Lesson Research Proposal for 3rd Year Higher Level – **Paving Patterns**

For lesson on 22/1/2018
At Deele College Raphoe, 3M Class
Instructor: Steven Gibson
Lesson plan developed by: Steven Gibson, Marilyn O'Riordan, Shauna Kelly, Paddy Flood

1. Title of the Lesson: Paving Patterns.

2. Brief description of the lesson

By examining patterns in various ways we will attempt to assist students in understanding key elements of linear patterns and in deriving connections between linear patterns, linear equations, linear graphs and rates of change.

3. Research Theme

At Deele College, we aim to develop students who engage purposefully in meaningful learning activities.
Also, we want our teachers to select and use teaching approaches appropriate to the learning intention and students' learning needs.
As a Mathematics Department we will actively support the achievement of these standards in the following ways:

a) by promoting among students an understanding of concepts and the ability to explain the purpose of the learning tasks they are engaged in, with a view to extending and developing the activity meaningfully.
b) by encouraging students to be actively engaged in self-discovery learning.
c) by delivering highly effective instruction which is directed at eliciting deep student engagement through various methodologies including instructional leadership strategies.
d) by meaningfully differentiating content and activities to challenge all students.

4. Background and Rationale

Why we chose the topic

This lesson is aimed at third year students. Teachers in our Maths Department have found that, in dealing with new syllabuses, students perform well when applying basic techniques separately. For example, finding slopes of lines and exploring patterns are generally well done in distinct situations. However, seeing the connections between these ideas and applying them in unfamiliar contexts presents difficulties. Clearly it is important that students develop competence in basic procedures. The onus is on the teachers at all stages of developing topics to highlight connections between areas and their applications. This may be to the point of teaching topics simultaneously.
We also find that students often struggle to deal with problems in real-life contexts or translating English language into mathematical models. Extensive practice is needed in this area. Finally, simple revision of basic concepts and discussion and sharing of ideas are incorporated in this lesson.

Our research findings

As a Maths Department we find that, in the application of Bloom's Taxonomy, our students excel at Stage 1 (Recall) but have lower competence in Stage 2 (Understanding / Comprehension) and Stage 3 (Application). Our lesson proposal tries to develop creative situations to promote student understanding using Instructional Leadership strategies and structured problem-solving approaches. A key objective is to foster confidence in applying concepts in unfamiliar contexts. We plan to use suitable teaching aids. Central to this approach is allowing students time in the lesson to consider the problem and debate the proposed solutions.

5. Relationship of the Unit to the Syllabus

Related prior learning outcomes	Learning outcomes for this unit	Related later learning outcomes
In primary school students are exposed to identifying relationships from fifth class onwards. In sixth class they should be able to • record verbal and simple symbolic rules for number patterns. • translate number sentences with a frame into word sentences and vice versa. • translate word problems with a variable into number sentences. • explore the concept of a variable in the context of simple patterns, tables and formulae. In first year the students learn to • coordinate the Cartesian plane and plot points. • use tables to represent a repeating pattern situation. • generalise and explain patterns and relations. in words and numbers • write arithmetic expressions for particular terms in a sequence. • use tables, diagrams and graphs as tools for analysing relations. • develop and use their own mathematical	In third year, students should be able to • use tables to represent a repeating pattern situation. • generalise and explain patterns and relations in words and numbers. • write arithmetic expressions for particular terms in a sequence. • find the underlying formula written in words and algebraically from which data related to linear relations are derived. • show that relations have features which can be represented in a number of ways. • distinguish those features that are especially useful to identify and point out how those features appear in different representations: in tables, graphs and formulae. • discuss rate of change and consider how this relates to the context from which a relationship is derived, and identify how this can appear in a table, graph or formula. • interpret simple graphs	At senior cycle students will need to use their learning and build on these skills to do the following: • use tables, diagrams and graphs as tools for representing and analysing linear patterns and relationships. • appreciate that processes can generate sequences of numbers or objects. • investigate patterns among these sequences. • use patterns to continue sequences. • generalise and explain patterns and relationships in algebraic form. • demonstrate that relations have features that can be represented in a variety of ways. • recognise whether sequences are arithmetic, geometric or neither. • verify and justify formulae from number patterns. • investigate arithmetic sequences and series. • solve problems involving slopes of lines. • examine, apply and interpret the concept of rate of change in a

strategies and ideas and consider those of others. • present and interpret solutions, explaining and justifying methods, inferences and reasoning.	• engage with the concept of a function. • explore and interpret properties of lines, including slope and equation. • investigate relations of the form $y = mx + c$. • make connections between the shape of a graph and the story of a phenomenon. • describe both quantity and change of quantity on a graph. • evaluate expressions • solve first degree equations and inequalities. • explore patterns and formulate conjectures. • explain findings. • justify conclusions. • communicate findings verbally and in written form. • apply their knowledge and skills to solve problems in familiar and unfamiliar contexts. • analyse information presented verbally and translate it into mathematical form. • devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.	variety of ways, including calculus.

6. Goals of the Unit

a) Students will recognize various types of patterns and their fundamental characteristics.
b) Students will understand the basic properties of linear patterns.
c) Students will apply problem solving strategies to explore the problem numerically, algebraically and graphically.
d) Students will recognise the significance of the connections between various approaches.
e) Students will have the opportunity to apply prior knowledge of key elements of coordinate geometry such as slope and equation of a line in different contexts.
f) Students will be encouraged to extend these ideas to the concept of rate of change.

7. Unit Plan

Lesson	Learning Goals and Tasks
1	Exploration of various types of simple patterns using numerical and pictorial strategies based on real life contexts. Emphasis on terminology and notation associated with patterns.
2	Study linear patterns. Focus on the key characteristics of linear patterns such as constant first difference and graphical properties. Write English sentences to represent linear patterns. Write algebraic expressions to represent linear models.
3	Extend the idea of algebraic expressions to include formulae expressed in the form $T_n = an + b$. Associate this form of expression with terms and tables and using this form to find terms and their position. Highlight the fact that *a* and *b* are constants. *Homework – Consider briefly key elements of the equation of a line.*
4 Research Lesson	Exploration of a real-life problem involving a linear pattern in a problem-solving context. Students will be asked to develop immediate, near and far away terms. Explain to students that they should consider a minimum of two different approaches to solving the problem. *Homework 1 – Some questions on linear sequences to apply knowledge of key features learned.* *Homework 2 – Examine the areas of the ponds and gardens. Are these patterns different in any way?*
5	Study of other types of patterns with characteristics which are different to those of linear patterns, e.g. quadratic and exponential. Emphasis on quadratic patterns. Use of various strategies (tables, graphs, etc.). Deducing algebraic formulae to represent quadratic patterns.
6	Study of exponential patterns. Use of various strategies (tables, graphs, algebraic formulae and others).

8. Goals of the Research Lesson

The design of this lesson includes mathematical goals and is underpinned by Junior Cycle Key Skills.

a) Mathematical Goals

- Students will consider and express linear patterns in multiple yet connected forms.
- Students will apply these various forms to find terms of linear patterns and positions of terms, whether the terms are “next”, “near” or “far away”.
- Students will recognise key elements and distinguishing features of different types of patterns.
- Students will deduce the significance of the slope of any graphical representation of a linear pattern.
- Students will build on this knowledge to explore ideas around rate of change.

b) Key Skills and Statements of Learning

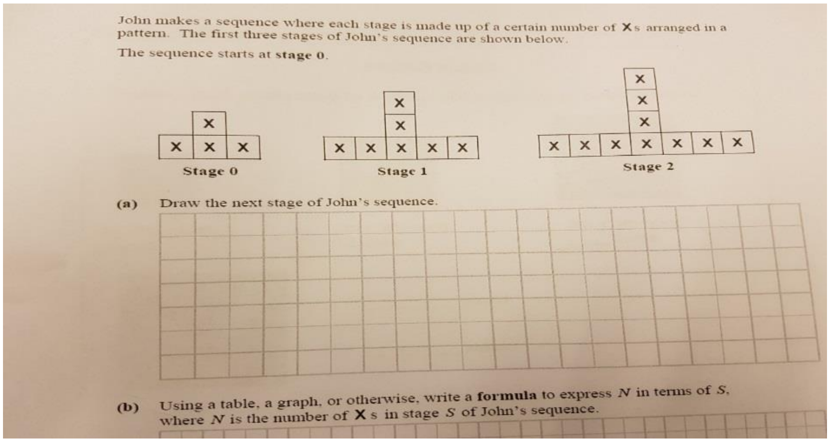
In the planning and design of this lesson Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will promote Key Skills in the following ways:

- Being numerate: seeing patterns trends and relationships.
- Being literate: expressing ideas clearly and accurately.
- Managing myself: students will have the opportunity to reflect on their own learning.
- Communicating: students will present, explain, justify and discuss their mathematical thinking and exploration of number.
- Being creative: students will be encouraged to explore options and alternatives and will actively participate in creative learning.
- Working with others: students will cooperate and will learn with and from each other.
- Managing information and thinking: students will be encouraged to be curious, to think creatively and critically, and to reflect on and evaluate their learning.

This lesson is also designed to meet the following Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The student describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge reasoning and skills.

Introductory Problem (see Section 9 “Introduction” below)


John makes a sequence where each stage is made up of a certain number of **X**s arranged in a pattern. The first three stages of John's sequence are shown below.

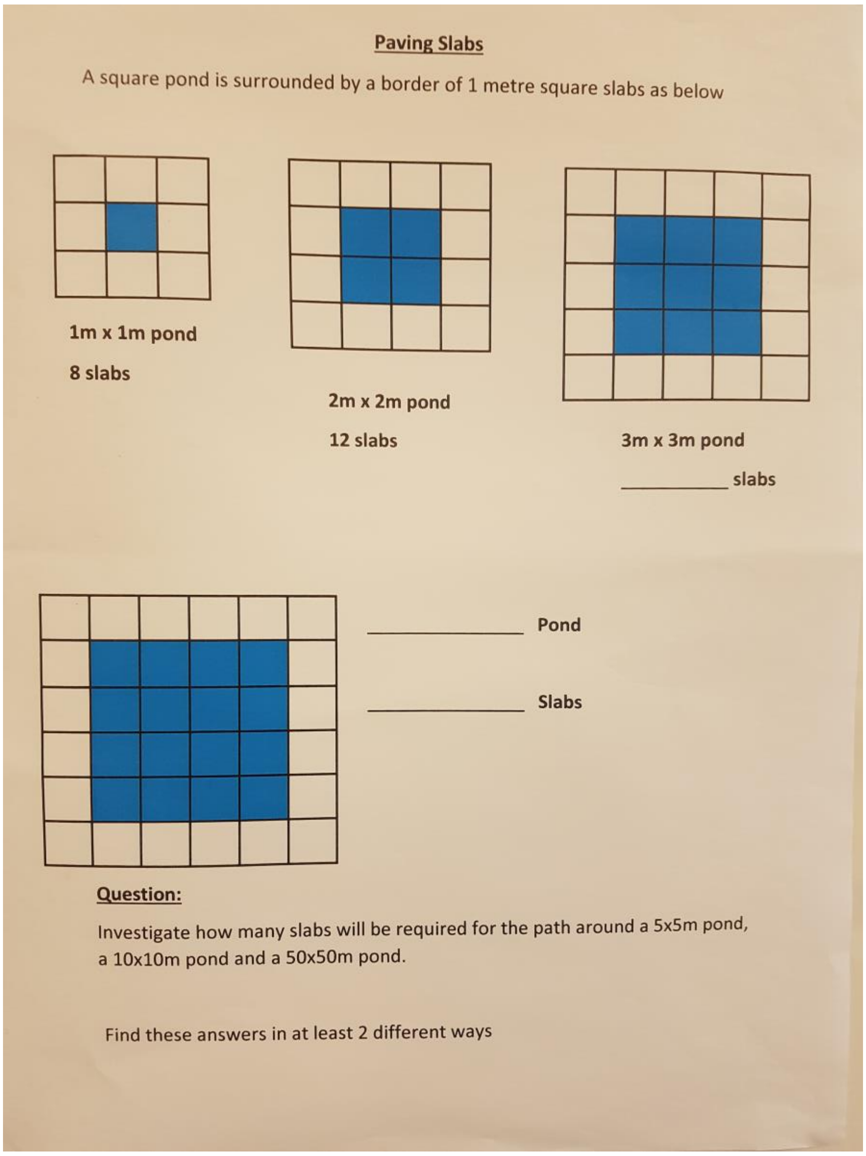
The sequence starts at **stage 0**.

Stage 0 Stage 1 Stage 2

(a) Draw the next stage of John's sequence.

(b) Using a table, a graph, or otherwise, write a **formula** to express N in terms of S, where N is the number of **X** s in stage S of John's sequence.

Main Problem (see Section 9 “Posing the Task” below)



Paving Slabs

A square pond is surrounded by a border of 1 metre square slabs as below

1m x 1m pond

8 slabs

2m x 2m pond

12 slabs

3m x 3m pond

__________ slabs

______________ Pond

______________ Slabs

Question:

Investigate how many slabs will be required for the path around a 5x5m pond, a 10x10m pond and a 50x50m pond.

Find these answers in at least 2 different ways

9. Flow of the Research Lesson

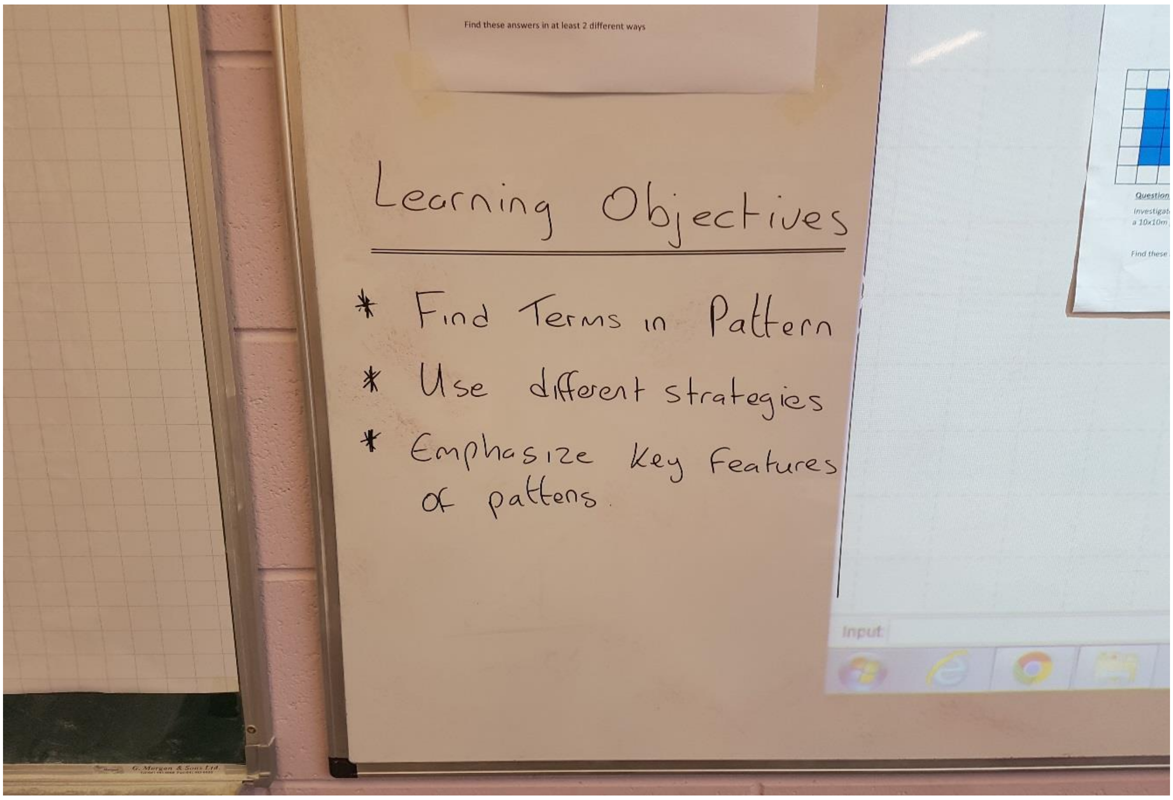
Steps, Learning Activities **Teacher's Questions and Expected Student Reactions**	**Teacher Support**	**Assessment**
Introduction [5 minutes] To start proceedings the teacher will introduce any guests and observers and will emphasise to students that class will be conducted as normal. The teacher will briefly explain the role of the observer (observing work, gathering information, taking photos as a record of work, etc.). The teacher begins by presenting the learning objectives of the lesson: • to find terms in patterns. • to use different strategies if possible to explore patterns. • to distinguish key features of particular types of patterns. "Before we start today's work I want to have a quick look back over some maths we looked at recently." "What do you see?" ❖ Count Xs in boxes. ❖ Look for pattern in diagrams / pictures. ❖ Look for pattern in number of Xs. ❖ Note that pattern is increasing. ❖ Note the constant increase in number of Xs. ❖ Highlight the linear pattern. "How would you try to examine the relationship between each of the diagrams?" ❖ Draw more diagrams. ❖ List the sequence. ❖ Make a table. ❖ Write an English statement. ❖ Form an equation ❖ Derive a formula of the form Tn = … . ❖ Draw a graph. "Keeping these ideas in mind, now let's look at today's problem."	At the start of the class the teacher will divide the class into groups of four at tables, ensuring differentiated levels of student abilities within each group. Place an image of the sequence of diagrams of Xs on the board. (using the data projector – see Section 8 for introductory "X's" problem). We want to be certain that students understand basic ideas and terms associated with patterns. We need to use effective questioning to ascertain students prior knowledge. In particular, we want to encourage the thought of using different approaches to deal with patterns.	Can students express to a reasonable level what they know about patterns and various ways of dealing with them?
Posing the Task [5 minutes] ***Square ponds are surrounded by a border of 1m² slabs. How many square slabs will be required for the path around the pond for a 5mx5m pond, a 10mx10m pond and a 50mx50m pond?*** ***Find these answers in at least two different ways.*** **Clarifying the problem** "For the 1m² pond how many slabs are required for the path?" "For the 2m² pond how many slabs are required for the path?" "For the 3m² pond how many slabs are required for the	Present an image of the problem on the board using the data projector and/or cut outs. Ensure that each student has a calculator, basic drawing equipment and markers or colouring pencils. Also students will be asked to bring their school diaires, which include "traffic lights" to indicate levels of understanding. Give each student an A4 copy of	Do the students understand the task? Do the students understand that they are fundamentally dealing with patterns? Do the students understand that they are required to look at the problem in different ways?

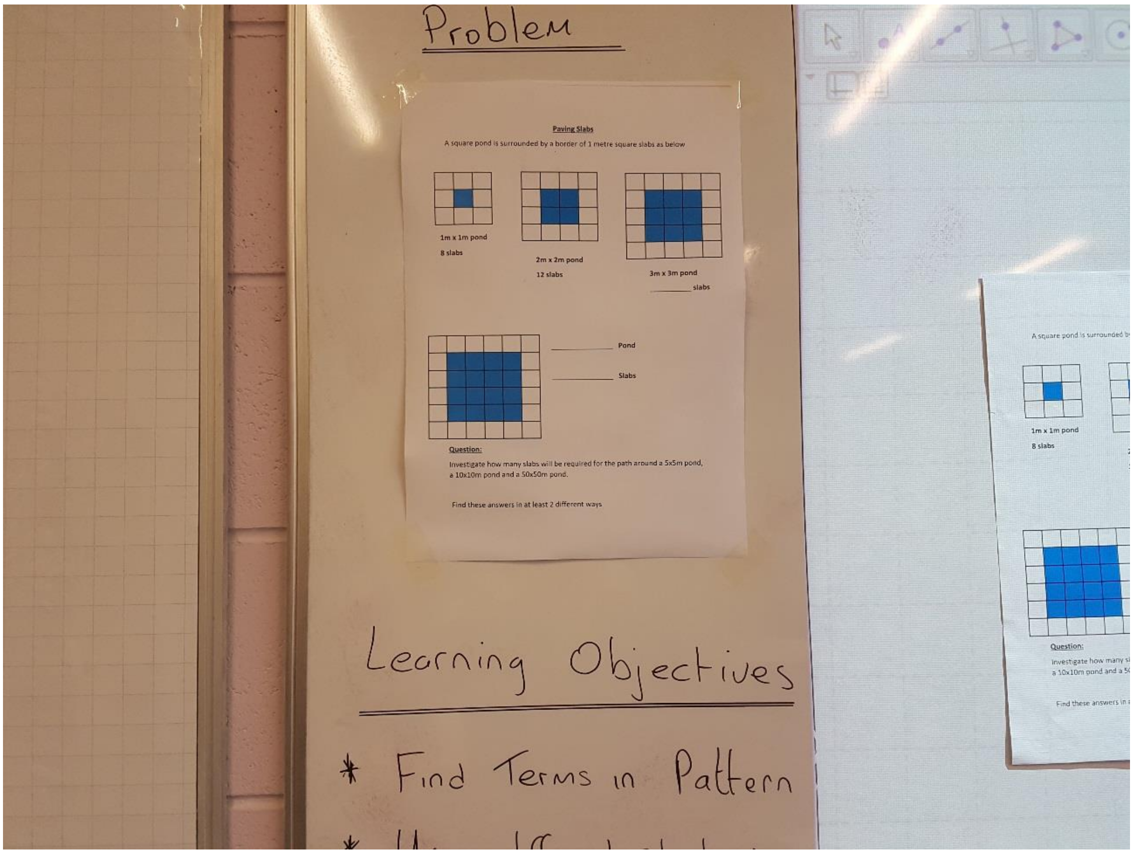
path?" "What are we trying to find out?" "What kind of changes are happening moving from one pattern to the next?" "Can you make any predictions based on what you've seen before we start working?" "Can we get the answers in different ways?" "I want you to now work on the problem individually for 10 minutes, after which we will work in groups at each table for a further period."	the problem and some sheets of A4 1cm² squared paper. Also one "Show Me" mini whiteboard each to work with. Give each table (group) of four an A3 squared paper template of the problem. Write any student answers and predictions on the board.	Are the students engaged in the work?
Student Individual [10 minutes] and Collaborative Group Work [10 minutes] *Student Response 1* Draw out next diagram and count squares in 5th diagram. *Student Response 2* Draw out some of the following diagrams and establish a sequence of numbers of squares. This may quickly enable finding the numbers of square slabs in the 5th and 10th diagrams. *Student Response 3* Express the successive numbers of square slabs as a numerical sequence. Write out the list of numbers of paving slabs needed in each case and try to make a connection. *Student Response 4* Lay out the successive numbers of square slabs as a table. Organise the information to establish a pattern and key features of the pattern. We would hope to look at the key elements of the pattern, i.e. the build up from the first term onwards using the constant difference between successive terms. *Student Response 5* Look and see that the number of square slabs required is the perimeter of the square pond plus the four corner squares every time. *Student Response 6* Express the behavior of the pattern of square slabs as an English statement. This would require some identification of the key elements of this particular pattern and of linear sequences in general. *Student Response 7* Create a map of "position to term". This may be in table form. When they find the common difference, they may recognise that all the terms are related to the multiples of this number. *Student Response 8* Derive a "rule" for the sequence of square slabs in algebraic form. We would aim again here to explore the key elements of this particular pattern and of linear	Use teacher's seating chart (or equivalent on Lesson Note app on iPad) to record the approach used by each student and each group. Note the order in which you will call a representative from each group up to the board during Ceardaíocht. Observers will use the observation template designed by the group of teachers (see end of Section 9) to gather and record relevant data. If students are having difficulties, guide them by asking appropriate questions. For example, "Can you see how the numbers of square slabs are changing?" or "Can you describe what changes are happening as we go from one diagram (or number of square slabs) to the next?" Students or groups may struggle to see different possible approaches. The teacher may gently guide them towards approaches that are not occurring to them after a period of time. For example, teacher may ask "Have we looked at patterns in other ways before?"; "Could we develop a mathematical statement or equation to reflect this pattern?"; "Why would this be useful?"; "Could we develop some picture or graphical representation of the pattern?" During "Ceardaíocht" phase discussion, students will be asked to comment, reflect and elaborate on what other students say. The teacher will use this and	Are students able to meaningfully engage with the problem? Do the students understand that this problem may be viewed as a question of patterns and sequences? For stronger students, can they demonstrate an ability to look at the problem in different ways, for example tables, equations, formulae, graphs? Are students identifying key features of a linear pattern? What limitations may apply to different approaches to solving the problem? Do all approaches facilitate finding the 5th term? Or the 10th term? Or the 50th term? What strengths and weaknesses does each proposed solution have? Do any approaches have so many advantages that we may wish to focus on them in future? Why are some approaches more useful than others? By identifying key features in each approach, could

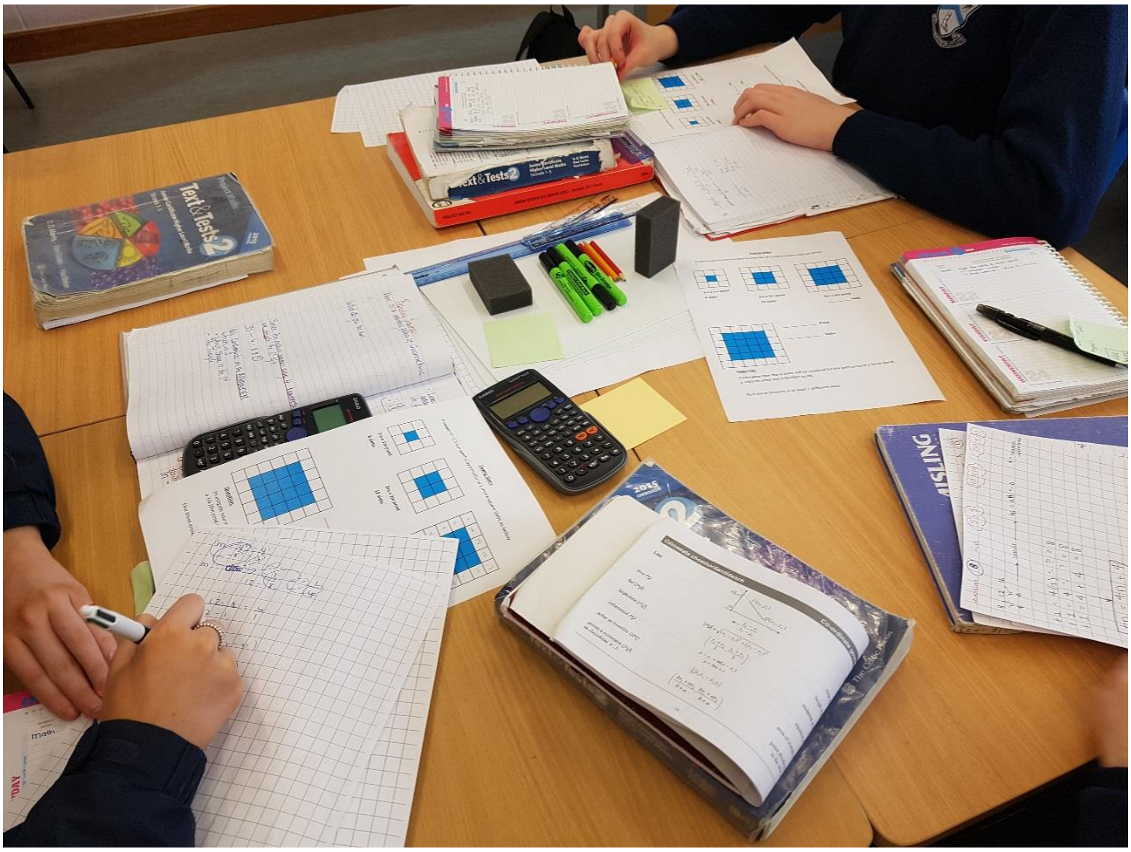
sequences in general, especially highlighting the role of the initial value or term, the rate of change and how these appear in any algebraic expression. *Student Response 9* Derive a "rule" for the sequence of square slabs in the form "$T_n = an + b$". To do this, students could find the common difference, multiply by *n* and then find what they add to common difference to make the first term. *Student Response 10* Students may derive a formula for the pattern using the "DiNO method" where Di is the difference N is the n term (position) O is the observation of what the previous term to the first term would be. *Student Response 11* Plotting position against numbers of square slabs in a graph to establish a linear pattern. *Student Response 12* Plotting position against numbers of square slabs in a graph to establish a linear pattern. Exploring the significance of the *y* intercept and the slope of any associated line. *Student Response 13* Plotting position against numbers of square slabs in a graph to establish a linear pattern. Finding and using the equation of any associated line. We should look at advantages and limitations of using a line and its equation here (e.g. pattern is discrete, not continuous). *Student Response 14* While unlikely at Junior Cert level, students may have been taught or may recognise the suitability of the $Tn = a+ (n-1)d$ formula. They could apply this formula to derive an expression for this sequence.	other strategies to ensure that all students remain engaged. The teacher will look out for opportunities to use student errors or misconceptions to consider and analyse common difficulties among students. Possible misconceptions or difficulties for students may include • misuse or misunderstanding of the constant increase. • not matching pond (position) number to slabs (term) value. After 10 minutes individual work, the teacher will ask students to work in groups. The teacher will assign positions in the group to each student to ensure each student contributes to the discussion and offers ideas. The aim of the groupwork is to debate the various approaches offered and to enable effective communication. This groupwork almost serves as a pre-Ceardaíocht.	solutions have wider applications?
Ceardaíocht /Comparing and Discussing [20-25 minutes] From the anticipated student responses, the teacher will select at least one solution to be presented at the board based on each of the following (where available), and in this order: • drawing extra diagrams • using a numerical table • describing the pattern in words or symbols • expressing the pattern as an algebraic formula • representing the pattern as a linear type graph. As students or groups are developing solutions, if it becomes apparent that some of the above approaches are not emanating from any student or group, the teacher may hint at or give some basic guidance towards considering them. The teacher must ask questions of the presenter and of	At the Ceardaíocht stage it is important to keep in mind the goals of the lesson. The focus of the teacher's questioning and direction of the discussion should be guided by these goals. Students may need to be reminded that a major goal of the lesson is to examine the problem in a variety of ways, not just to find a solution. This also allows for differentiated work, allowing stronger students to be challenged while weaker students can maintain confidence in their work. For each anticipated student	Can each student clearly explain his/her approach to solving the problem when up at the board? Can each student justify elements of his/her solution and communicate this effectively? Can students express an understanding of solutions developed by others? Do students recognise similarities or differences between their own

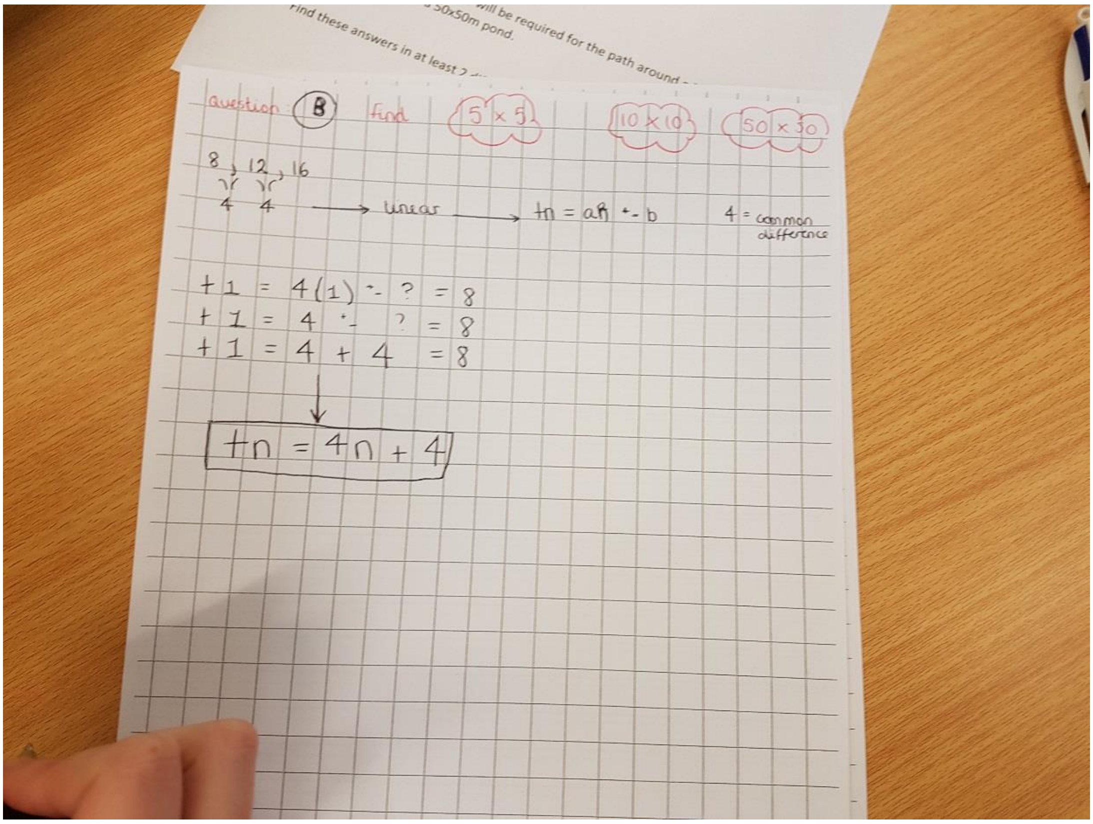
other students, to try to ascertain levels of understanding of key elements of solutions. It is important that the whole class is engaged and that students understand they may be called on at any stage to reflect on what another student said. The teacher must keep in mind that clear student communication and understanding are goals of the lesson. In keeping with the goals of the lesson, teacher questioning will focus on the most important properties of linear patterns arising from each solution. These include the role of the first term, the significance of the constant change, how these impact on algebraic and graphical interpretations and the concept of rate of change. For example, "How is pattern built from start?"; "Could anyone suggest a better structure for a table in this case?"; "Can anyone trace where this coefficient is coming from?"; "Why are sequences like this called linear patterns?"; "What is the significance of the slope of the line?" The teacher should highlight comparable features in different solutions. For example, "Can anyone see any similarities between what's happening in the table and in the English description (or the formula or the graph)?" Teacher questioning should lead debate on interesting features, merits or misconceptions of solutions. When all solutions have been presented, the teacher may pose the question "What's the best solution? Why?". The teacher could also ask students to consider if the comparisons between solutions may apply generally to linear patterns (although this may be work for another day). To finish the Ceardaíocht stage, the teacher will ask students to move around classroom on a "one stay rest stray" basis, i.e. one student from each group will remain at his/her table to answer any questions, while the other students move around to look briefly at the work of others.	response the teacher (in conjunction with the full Lesson Study group) will have prepared a basic initial position (typically an appropriate diagram) as a starting point for the student to build on and develop his/her solution on the board. When a student presents work on a board make sure to attach his/her name (or a group's name) to it. Ask other students to raise their hands if they used the same method. When each anticipated solution has been presented and discussed, ask did anybody use a different approach. The teacher should use any opportunity in the discussion to reinforce understanding of varied terminology, e.g. patterns, sequences, coefficients, rate of change.	approach and that presented on the board? Do students offer alternative approaches to solving the problem? What major misconceptions or errors are arising as discussion of solutions is proceeding? If a solution does not work out, can students see exactly where the problem arose?
Extending Students' Learning [5 minutes] The teacher may ask students to follow up the work of the class with some of the following: • any anticipated solution that is unused. • any interesting issue that arises in Ceardaíocht. • application of key features of linear patterns learned during class to other linear sequences. • an examination of the area of the pond. • an examination of the total area of the pond and path (rectangular quadratic pattern). • use of these to develop ideas on quadratic patterns. • derivation of the formula $T_n = a + (n-1)d$.		
Summing Up and Reflection [5 minutes] The teacher will sum up the lesson by reminding students	The teacher will use the layout of the boardwork to help provide	

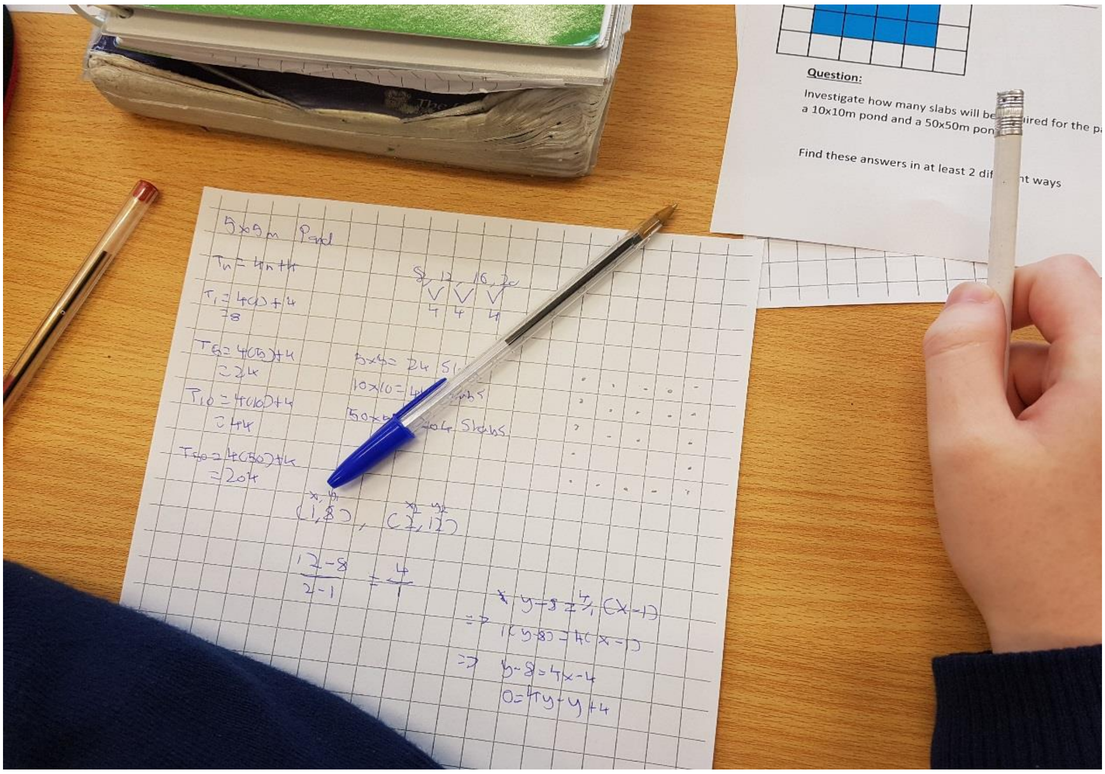
of the learning objectives outlined at the start of the lesson. The teacher will highlight any key features and difficulties encountered in working with linear patterns. To finish the class, each student will be asked to reflect on what they learned in the class. Each student will be asked to write two or three brief points on post-its, indicating what they consider important or interesting from the class. To assist students, they teacher may suggest phrases to use such as "Today one thing I learned was ..." "Today one problem I had was …" "Today I understood ..." "Today I noticed …" "Today the questions I have are …" "Today I learned from my classmates' ideas that …" "Today I was unsure of ..." "Today the thing I found most helpful was …." Students may put their names to their reflections or they may remain anonymous. The students will stick their post-its on a board when leaving the class. The teacher will thank all involved at end of lesson.	students with a summary of the progression in their learning. Ideally here, this will progress from diagrammatic solutions to numerical sequences to tables to algebraic formulae to graphical representations (if each of these has featured in boardwork).	

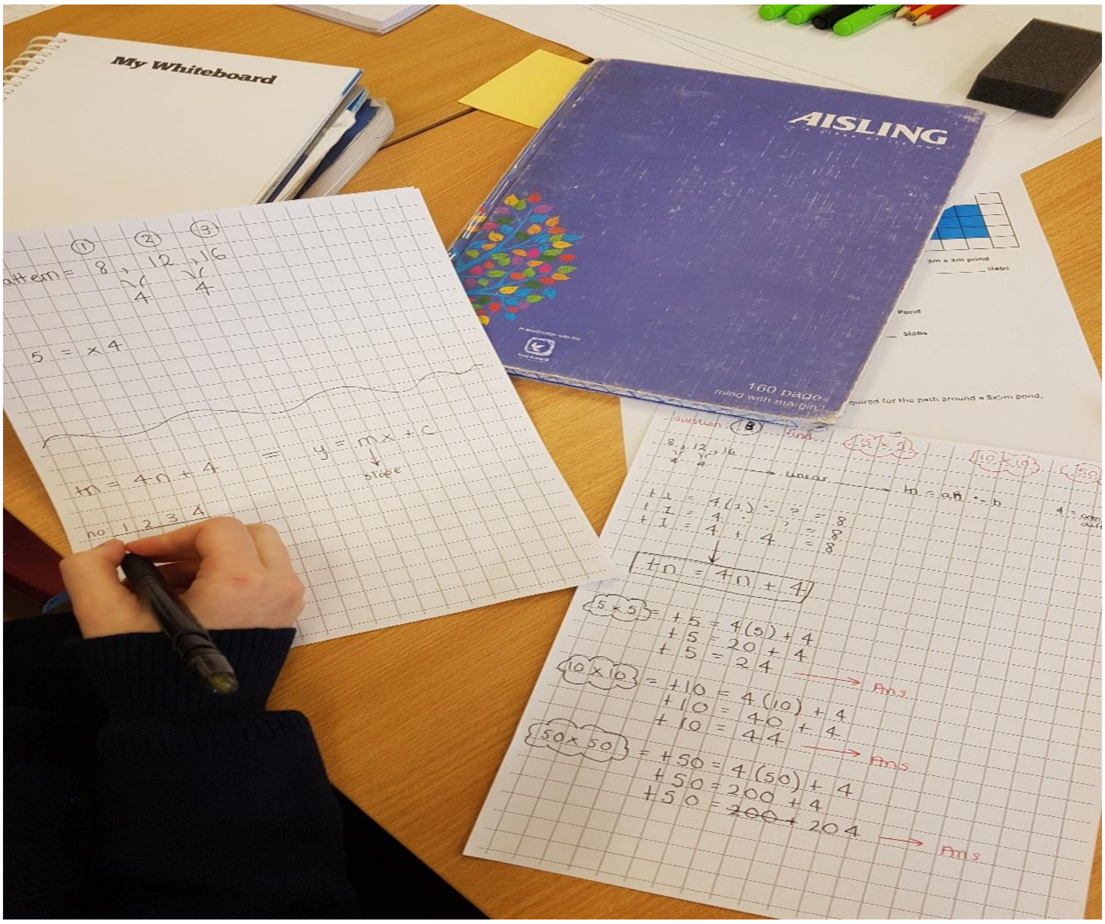

Find these answers in at least 2 different ways
Learning Objectives
* Find Terms in Pattern
* Use different strategies
* Emphasize Key Features of pattens

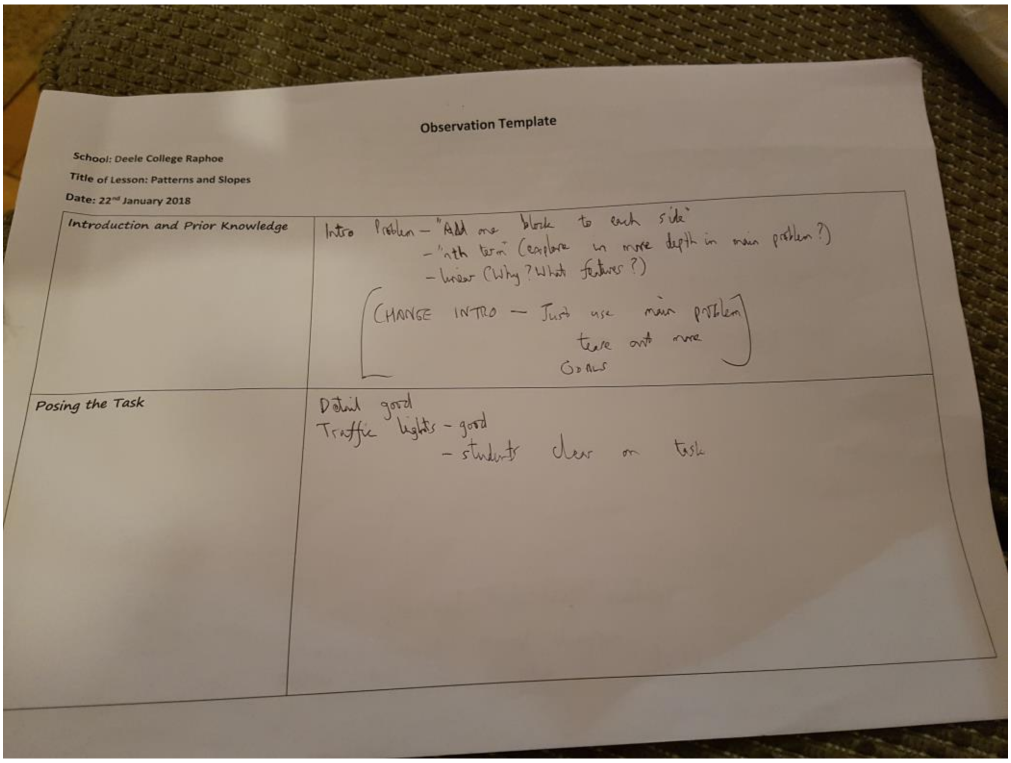

Problem
Paving Slabs
A square pond is surrounded by a border of 1 metre square slabs as below
1m x 1m pond
8 slabs
2m x 2m pond
12 slabs
3m x 3m pond
slabs
Pond
Slabs
Question:
Investigate how many slabs will be required for the path around a 5x5m pond, a 10x10m pond and a 50x50m pond.
Find these answers in at least 2 different ways
Learning Objectives
* Find Terms in Pattern




Find these answers in at least 2
will be required for the path around
50x50m pond.
Question 8 find 5 x 5 10 x 10 50 x 50
8, 12, 16
4 4
linear
tn = an +- b
4 = common difference
t1 = 4(1) +- ? = 8
t1 = 4 +- ? = 8
t1 = 4 + 4 = 8
tn = 4n + 4


Question:
Find these answers in at least 2 dif


My Whiteboard
AISLING
160 page



Observation Template

School: Deele College Raphoe

Title of Lesson: Patterns and Slopes

Date: 22nd January 2018

Introduction and Prior Knowledge	Intro Problem – "Add one block to each side" – "nth term" (explore in more depth in main problem?) – linear (Why? What features?) (CHANGE INTRO – Just use main problem tease out more GOALS)
Posing the Task	Detail good Traffic lights – good – students clear on task

Student Individual Work

- What methods did the students use to solve the problem?
- Were any anticipated solutions not used?
- What comments or questions did the students have?
- Did students show a good understanding of the key features associated with linear patterns?
- To what extent were students able to use various approaches and link them?
- What were the common misconceptions and misunderstandings?

(unusual?)

Initial exploration

S3 Numerical sequence with difference [2 - Orin ~~Leah~~ James ; 1 - Kian, Ellie

S2 Diagram (~~It~~ ...) (1 - Francis)

S9 $T_n =$ (using key features rather than simul. eqs.) [1 - Kian, Ellie]
James [2 - Leah (uncertain)

S8 $T_1 =$, $T_2 =$, Informal $T_5 = 4(5)+4$, etc. [2 - Orin, James] (Used table function on calculator)

S12 Slope/ [2 - James] [2 - Orin (uncertain exactly how slope applied)
Equation

*

2 James finished 2/3 ways quickly.
1 Francis unsure beyond diagram
2 Orin - Used Table function on calculator

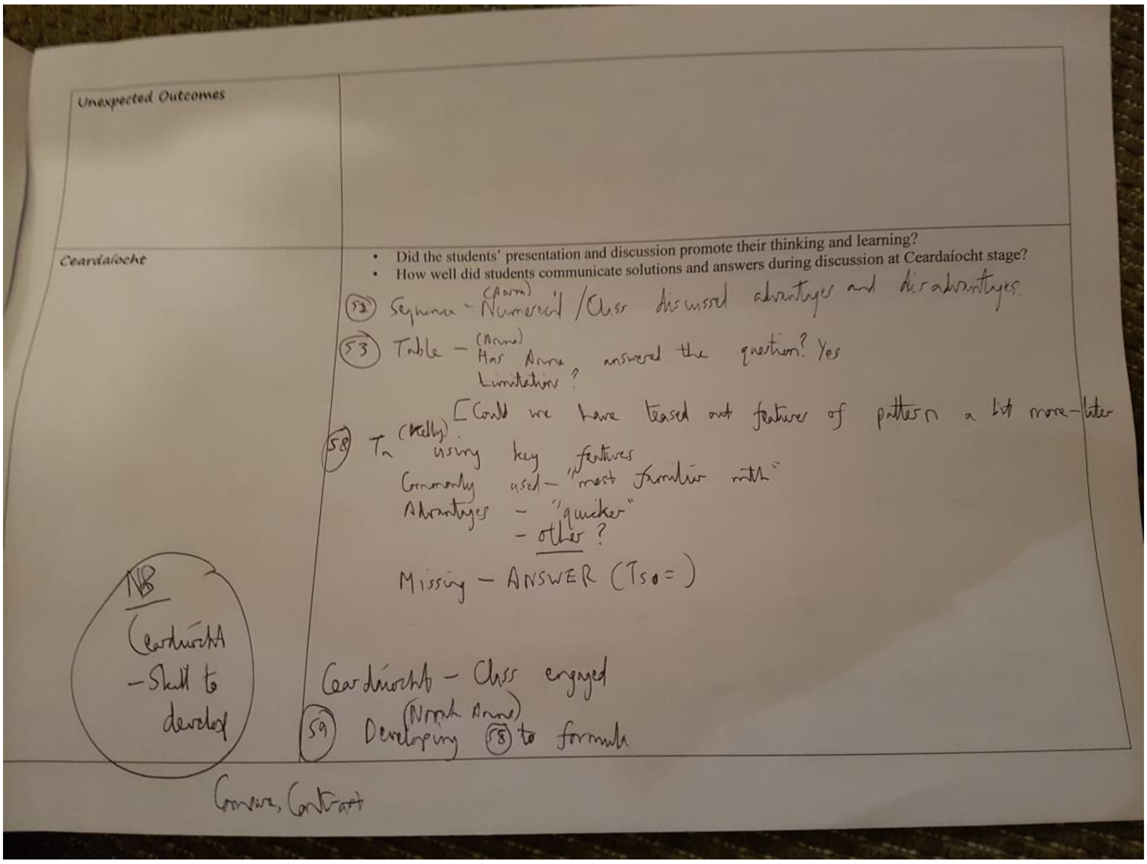
Collaborative Group work

(Shorter - 5 mins)

- How and when did students' understanding change?
- Did any students demonstrate a continued lack of understanding or progress? How could this be addressed?
- How well did students collaborate at groupwork stage?
- How successful was differentiation of students' work?

1 - Discussing, active immediately (Ellie leading - good at explaining) (Kian curious, learning) (Francis less engaged, continuing her own)
↳ good questions ("I get it now" for slope/line approach)
2 - Quieter, slower to start, less collaboration
James, Leah similar answers, explaining to each other
Orin a bit separate - focussed on table/numerical.

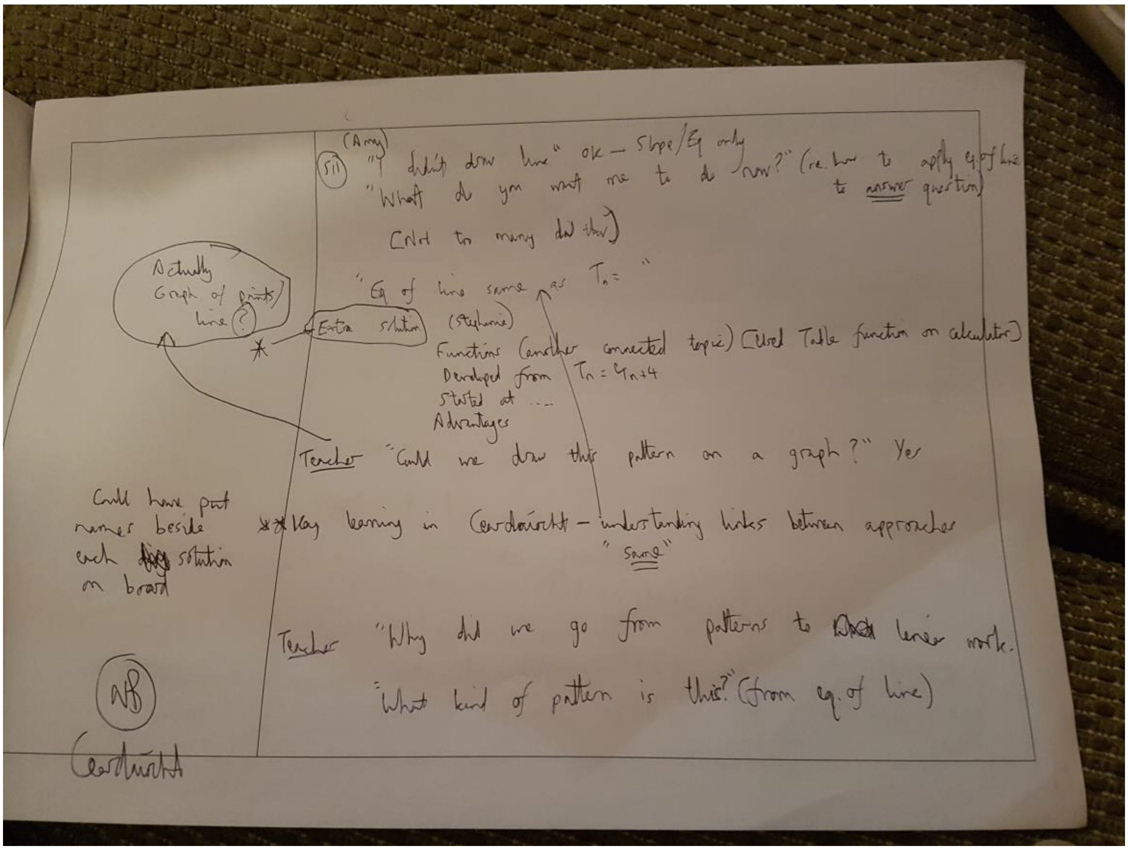
Teacher - hints at groups



Unexpected Outcomes	
Ceardaíocht	• Did the students' presentation and discussion promote their thinking and learning? • How well did students communicate solutions and answers during discussion at Ceardaíocht stage? (52) Sequence - (Anna) Numerical / Class discussed advantages and disadvantages (53) Table - (Anna) Has Anna answered the question? Yes Limitations? [Could we have teased out features of pattern a bit more - later (58) T_n (Kelly) using key features Commonly used - "most familiar with" Advantages - "quicker" - other? Missing - ANSWER (T_{50} =) Ceardaíocht - Class engaged (59) Developing (Norah Anne) (58) to formula

NB
Ceardaíocht
- Skill to develop

Compare, Contrast



(511) (Amy) "I didn't draw line" ok - Slope / Eq only
"What do you want me to do now?" (re. how to apply eq of line to answer question)

(Not too many did this)

"Eq of line same as T_n = "

Actually Graph of points / line (?)

Extra solution (Stephanie)
Functions (another connected topic) (Used Table function on calculator)
Developed from $T_n = 4n+4$
Started at ...
Advantages

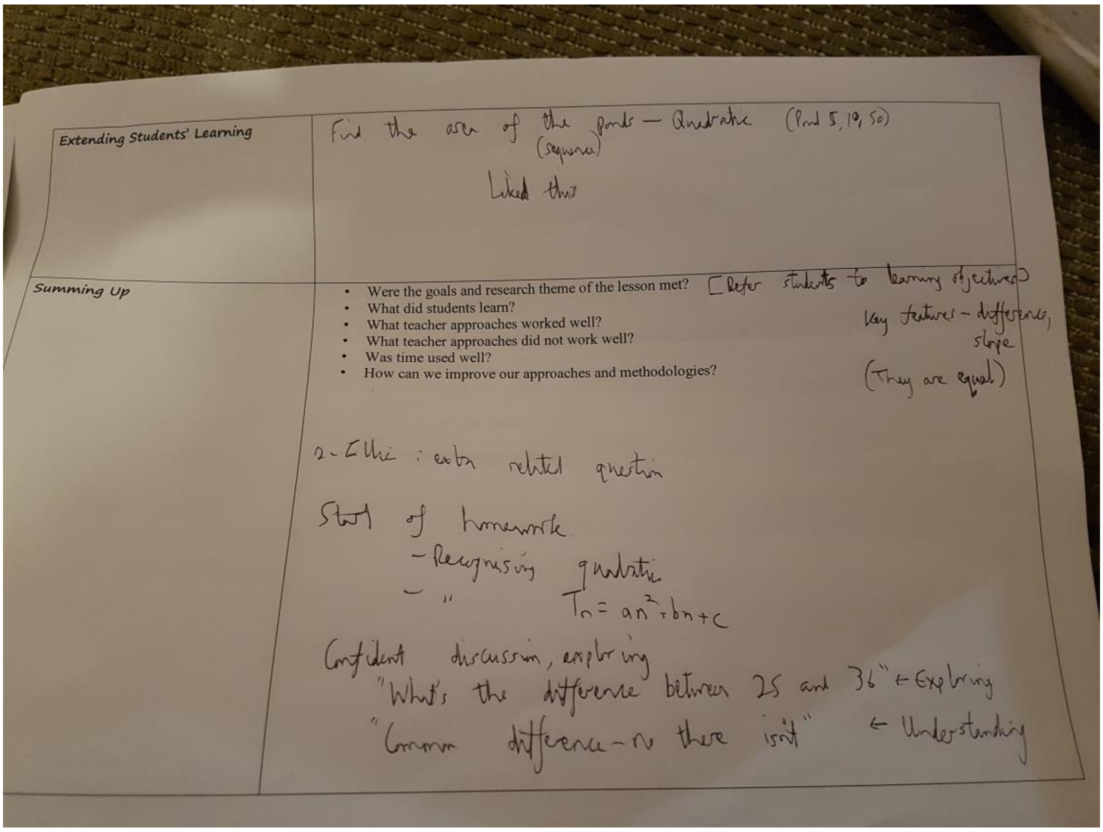
Teacher "Could we draw this pattern on a graph?" Yes

Could have put names beside each solution on board

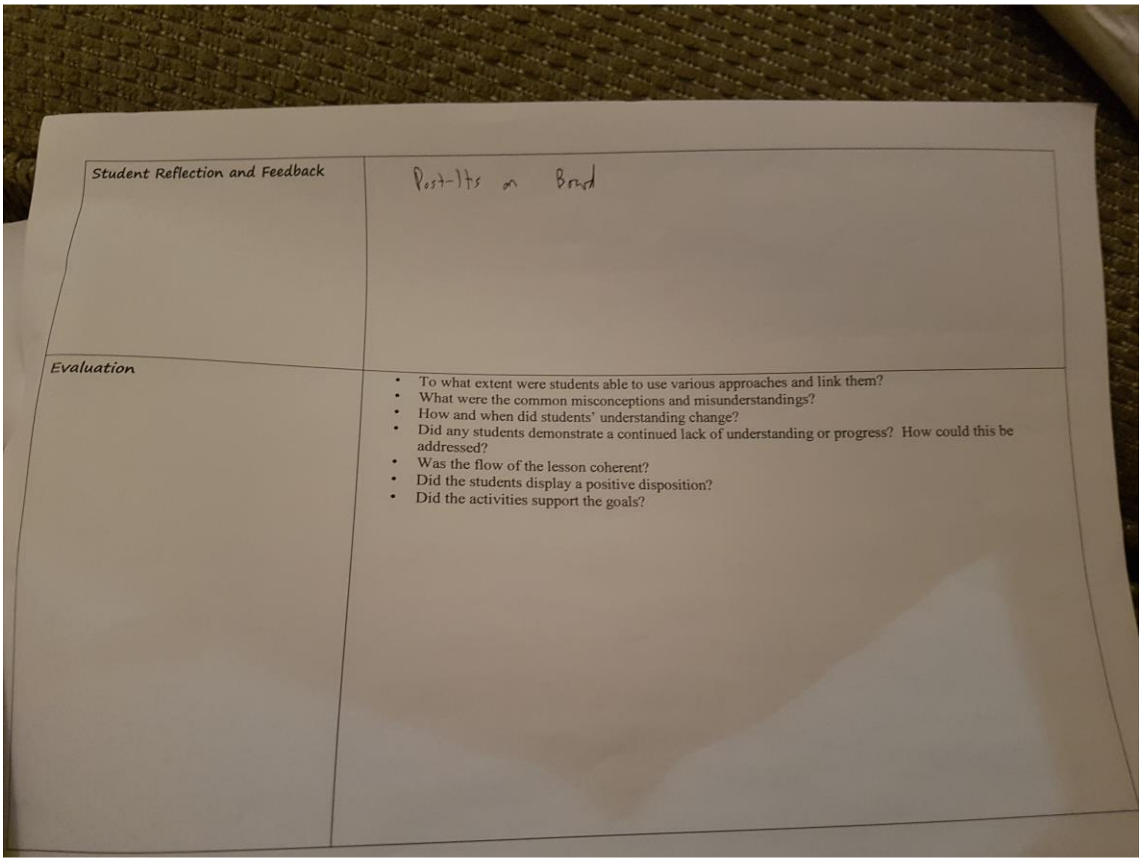
** Key learning in Ceardaíocht - understanding links between approaches
"Same"

Teacher "Why did we go from patterns to linear work.
"What kind of pattern is this?" (from eq. of line)

NB
Ceardaíocht



Extending Students' Learning	Find the area of the ponds — Quadratic (sequence) (Pond 5, 19, 50) Liked this
Summing Up	• Were the goals and research theme of the lesson met? [Refer students to learning objectives] • What did students learn? • What teacher approaches worked well? Key features – difference, slope • What teacher approaches did not work well? • Was time used well? (They are equal) • How can we improve our approaches and methodologies? 2 – Ellie : extra related question Start of homework – Recognising quadratic – " $T_n = an^2 + bn + c$ Confident discussion, exploring "What's the difference between 25 and 36" ← Exploring "Common difference – no there isn't" ← Understanding



Student Reflection and Feedback	Post-Its on Board
Evaluation	• To what extent were students able to use various approaches and link them? • What were the common misconceptions and misunderstandings? • How and when did students' understanding change? • Did any students demonstrate a continued lack of understanding or progress? How could this be addressed? • Was the flow of the lesson coherent? • Did the students display a positive disposition? • Did the activities support the goals?

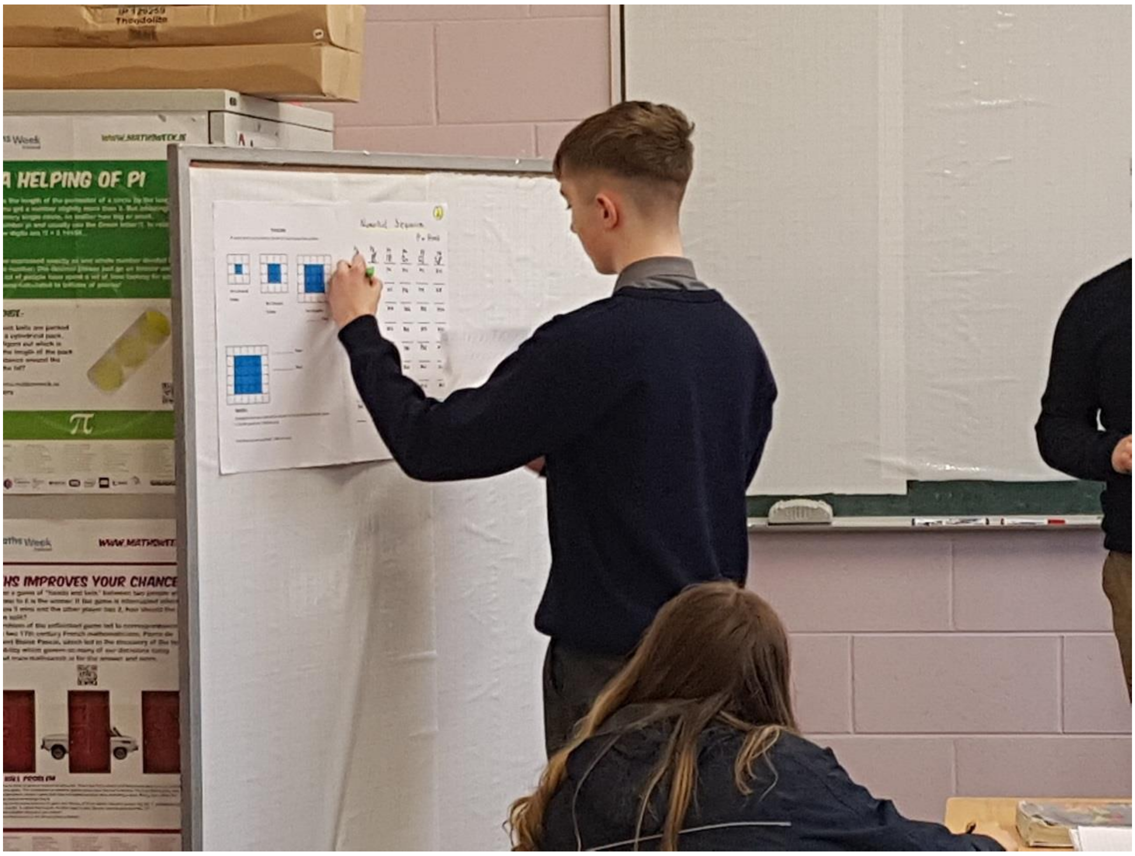

Problem
$T_N = 4(N) + 4$

10. Board Plan

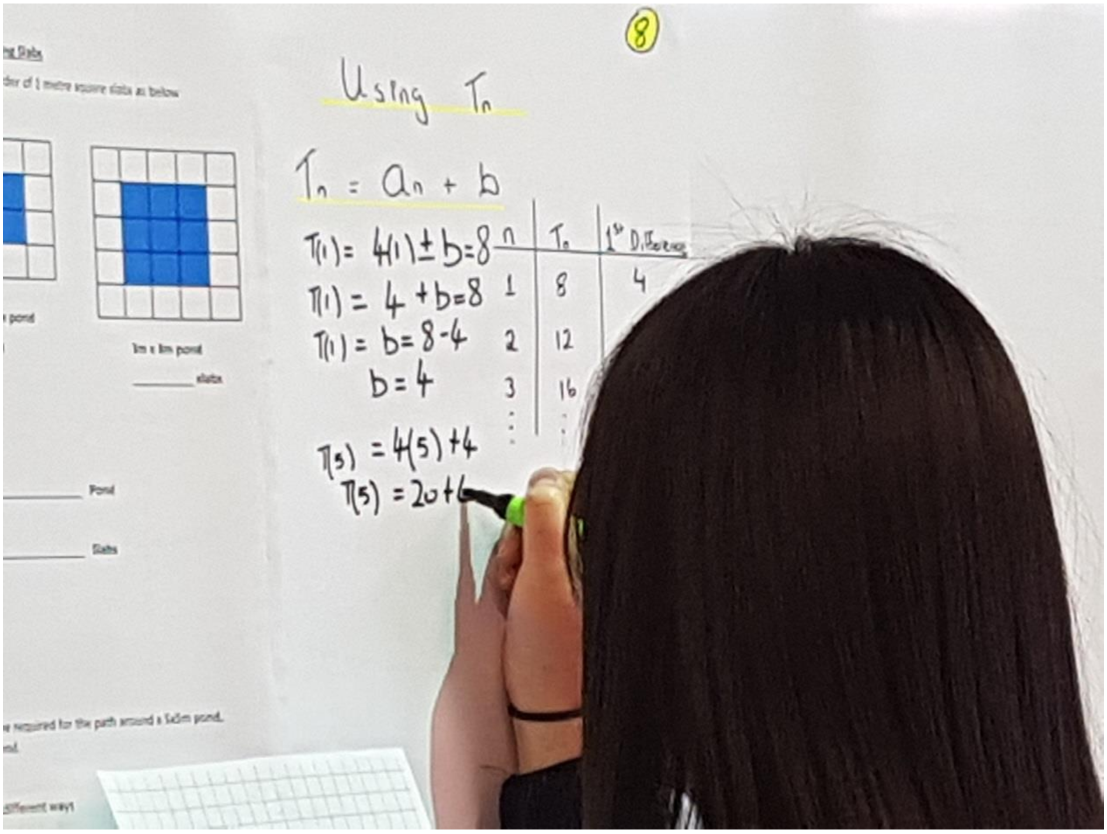
The primary aim of our boardwork was to provide a clear record for students of how learning progressed throughout and to highlight connections across seemingly disparate areas of mathematics. We planned to arrange solutions in order from least sophisticated to most sophisticated, ensuring we displayed at least one of each of the main types of solutions:

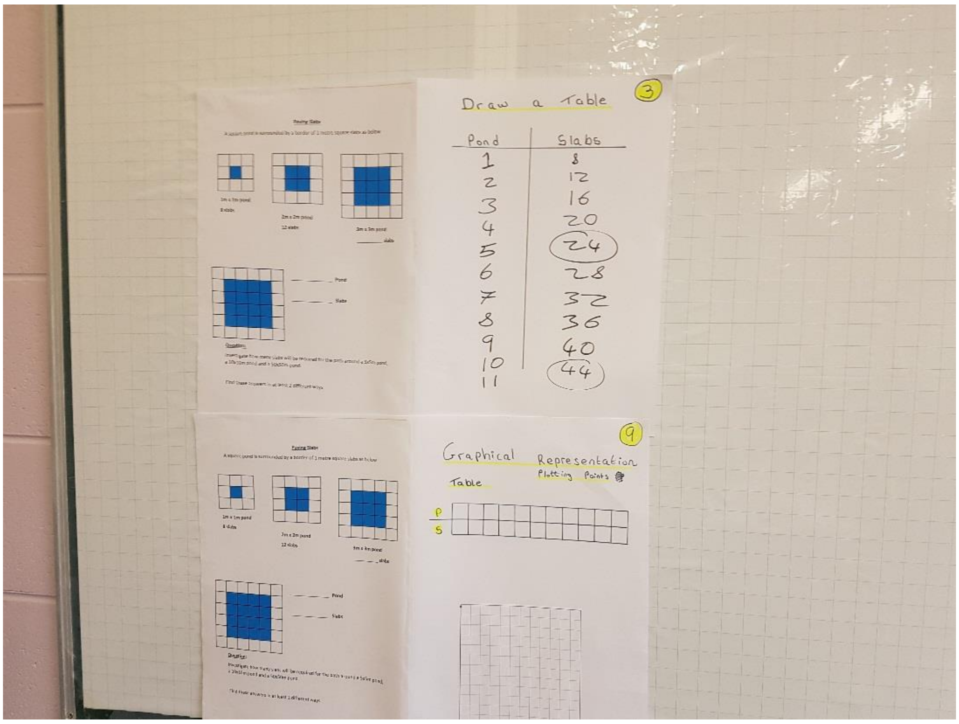
- drawing extra diagrams.
- using a numerical table.
- describing the pattern in words or symbols.
- expressing the pattern as an algebraic formula.
- representing the pattern as a linear type graph.

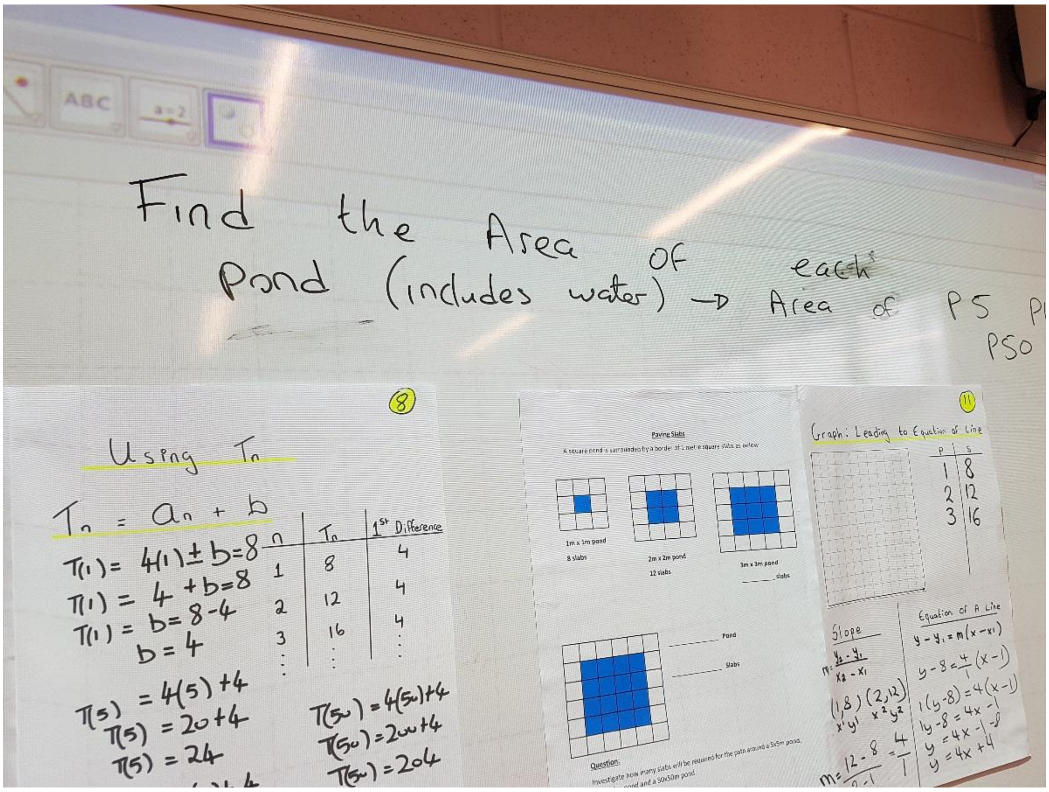
We prepared a basis of each anticipated solution to put up on the board as a starting point for each student to work with. We practiced two in advance (but in hindsight we probably concentrated on the physical work at the board rather than emphasising student explanation and communication).

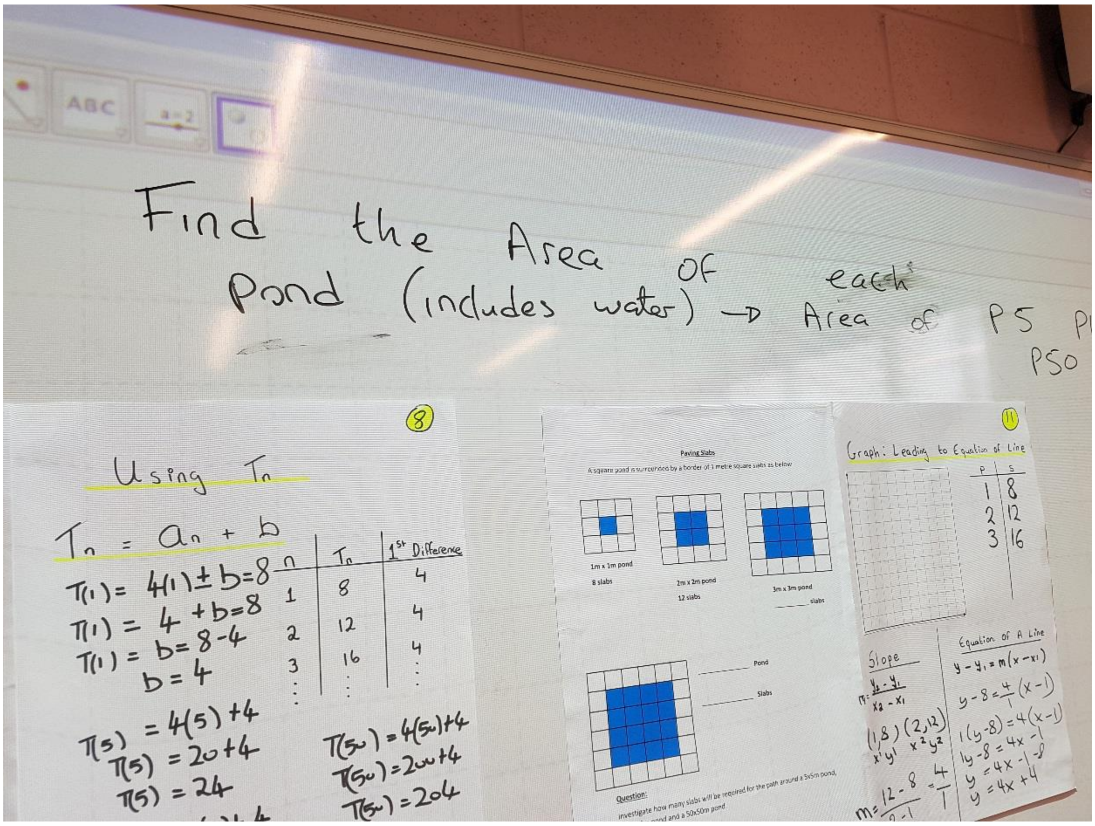


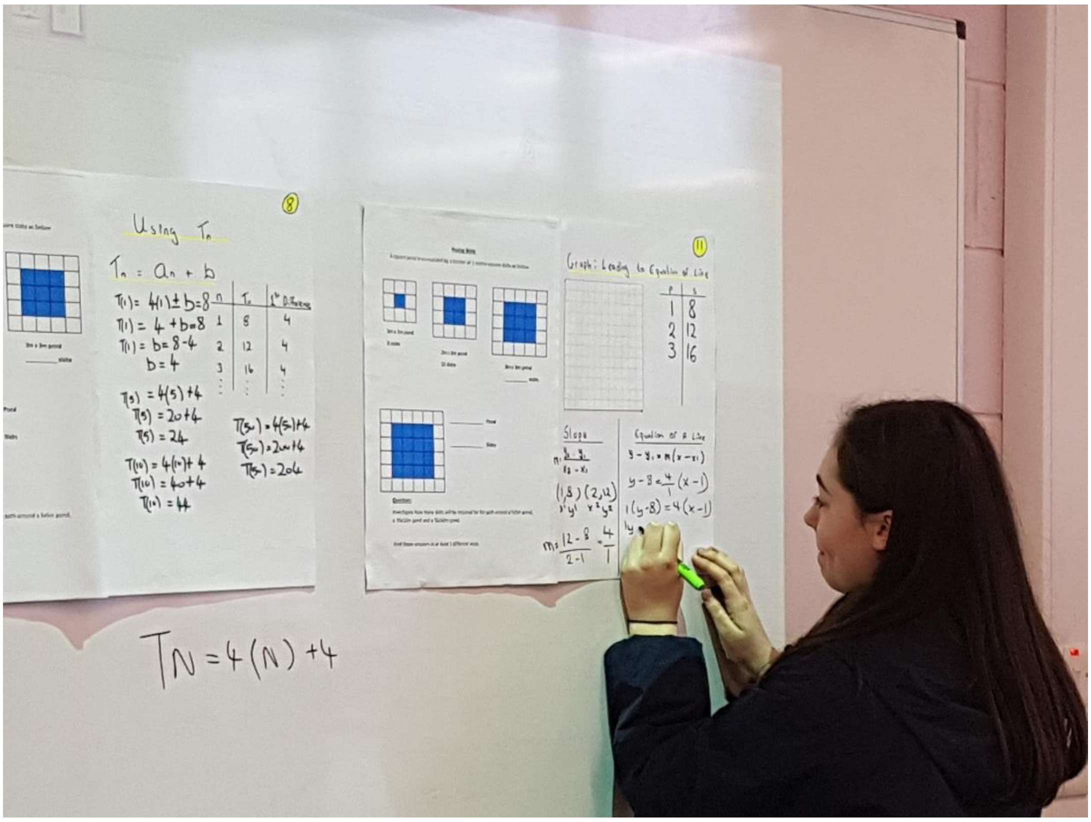

Draw a Table
Pond
Slabs
1 8
2 12
3 16
4 20
5 24
Problem
Learning Objectives
* Find Terms in Pattern
* Use different strategies
* Emphasize Key Features of patterns

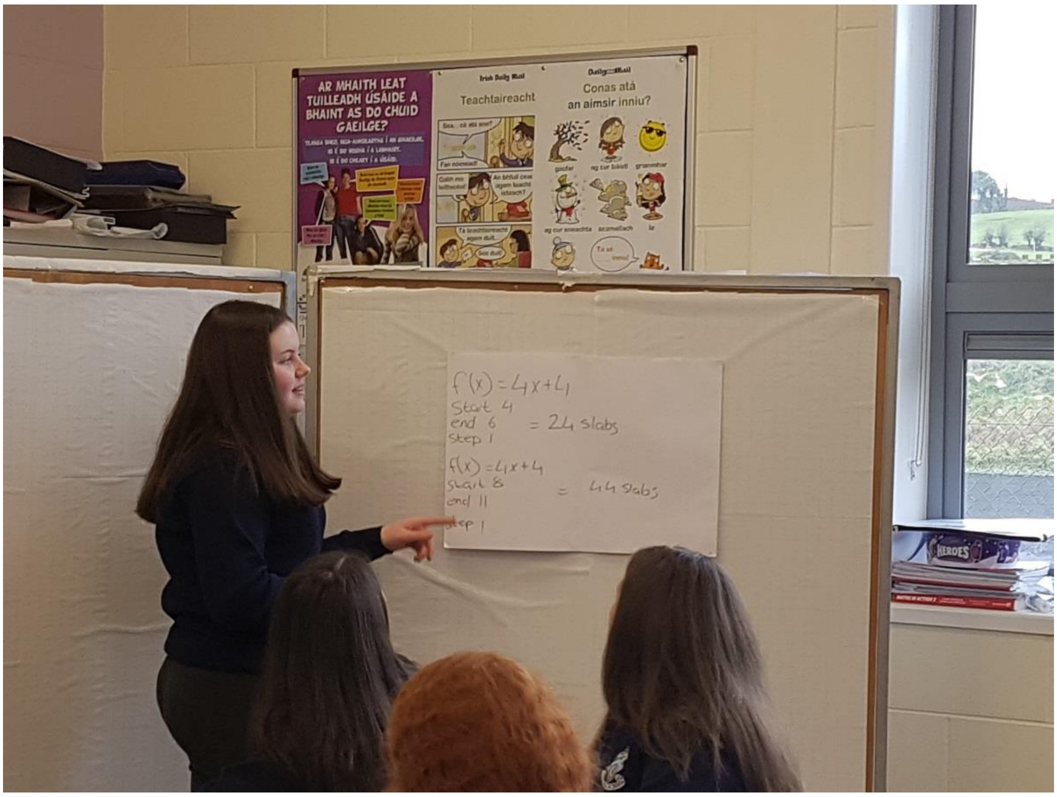

8
Using Tn
Tn = an + b
T(1) = 4(1) ± b = 8
T(1) = 4 + b = 8
T(1) = b = 8 - 4
b = 4
T(5) = 4(5) + 4
n Tn 1st Difference
1 8 4
2 12
3 16
1m x 1m pond
slabs
Pond
Slabs
different ways


Draw a Table
Pond Slabs
1 8
2 12
3 16
4 20
5 24
6 28
7 32
8 36
9 40
10 44
11
Graphical Representation
Plotting Points
Table
P
S

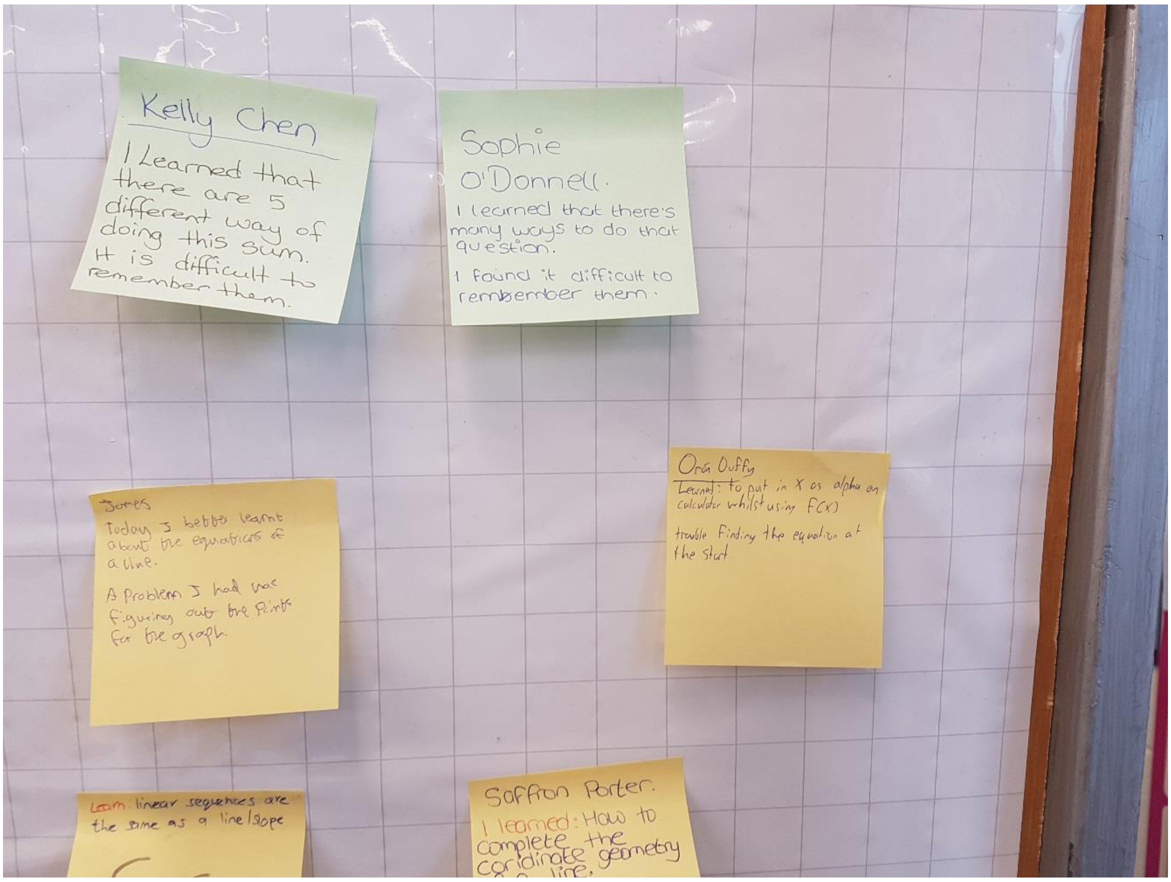

Find the Area of each
Pond (includes water) → Area of P5
P50
Using Tn
Tn = an + b
n Tn 1st Difference
1 8 4
2 12 4
3 16 4
T(1) = 4(1) + b = 8
T(1) = 4 + b = 8
T(1) = b = 8 - 4
b = 4
T(5) = 4(5) + 4
T(5) = 20 + 4
T(5) = 24
T(50) = 4(50) + 4
T(50) = 200 + 4
T(50) = 204
Paving Slabs
Graph: Leading to Equation of Line
P S
1 8
2 12
3 16
Slope
Equation of A Line
y - y1 = m(x - x1)
y - 8 = 4/1 (x - 1)
1(y - 8) = 4(x - 1)
1y - 8 = 4x - 1
y = 4x - 1 + 8
y = 4x + 4


Find the Area of each
Pond (includes water) → Area of P5 P
P50
Using Tn
Tn = an + b
T(1) = 4(1) ± b = 8
T(1) = 4 + b = 8
T(1) = b = 8 - 4
b = 4
T(5) = 4(5) + 4
T(5) = 20 + 4
T(5) = 24
T(50) = 4(50) + 4
T(50) = 200 + 4
T(50) = 204
Paving Slabs
Graph: Leading to Equation of Line
Slope
Equation of A Line
y - y1 = m(x - x1)
y - 8 = 4/1 (x - 1)
1(y - 8) = 4(x - 1)
y - 8 = 4x - 1
y = 4x - 1 - 8
y = 4x + 4


Using Tn
TN = 4(N) + 4



AR MHAITH LEAT TUILLEADH ÚSÁIDE A BHAINT AS DO CHUID GAEILGE?
Teachtaireacht
Conas atá an aimsir inniu?
f(x) = 4x+4
Start 4
end 6
step 1
= 24 slabs
f(x) = 4x+4
start 8
end 11
step 1
= 44 slabs



Kelly Chen
I Learned that there are 5 different way of doing this sum.
It is difficult to remember them.
Sophie O'Donnell.
I learned that there's many ways to do that question.
I found it difficult to rembember them.
James
Today I better learnt about the equations of a line.
A Problem I had was figuring out the points for the graph.
Learned: to put in X as alpha on calculator whilst using F(x)
trouble finding the equation at the start
Learn: linear sequences are the same as a line/slope
Saffron Porter.
I learned: How to complete the coridinate geometry

11. Evaluation

As part of trying to maintain focus on our research themes and goals of the lesson, we looked at how students were linking up various elements of the course while dealing with patterns. The teacher and all observers felt that the students understood where they were going with the problem posed and were familiar with what the question was looking for. Most went for a T_n approach as expected. Asking for multiple solutions required them to come up with other ideas, which individual students were all able to attempt. Some students found this demand for multiple strategies strange initially. Mostly they stuck with $T_n = an + b$, but there were some other methods. Some discussions took place in groups as to whether two methods were essentially the same, which showed good understanding. An objective throughout the process was for students to see various solutions as connected parts of a whole, seeing how key features of solutions are linked.

Students developed an understanding of the limitations of using certain methods (e.g. counting up takes too long, tables or graphs alone make dealing with later terms difficult). A huge benefit was covering what was traditionally treated as three separate topics in one question. As any teacher would hope for, some students demonstrated the creativity to offer solutions that we did not anticipate. One group adopted a function approach, reaching $f(x) = 4x+4$, making it easy to see the connection between this branch of mathematics and patterns, sequences, algebra and coordinate geometry. Another group used the table function on their calculators (Casio fx83) to develop terms from their algebraic formula.

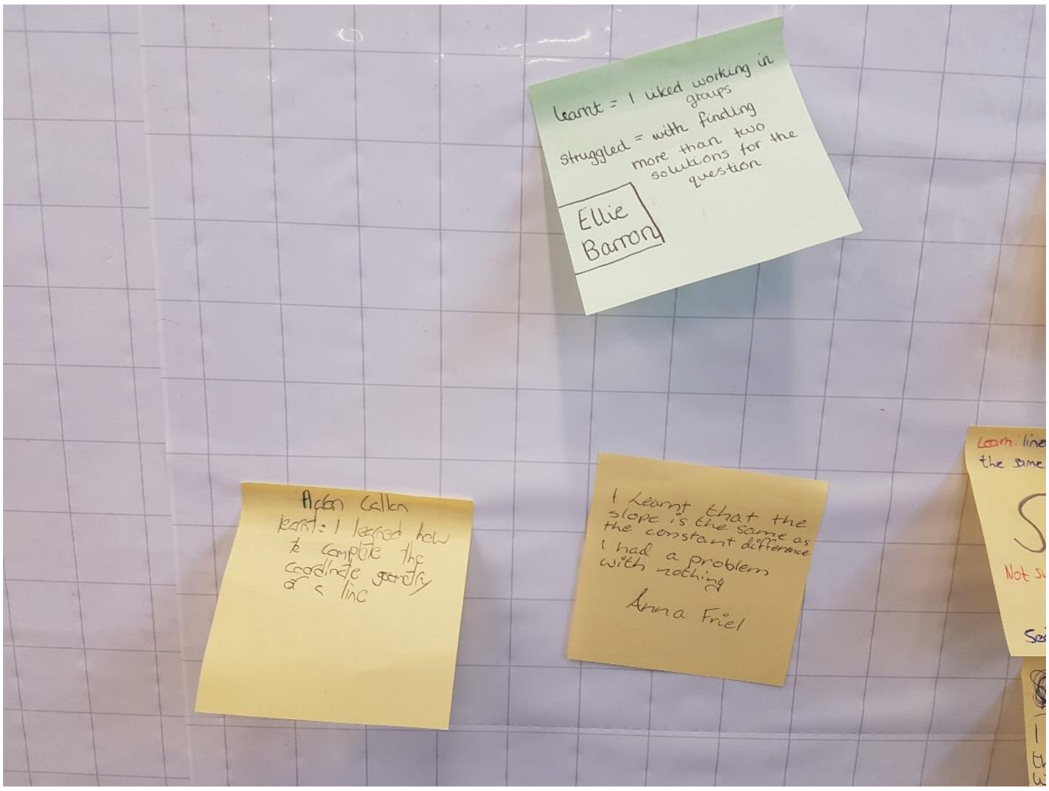
Group work did facilitate communication after the individual work stage, insofar as students had the opportunity to explain their work and question their colleagues in a smaller, less intimidating setting. Mixed ability grouping often led to better understanding because stronger students were able to explain solutions to weaker partners. In one particular pair, the student who was struggling had the confidence to persist with questioning her friend until she could say “I get it now”. The stronger student also benefitted from this process in having to analyse her solution and clarify her communication. It was almost like a pre-Ceardaíocht for both. We felt afterwards that maybe paired work would be better than groups of four, as in most groups some students were slightly detached. Groupwork did promote differentiation of levels of work, with weaker students having to stand over their solutions, even allowing for less sophisticated approaches. Where groups worked well all members benefitted from more varied solutions. In the less productive groups some individuals were ”left behind” to a certain extent and tended to be limited to fewer and less sophisticated solutions. This undermined some key lesson goals. Within the groups, Marilyn O’Riordan observed how some students were coming up with solutions but didn’t have the confidence to voice them. Certain students did the problem in a particular way but didn’t have the confidence to explain. All individual students made good use of notes, but in a group setting some were happy to hold on to their own individual answers and not fully engage in discussion. In future we will plan to offer reinforcement to effective groupwork (e.g. requiring regular checking of what others are doing). Most students could have used more resources. All tables were provided with a variety of equipment to suggest different strategies, but students tended to stay with a limited range of solution types.

We had two “knowledgeable others” observing the class, Anne Brosnan and Stephen Gammell. Anne recommended spending more time in the introduction on basic physical properties associated with the diagrams. While the introduction went well and all students were clear on key prior knowledge and the task, we all agreed that it may have been better to discard the brief introductory problem (with Xs) and to use our main problem with more depth in the introductory phase. We could have planned to spend more time on basic detailed questioning around the patterns (e.g. “How many slabs do you see?”, “Where do you see them?”). It was notable that the class did not come up with some anticipated solutions. Rather than seeing common misconceptions or errors, this was the main difficulty encountered. More extensive preparation of discussion during Ceardaíocht at our planning stage may have helped the class here. Misconceptions were less frequent than expected, possibly because this class is coming near to the end of the Junior Cycle and has reached a level of separate competence in these linked areas of the course already. While appreciating the generally effective questioning during the Ceardaíocht developed by the teaching group and implemented by the teacher of the lesson, Anne did query why all solutions that we were expecting weren’t shown on the board. We suggested that this happened because they weren’t being offered by students, but we agreed that with more detailed questioning they could be teased out.

Steven Gibson (the teacher of the lesson) and all observers felt after the lesson that students who were presenting solutions at the board need to communicate more effectively. If allowed to do so, students at the board will just write. They need to turn around and explain their thinking out loud to the classroom. Students didn’t understand initially that, when a pupil was finished explaining a solution at the board, other pupils in the group could add to the explanation. The teacher clarified this early on and it produced good insights. Putting each student’s name on his or her solution is important to add ownership and a sense of worth to their work. In any future lesson planning, we will be very aware of the need for this communication at our planning stage.

The structure and flow of the lesson worked well. Shauna Kelly highlighted that some pupils were quiet and initially at times did not see the key features of patterns and connections which we were trying to emphasise. However, good effective questioning in the Ceardaíocht was critical to helping them to go in the right direction. When a pupil came up to the board to develop the link between the formula and the table, the class showed a good understanding of this when questioned during the discussion. Towards the end of the class, one group saw that the equation of the line is essentially the same as T_n. It was good that links were formed through thoughtful questioning. Teasing out solutions was generally well done. One major lesson we as teachers learned from the whole experience is the essential value of detailed consideration before the lesson of how we want to conduct Ceardaíocht. Where we had planned this well the results of Steven Gibson's questioning during the class were very productive in achieving our aims of student communication and understanding. As a group we saw afterwards that we left a few good opportunities for learning behind that we could have taken with stronger planning. More valuable time during Ceardaíocht could have been spent on driving home key features of some solutions and the most important connections. Stephen Gammell particularly wanted to highlight the importance of Student Response 5 to show the physical relationship between the slabs and the algebra. When they get to $T_n = 4n + 4$, we could ask questions like "Where is each *4* coming from?"; "Where do they feature in the diagrams?"; "What is *n*?". Algebra is less meaningful to learners if they do not see a link to comparable physical situations to deepen their understanding. It is important to move them on from explaining what they do to why they do it (e.g. "I did this because . . ."; "So what about this part of your answer?"). These were major goals of our lesson. At times students need to be pushed harder to explain clearly and justify their reasoning. In short, at the planning stage of lessons in future more time will be devoted to considering how to tease out the deepest understanding possible. We may place more demands on students during our lessons. The Ceardaíocht was very successful in getting all students to see key features of linear patterns. One or two anticipated solutions did not fully originate from the groups. Towards the end the teacher directed the discussion towards further development of the graphical solutions. However, the class did not greatly emphasise the role of the slope of the line in linear patterns. As stated before, they preferred to focus on the algebraic elements of formulae, being very comfortable with these. Also, while many students used the constant difference of *4* to set up the coefficient of n in $T_n = 4n + b$, nobody approached the problem via $T_n = an + b$ and subsequently using simultaneous equations. A reasonable explanation for this would be that students felt the former approach was much the better of the two. Expressing the common difference or slope as the rate of change may have been worth exploring more.

The boardwork was very effective in two respects. The part answers prepared in advance by Steven Gibson provided a helpful basis for all the solutions developed (other than the functions approach which we had not anticipated). Furthermore, students could clearly see the progression of solutions during and at the end of the lesson. In their comments afterwards, students showed that this sequencing helped them see the connections between the approaches and how key features of linear patterns featured in each one. At the stage of the year when this lesson occurred (late January of 3rd Year), this lesson was very valuable as a broad revision before the mock exams. The boardwork was clear and visible to all in the class. It was clear from the student feedback at the end of the lesson (where students noted briefly on post-its the main learning they enjoyed and difficulties they faced) that the main aims of the lesson were largely achieved. This feedback is very useful for informing future lessons with this class for Steven Gibson and also for our Mathematics Department in future planning of Lesson Study.



The extended learning looking at the area of the pond for homework was immediately promising. In the brief exploration directed by the teacher near the end of the class, already some students had considered "What's the common difference?" and had recognized that "There isn't one". This was already the basis of seeing a difference in this type of pattern.

12. Reflection

The main features of the lesson which stand out for us are the huge benefits of using a well-planned problem-solving approach to examining mathematical situations. Enabling students to be truly actively engaged in thinking about how to approach problems, in constructing multiple solutions to the same problem and in considering links between various strategies really enhanced understanding and confidence. We had hoped that students would demonstrate improved understanding of the key elements of linear patterns as the lesson progressed. This certainly happened. Good questioning by the teacher and well-structured boardwork aided this.

As outlined above, as a group of teachers planning a Lesson Study class, we must consider a number of important issues for the future. Firstly, at the planning stage, we must practice our boardwork more to emphasise the importance of students genuinely communicating as they present answers at the board. The development of this skill takes time and practice for both the teacher and students. This communication cannot be perfunctory. It is a central objective of the process and must be thorough and searching. The most important part of the whole Lesson Study concept is the Ceardaíocht stage. We learned during the lesson that, where we considered this discussion in depth in advance of the class and practiced extensively the type of questioning we judged to be essential, the outcome of our Ceardaíocht was more productive. For any anticipated solutions where we did less planning on what discussion may arise, we found that we got less from the subsequent interaction in the class. The potential benefits of such student work is too valuable to miss out on. We had the benefit of two "knowledgeable others" with extensive experience in this field. They were very happy with the flow and outcomes of the class, but they were keen to highlight the centrality of Ceardaíocht. Our teaching group recognises that our preparation must be directed to get the most from this stage. In short, more time at planning should be spent on core elements of the Lesson Study process. These core elements arise from the research theme and the goals of the lesson, which we must keep in mind when allocating our planning time.

Engaging in the process of Lesson Study over the last four years has brought real benefits to the students and maths teachers of Deele College. We have found that the supports developed by the Project Maths team over that time have enabled us to make much better use of the process. The learning outcomes for students in terms of better communication and deeper understanding of concepts are clear. Collaborative planning, critical evaluation and peer observation embedded in Lesson Study have offered our teachers valuable opportunities to reflect on and improve the quality of maths teaching in our school.