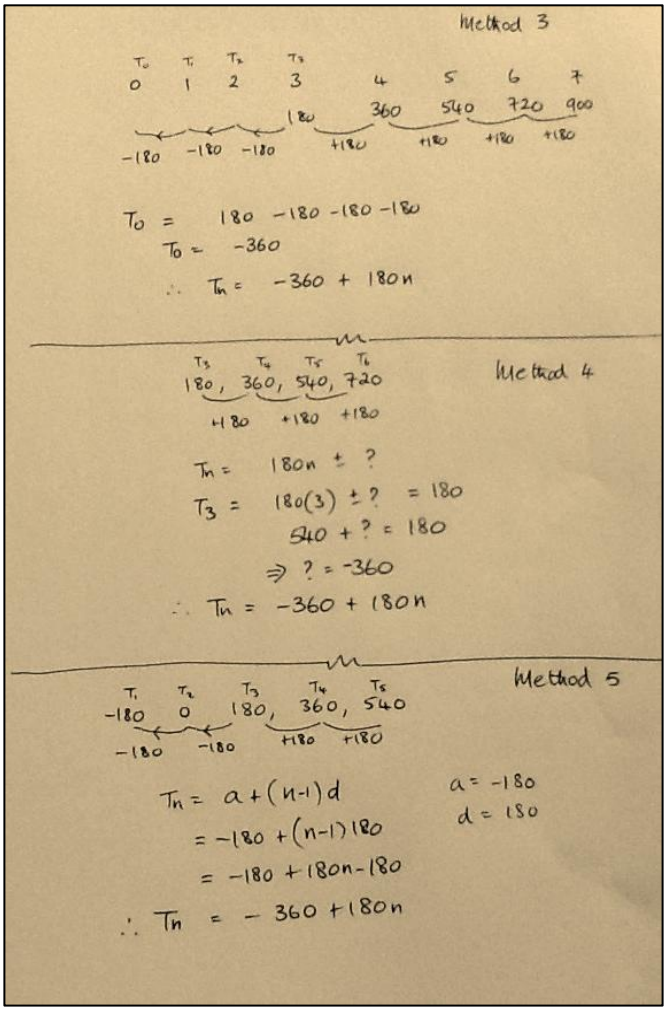
Method 1

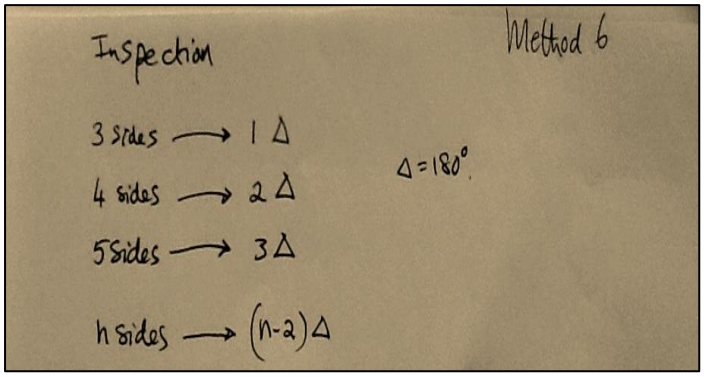


Method 2



Method 3, 4 & 5

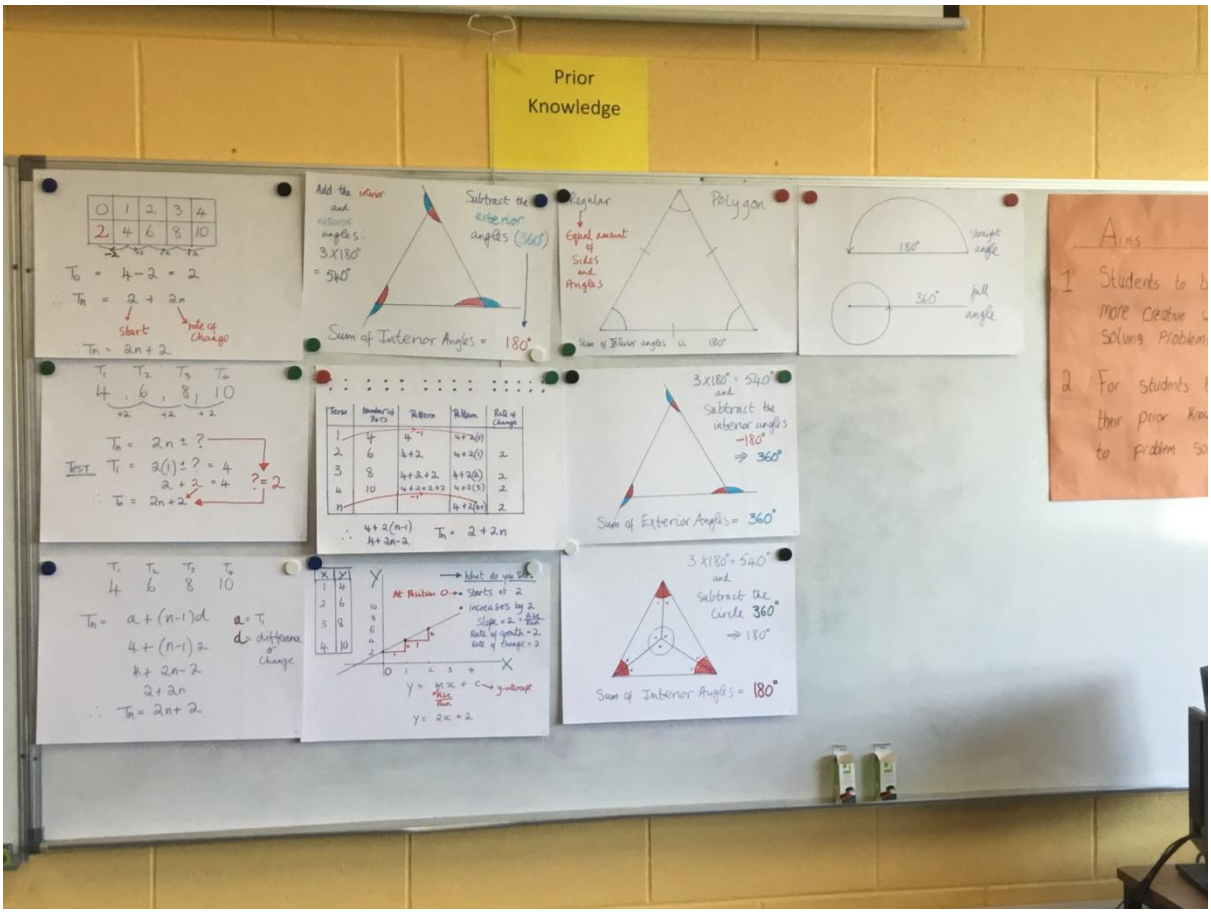


Method 6  		
Ceardaíocht /Comparing and Discussing (15mins) As a result of walking around the room and taking note of the students work, we will now ask students to come up to the board and show their different methods to the class.	Bring students to the board starting with the most basic approach to the most advanced approach. Ensure to summarise each solution once the individual student has presented their work. Question the other students constantly while doing this. Guide students through the solutions that they may not have got.	

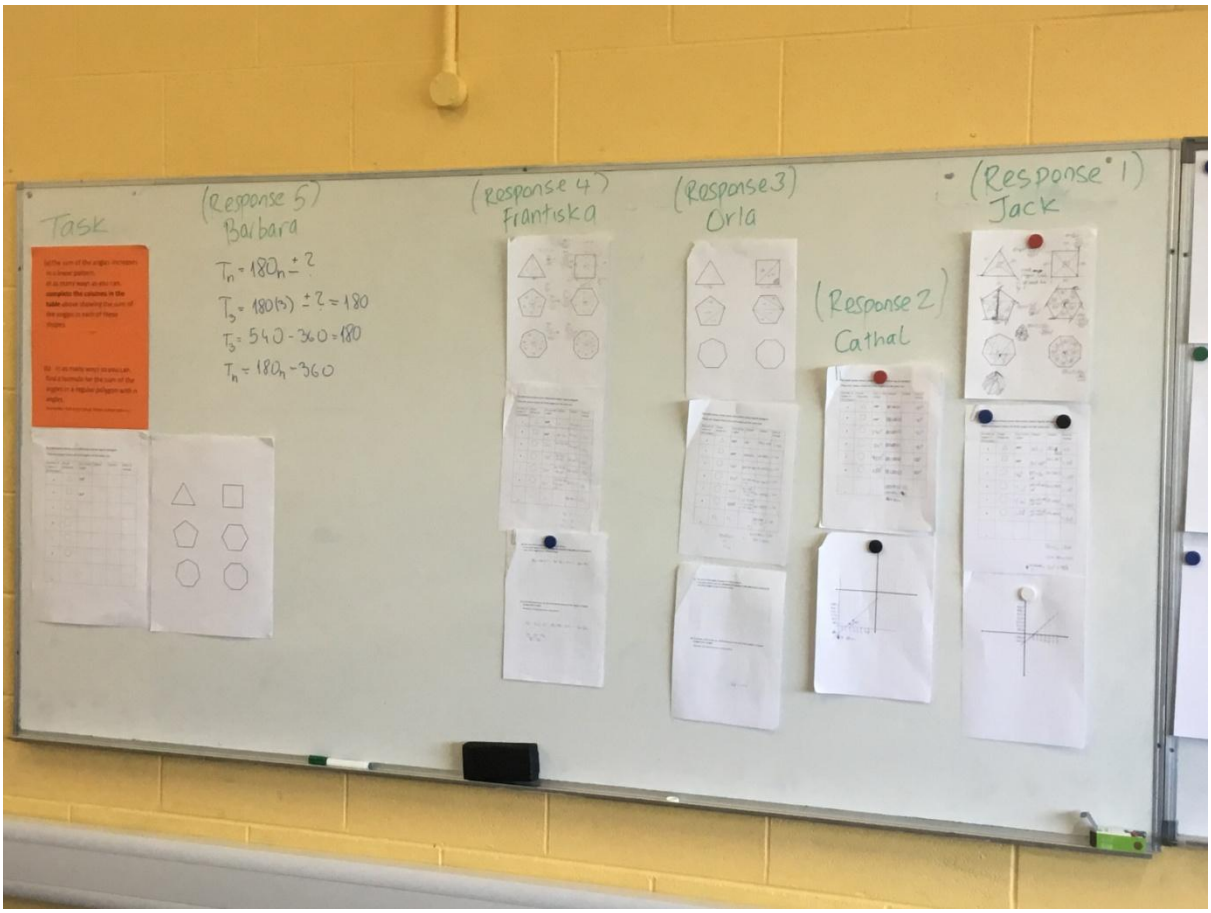
Summing up & Reflection (5mins) The teacher will recap the lesson by: - Discussing the different types of polygons. - Deriving the sum of the angles in each polygon. - Identifying a pattern to match the sum of the angles in the polygon - Describe the type of pattern to be linear which means constant rate of change which can also mean rise/run. **<u>Homework Q:</u>** As n goes to infinity can a polygon ever become a circle?	Ensure to reinforce the important aspects of today's class.	

10. Board Plan

Prior Knowledge



Students Responses



11. Evaluation

a) Research Theme

It was broadly felt that the research theme was met during the teaching of this research lesson. In particular we were delighted to observe motivated and determined students progressing through the lesson and achieving with various degrees of success the challenge at hand. The students were thoroughly engaged during the presentation and explanation of student work during Ceardaiocht.

The students had a firm understanding of the question which was aided by the prior knowledge prompting at the start of the lesson. This contributed to the students' confidence and success throughout the lesson.

b) Goals of the Lesson

The goals of the lesson were partially achieved. Almost all students understood the characteristics of triangles and applied them in most cases successfully to develop a pattern within the polygons.

The numerical table was completed successfully by almost all students indicating that they could observe, identify and extend the pattern. Most students attempted to plot the linear pattern but a number started at the origin n=0 instead of n=3. The Ceardaíocht developed this misconception in an eloquent way that met with widespread acceptance.

c) Data Collection

Evidence of students work was collected at the end of the lesson along with an individual reflection on what the students had learnt and found most beneficial. The Lesson Note App was used to record the breakdown of the hour long lesson and to record photographic evidence of student work. The team were most familiar with the research theme and goals of the lesson and were on the lookout throughout for evidence to support.

12. Reflection

a) *What the team had hoped to observe during the lesson*

The team at Colaiste Chill Mhantain hoped to observe students enjoying, achieving and reflecting on their mathematical learning during their research lesson.

b) *What was actually observed during the lesson, by the team members and others*

This was evident throughout the lesson. Students received the lesson positively, were actively engaged and made precise contributions and observations throughout the sixty minute lesson, allowing them to make progress as learners. This was evident by their enthusiastic participation and their positive comments in their personal reflections.

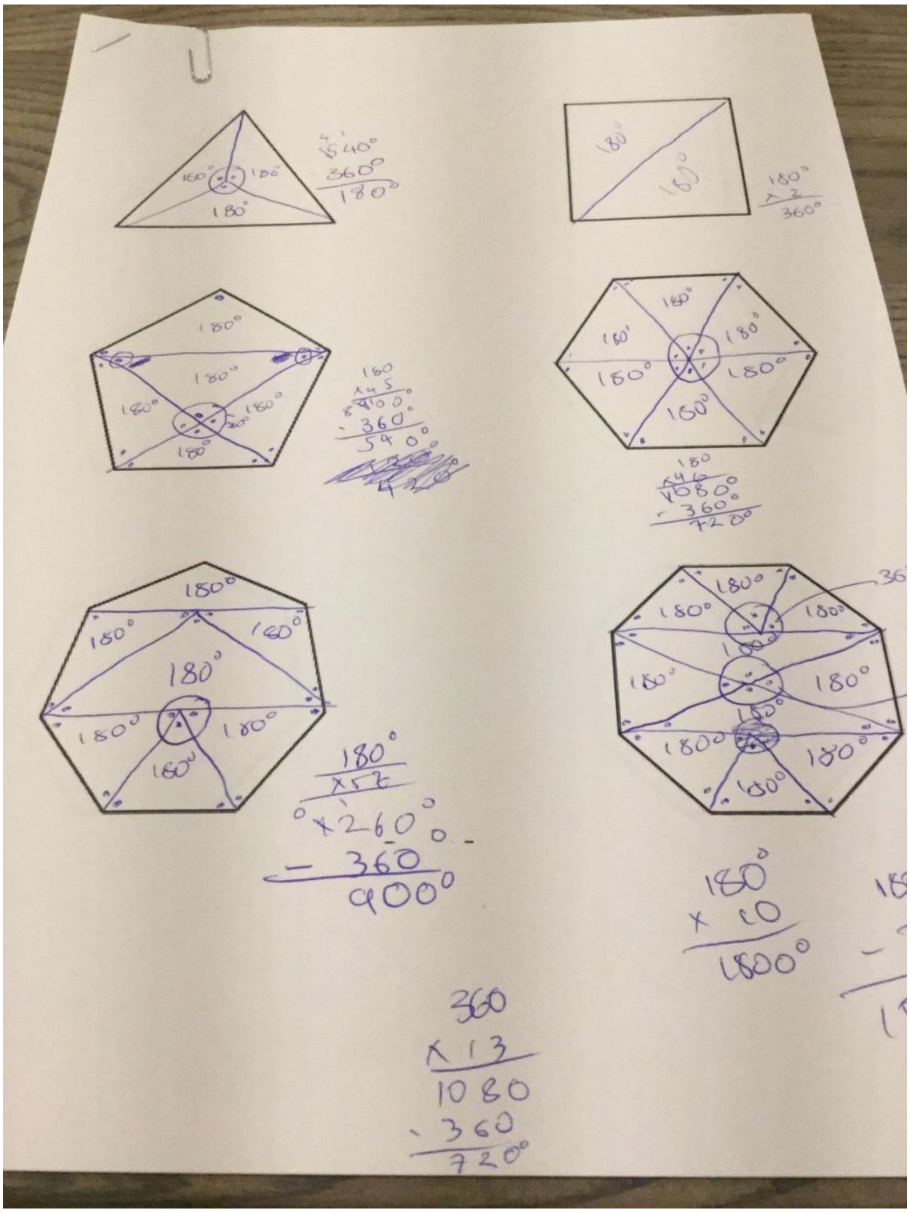
All students displayed a concrete understanding of the characteristics of triangles and attempted to apply this knowledge to determine the sum of the interior angles of larger polygons. Students examined their respective patterns and attempted to complete the table and derive a formula by numerous means for the general term of the pattern. The number of methodologies used, including graphs and tables, far exceeded the number of examples set aside and developed in our Ceardaíocht. This presented a vast resource of student work and also presented a manageable

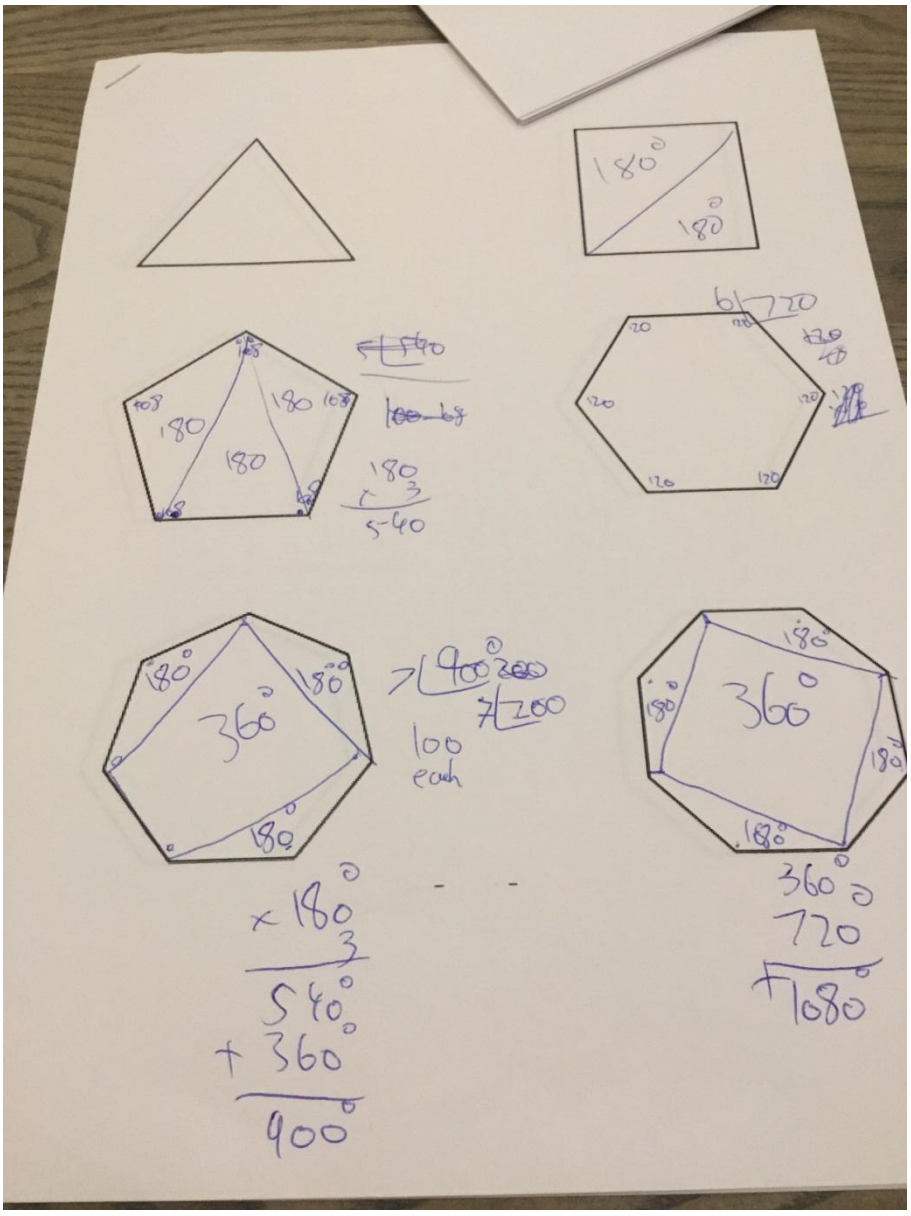
challenge for the teacher. However it put an additional strain on the most valuable of commodities, time.

c) *Major points raised during the post lesson discussion, and the team's own opinions/*

The team felt that the task firmly supported the goals of the lesson. Although some students had difficulty dividing polygons into triangles most had a firm understanding and proceeded with various degrees of success to achieving a general formula.

Photo 1 & 2 –example of correct and incorrect dividing of polygons



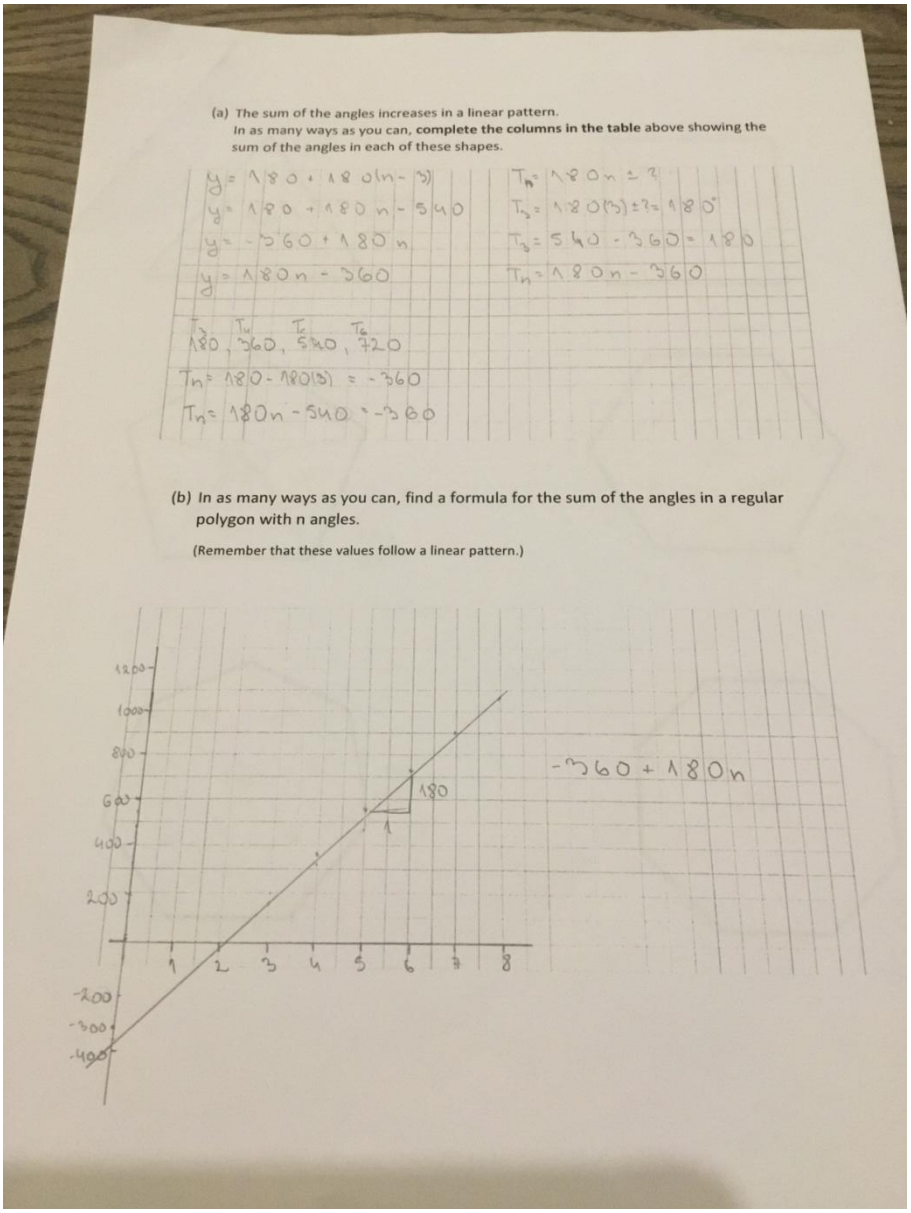


There are numerous methods for developing this formula which encouraged successful students to devise additional methods. This list is not exhaustive and allows for students to explore methodologies and concepts beyond the scope of the current Junior Cert Higher Level Syllabus. The open ended nature of the task accommodates higher order thinking and facilitates the exceeding expectations goals of the new Junior Cycle Syllabus.

The flow of the lesson from prior knowledge to posing the task and to Ceardaiocht, flowed coherently. During Ceardaíocht the students' presentation and answers to the teacher's questions promoted broad understanding in the classroom. A small number of students completed the table numerically and did not examine or divide the attached polygons correctly. The majority of students did however, by diving them into triangles or by marking a point at the centre and constructing equilateral triangles from there.

Almost all students were successful in continuing the table for the Sum of The Angles, many failed to further develop the correct pattern. This was manifest in a number of cases where students struggled to break up the polygons into triangles and thereby develop a formula (Tn = 180n – 360), and as a result failed to find the correct sum of the angles.

Photographs 3 & 4 showing successful and unsuccessful graphs



(a) The sum of the angles increases in a linear pattern.
In as many ways as you can, **complete the columns in the table** above showing the sum of the angles in each of these shapes.

$y = 180 + 180(n-3)$
$y = 180 + 180n - 540$
$y = -360 + 180n$
$y = 180n - 360$

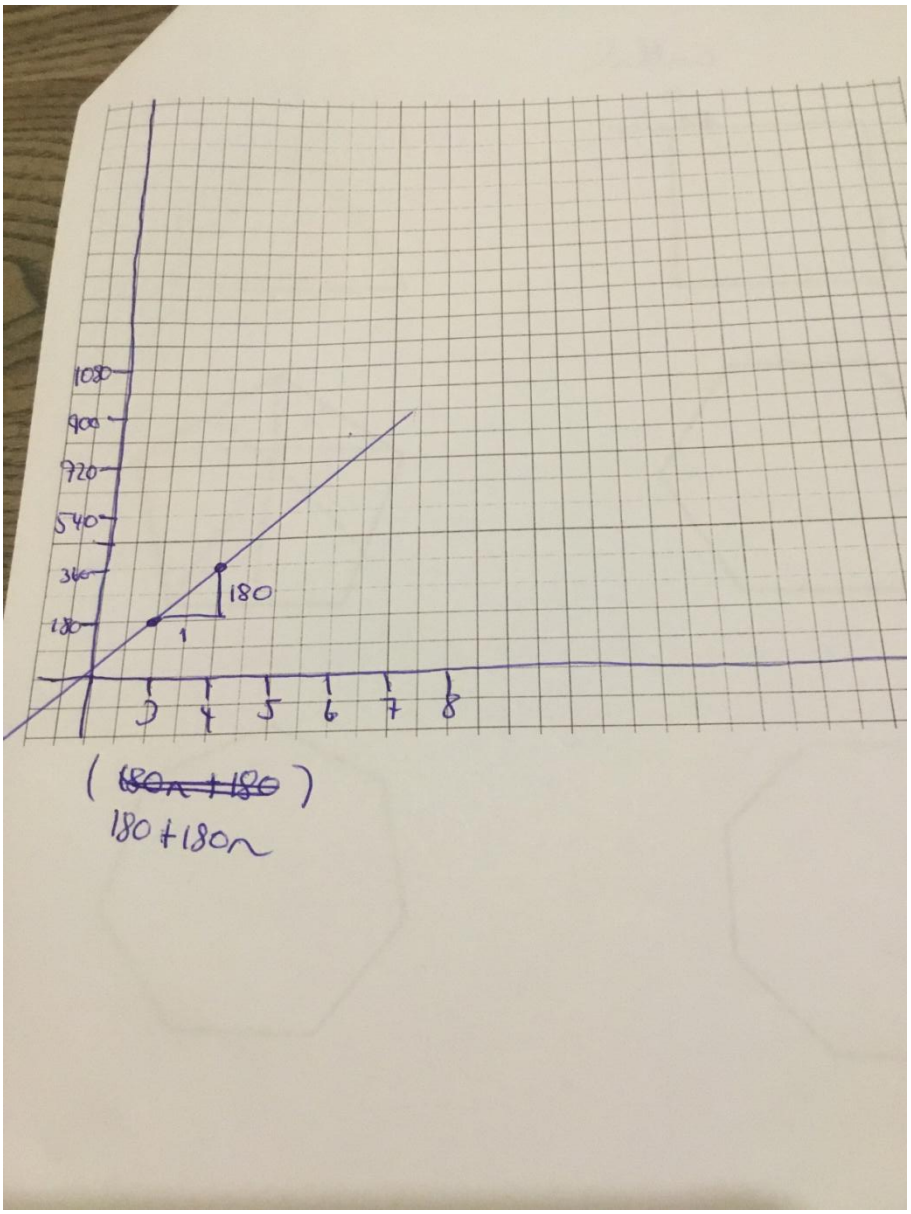
$T_n = 180n - ?$
$T_3 = 180(3) - ? = 180°$
$T_3 = 540 - 360 = 180$
$T_n = 180n - 360$

T_3 T_4 T_5 T_6
180, 360, 540, 720
$T_n = 180 - 180(3) = -360$
$T_n = 180n - 540 = -360$

(b) In as many ways as you can, find a formula for the sum of the angles in a regular polygon with n angles.

(Remember that these values follow a linear pattern.)

1200
1000
800
600
400
200
-200
-300
-400
1 2 3 4 5 6 7 8
180
1
$-360 + 180n$



One of the skills highlighted in the prior knowledge was to determine the sum of the three angles in a triangle by adding the interior and exterior angles less 360, which gave the sum of the interior angles. A number of students followed this procedure (Method 2) during the lesson but made mistakes with summing the interior angles in more complicated polygons.

Method 6, producing the pattern by inspection was not developed in Ceardaíocht due to time pressures. This is an eloquent and simplistic method that may have resonated with the students.

Many students opted to plot a linear graph to represent the pattern. These graphs all demonstrated the slope (rate of change) being 180 but some commenced at the origin for n=0 instead of for n=3.

d) Ideas for further study

Being part of the Lesson Study process has enabled participants to share creative ideas together with regard to anticipated student responses and difficulties. The Board Work allows for students to examine and absorb additional methods for solving the problem and determining Tn.

A natural extension would be to extend the pattern to the twelfth term namely a dodecahedron, and even posing the question as n tends to infinity can a circle be a polygon?