Lesson Research Proposal for 2nd Year Geometry

For the lesson on 25th of January 2018
At Coláiste Na Toirbhirte, Eimear White class 2nd year
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1. Title of the Lesson: Plane Clothes

2. Brief description of the lesson

Combine students understanding of the links between the properties of 2D shapes (triangle) and co-ordinate geometry.

3. Research Theme

Learner outcomes – We want students to enjoy their learning, are motivated to learn, and expect to achieve as learners have the necessary knowledge and skills to understand themselves and their relationships

Teachers' collaborative practice – As teachers we want to value and engage in professional development and professional collaboration

As mathematics teachers, we will actively support the achievement of these goals by paying attention to the following entry points in my every day classes:

- Give pupils opportunities to design, present and explain their own reasoning in coming to one solution of a problem that is of relevance to the pupils
- Teachers partake in collaborative practice both informally and more formal settings such as lesson study

4. Background & Rationale

a) Why you chose the topic

We as a math's department have identified that pupils find it difficult to relate their knowledge of co-ordinate with properties of 2-D shapes. Our pupils learn a method and replicate for future problems without fully appreciating the relationships with existing knowledge and understanding.

Usually pupils do not get the opportunity to apply their knowledge of 2-D shapes to the Cartesian plane until 3rd year when they start to work through exam papers.

We hope that pupils' understanding of both topics will be improved while also giving the opportunity to look at exam type questions. As a result, we would hope to improve the confidence levels of students and their attitudes towards mathematics. We also want to facilitate different learning types in relating the physical properties of 2-D shapes and mathematical content.

b) Your research findings

As a department we teach Co-ordinate geometry at the beginning of 2nd year. Some of the properties of 2-D shapes are studied at the end of 1st year (time dependent) and the rest is completed in 3rd year.

Through discussions of members of the maths department we realise we currently do not explicitly make links between the two topics in enough detail. From analyzing past examination papers, we noted that several topics are inter-linked within questions with many questions relating to real life scenarios. Coordinate Geometry is taught from a procedural point of view - plotting points, slope and midpoint, equation of the line and ending with points of intersections of lines, parallel lines and perpendicular lines. We do not link back to area in geometry as it does not form part of our scheme of work.

Due to these deficits we have decided to integrate teaching co-ordinate geometry with properties of 2-D shapes using a problem-solving situation which naturally gives rise to identifying the relationships of 2-D shapes and co-ordinate geometry.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
By the time students complete 6th class in primary school they should be able to: • Make informal deductions about 2-D shapes and their properties • Use angle and line properties to classify and describe triangles and	In this unit of work students will revise how to: • Calculate the area of a rectangle and square • Calculate the area of a triangle (right-angled and non-right-angled) • Calculate the area of other quadrilaterals	There are a number of learning outcomes in Leaving-Certificate maths which are related to student learning in this unit. These include: • Applying the result of the Theorem of Pythagoras to solve right-angled triangle problems of a simple nature

quadrilaterals • Use 2-D shapes and properties to solve problems • Plot simple co-ordinates and apply where appropriate use • geoboards and squared paper • Discover that the area of a rectangle is length by breadth • Estimate and measure the area of regular and irregular 2-D shapes • Calculate area using square centimetres, square metres, ares and hectares • Recognise that the length of the perimeter of a rectangular shape foes not determine the area of the shape • Measure the surface area of specified 3-D shapes • Find the area of a room from a scale plan By the time they complete first year in secondary school students should be able to • Coordinate the plane • Locate points on the plane using coordinates	• Locate points on the coordinated plane • Plot points on the coordinated plane • Calculate the length of a line segment • Calculate the slope of a line segment • Find the equation of a line • Find the intersection of two lines • Identify two lines as being perpendicular based on their slopes Students will also learn how to: • Apply the skills of coordinate geometry to geometry problems • Recognise that the application of algebra to geometry (that is the use of coordinate geometry) offers increased accuracy • Use surd relationships to simply numerical expressions • Use Pythagoras's Theorem to determine if a triangle is right-angled	involving height and distance • Using trigonometric ratios to solve real world problems involving angles • Solve problems involving the perpendicular distance from a point to a line and the angle between two lines • Solve problems involving slopes of lines • Use slopes to show that two lines are parallel or perpendicular • Solve problems involving a line and a circle • Use suitable strategies to find the length of the perimeter and the area of the following place figures: disc, triangle, rectangle, square and figures made from combinations of these • Select and use suitable strategies to estimate the area of a combination of regular and irregular shapes • Draw and interpret scaled diagrams

6. Goals of the Unit

- Students will understand that some types of problems do not have a single method to find the solution, rather they have a finite (or infinite) set of solutions methods.

- Students may then apply and combine their prior knowledge of properties of 2-D shapes with co-ordinate geometry. This will increase their awareness of links between topics and expose them to exam style questions earlier in 2nd year.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	2-D shapes- triangles
2	2-D shapes-quadrilaterals
3	Revision on co-ordinate geometry
4	The research lesson
5	Further exploration of problem solving with 2-D shapes on the Cartesian plane

8. Goals of the Research Lesson:

a) Mathematical Goals

Students will:

- understand the concept how 2-D shapes can be analysed on the Cartesian plane
- understand the need to apply the distance formula to find the area of a triangle
- apply slope formula to verify given triangle is right angled
- the advantage of using formulas from coordinate geometry instead of using counting methods such as counting boxes, using ruler etc.
- Importance of accuracy in answers.

b) Key Skills and Statements of Learning

In the planning and design of this lesson the Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will implement and promote JC Key Skills in the following ways:

1. Being Literate: Students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: It will develop a positive disposition towards problem solving.
3. Managing Myself: Student's will have the opportunity to reflect on their own learning.
4. Staying Well: Students' confidence and positive disposition to learning will be promoted.
5. Communicating: Students will present and discuss their mathematical thinking.

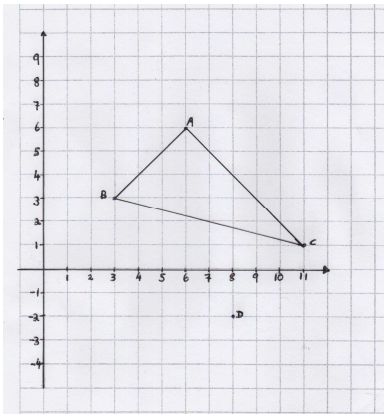
6. Being Creative: Students' will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with Others: Students will learn with and from each other.
8. Managing information and thinking: Students will be encouraged to think creatively and critically.

This lesson is also designed to meet the following JC Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The students describe, illustrates, interprets, predicts and explains patterns and relationships.
17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.
18. Observes and evaluates empirical events and processes and draws valid deductions and conclusions

9. Flow of the Research Lesson:

Steps, Learning Activities **Teacher's Questions and Expected Student Reactions**	**Teacher Support**	**Assessment**
Introduction: To begin I will inform students of the learning intentions of today's lesson. We are going to use our mathematical knowledge to solve a problem. We will try to solve the problem by ourselves and then we will come together as a class to learn something new. We will complete a short refresher of student's knowledge before they begin the problem. I intend doing this recap by providing the diagram from the problem. Students will be required to state what they see in the diagram. After this I will ask them what they could be asked to do, find, calculate on this diagram.	To help students to stay on task I will write the keywords on the board as they are mentioned. This will speed up the revision ensuring no repetition while also building on each student's answers.	The use of questioning will involve all students, thus insuring an inclusive classroom, while also giving the students ownership of their learning.

Posing the Task A clothes designer has launched a new brand of clothing. The designer's logo, a triangle, will appear on all the clothes she produces. This logo is outlined below. Find the area of the triangle, ABC, using a variety of methods.  	The question and diagram will be given to each student on a handout while also receiving a free-standing triangle with the same dimensions as that on the coordinate plane. The diagram will be displayed on the board on GeoGebra while A2 copies of the diagram will be posted on the walls in the classroom. Students will also receive an answer sheet on which the diagram also appears to aid them to see the several methods to solving this problem.	To make sure students understand the task at hand they will be encouraged to read the question several times. The answer sheet will not be handed out at the same time to aid their comprehension and make sure they are disciplined to read before starting task.
Student Individual Work Student Response 1: "Counting boxes" Students Response 2: "Measurement using a ruler" Student Response 3: "Placing the free-standing triangle on the origin and Axis for the measurements."	Is that the most accurate way of measuring distance/area? Can you think of another method? Are there more methods to finding the area? Is this going to work for any triangle or is this triangle special? Why did you use that method? Why did that method work? Would it work for every triangle?	Do students understand that area is the number of square units Do pupils understand that this is an estimate Pupils use a ruler to measure the base and perpendicular heights and apply area of triangle formula Do pupils understand that the Cartesian plane can be used to measure distance from the x and y axes

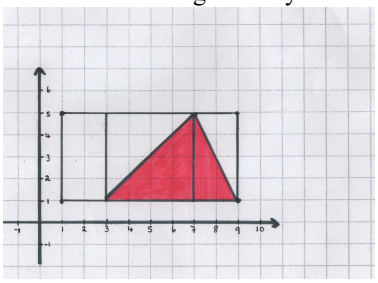
Student Response 4: "Distance formula to calculate the lengths of the sides"	How do you measure distance on the Cartesian plane? Is there a formula for calculating distance	Pupils have correctly labeled and inserted co-ordinates into the distance formula
Student Response 5: "Forming a rectangle with free standing triangles, calculating its area and then dividing by 2 to get area of the triangle."	Can you use more than 1 of the free triangles to help find the area? What is the relationship between this rectangle and the triangle What is the advantage of turning it into a rectangle? Are opposite sides parallel and equal? Could you verify this?	Pupils align free triangles on x and y axes or parallel lines to read off length
Student Response 6: "Verifying the triangle is Right Angled by calculating the slopes of the line or using a protractor or set-square, then proceeding to find the length and area of the triangle."	How can you check it is a right angle/perpendicular height?	Pupils use $m_1 \times m_2 = -1$ Can you use your protractor?
Student Response 7: "Forming a rectangle by using coordinate D in diagram, measuring sides using ruler and calculating its area, then dividing by 2 to get area of the triangle."	Is there a more accurate mathematical method (formula) for measuring length/distance? What is special about co-ordinate D	Pupils have connected co-ordinate D to C and B forming a rectangle
Student Response 8: "Forming a rectangle, calculating its area having measured lengths of sides using formula using these to find area then dividing by 2 to get area of the triangle."	How can perpendicular angles be verified? Why did you move the triangle? Will this transformation maintain area?	Pupils use the free triangle to verify triangle is perpendicular/right angled Pupils use distance formula to get length of sides and half the answer

Student Response 9: "Rotating the triangle or moving it to another place on graph for it to look more apparent that it is a right-angle triangle."	Are the X and Y axes perpendicular? Are all lines parallel to X and Y axes also parallel?	Pupils have moved free triangle onto (parallel to) X and Y axes
Student Response 10: "Forming smaller right-angle triangles and large rectangle around ABC, calculating their areas and then subtracting the area of the triangles from the area of the rectangle."	What is the advantage of using this method? (is this a better method of a non-right angled triangle)	Pupils form a rectangle with each vertex of triangle touching side of rectangle. Sides are parallel to X and Y axes
Student response 11: Using the formula $\frac{1}{2} \lvert x_1y_2 - x_2y_1 \rvert$ from the log table	When is this formula/method applicable? Why is there only 2 co-ordinates in this formula when a triangle has 3 vertices/co-ordinates	Pupils have transformed the triangle with one vertex on (0,0). labelled other 2 vertices and substituted these co-ordinates into formula
Ceardaíocht /Comparing and Discussing	General- What do you think? Why is that? Did anyone else solve it the same way? Can you explain this method? Is this method accurate? Did you make any assumptions using this method? Where did you find this formula?	Pupils are engaging in the discussion Pupils are defending their methods and analyzing the merit of other methods
Student Response 1: "Counting boxes"	How did you count the non-full boxes? Were there any other ways of counting the non-full boxes? Is this accurate?	Pupils understand with shapes on a Cartesian plane it is not possible to count whole boxes
Student Response 2: "Measurement using a ruler"	Is this an efficient way of calculating area?	Pupils accept that a mathematical solution is more accurate than a measured solution

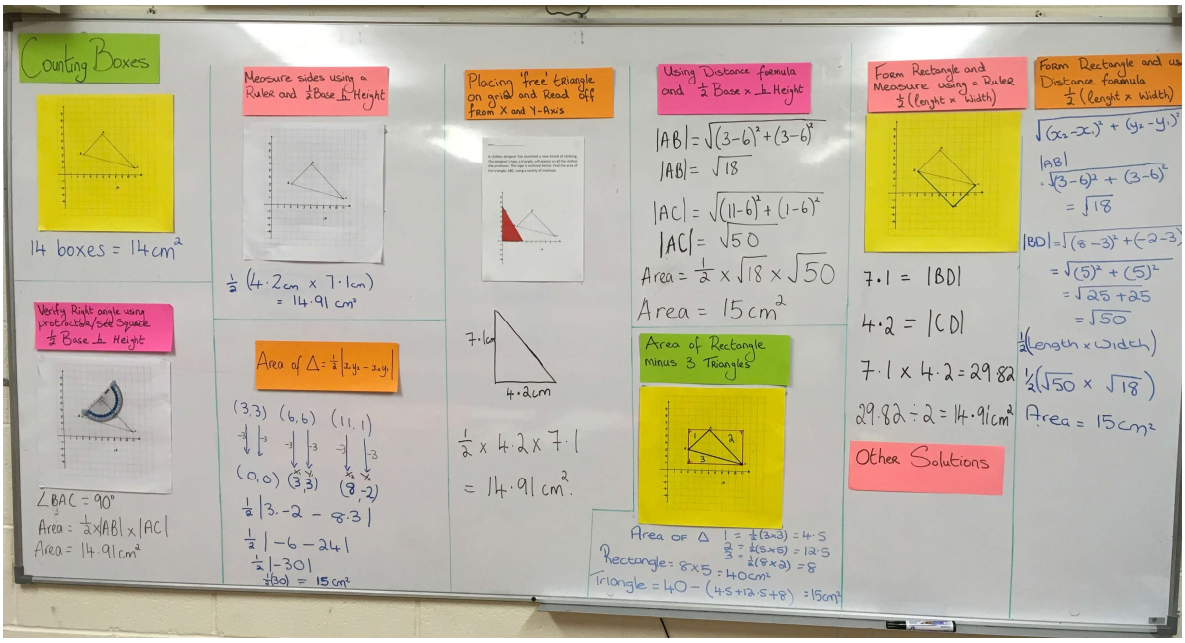
Student Response 3: "Placing the free-standing triangle on the grid for the measurements."	Why did that corner of the triangle line up with x and y axes? How could you check it is a 90 degree angle?	Pupils realize that the x and y axes are perpendicular to one another Pupils recall $m_1 \times m_2 = -1$
Student Response 4: "Distance formula to calculate the lengths of the sides"	Why did you choose to find the distance of these 2 sides of the triangle? What is the relationship between perpendicular distance and this angle?	Pupils will note that for the area of a triangle the height must be the perpendicular height
Student Response 5: "Forming a rectangle with free standing triangles, Measuring sides with a ruler and calculating its area. Then dividing by 2 to get area of the triangle."	Why did you form a rectangle? What is the relationship between this rectangle and our triangle? Are there other rectangles that could be formed from 2 of our triangles?	Pupils will see that the triangle may be doubled to form a rectangle, this will have easier to discern measurements Pupils will discuss importance of the need to verify assumptions
Student Response 6: "Verifying the triangle is Right Angled by calculating the slopes of the lines, then proceeding to find the length and area of the triangle."	Why is it important to verify this is 90 degrees? What way did you calculate slope? Are there other ways of calculating slope?	Pupils will offer alternate methods to calculate slope. Pupils will explain why the area of the rectangle is double the area of the triangle
Student Response 7: "Forming a rectangle by using coordinate D in diagram, calculating its area and then dividing by 2 to get area of the triangle."	How could you ensure that opposite sides and opposite angles are equal? How do you know co-ordinate D will for a regular rectangle? Can you verify this?	Pupils will agree distance formula is a more accurate way of measuring length. Can you move the triangle on the Cartesian

		plane mathematically? Does this correspond with the co-ordinates when moved?
Student Response 8: "Forming a rectangle, calculating its area having measured lengths of sides using distance formula and then dividing by 2 to get area of the triangle."	Is this a more accurate way of measuring the length of the sides of the rectangle?	
Student Response 9: "Rotating the triangle or moving it to another place on graph for it to look more apparent that it is a right-angle triangle."	Why did you need to verify it was a right angled triangle? What is the relationship between the x and y axes?	Pupils will discuss if the main triangle needs to be a right angled triangle for this method to work
Student Response 10: "Forming smaller right-angle triangles and large rectangle around ABC, calculating their areas and then subtracting the area of the triangles from the area of the rectangle."	What is the advantage to this method? Did you have to verify the triangle was a right angled triangle? Would this work for a non-right angled triangle?	Pupils will be able to discuss which methods have similar approaches.
Student response 11: Using the formula $\frac{1}{2} \lvert x_1y_2 - x_2y_1 \rvert$ from the log table	Are there any formulas in the log tables that could calculate area of a triangle? What does this formula only have 2 co-ordinates (when a triangle has 3)	Pupils make use of their log tables Pupils have attempted translation Pupils able to relate answer to diagram/estimate/other solutions
Categorising approaches.	What methods have common approaches? Which are the least accurate? Which methods are the most accurate?	Pupils are able to discuss what is common or different in the solutions
Summing up & Reflection We learned that:	Ask pupils to comment on solutions?	Pupils are actively involved in discussion on

It is possible to combine knowledge of topics in answering of a problem There are links between topics (Co-ordinate geometry and 2-D shapes) There are many ways to find the area of the triangle We should examine the question from all aspects Using co-ordinate geometry is more accurate An estimation is not an accurate solution We should attempt everything The importance of verifying any assumptions How to deal with varying number systems (surds vs decimals) The following extension question will be used to further enhance pupil learning and understanding Similar methods **Estimation-** • Counting boxes • Measurement using a ruler • Placing the free-standing triangle on the grid for the measurements • Forming a rectangle with free standing triangles, measuring sides with a ruler and calculating its area. Then dividing by 2 to get area of the triangle • Forming a rectangle by using coordinate D in diagram, measure using ruler and calculating its area and then dividing by 2 to get area of the triangle **Co-ordinate Geometry-** • Distance formula to calculate the lengths of the sides" • Verifying the triangle is Right Angled by calculating the slopes of the lines, then proceeding to find the length and area of the triangle • Forming a rectangle, calculating its area having measured lengths of sides using distance formula and then dividing by 2 to get area of the triangle	What is the same? What is different? What methods are more accurate? What methods do you prefer?	different methods and solutions Pupils are taking ownership of their learning and their work Pupils are communicating and justifying the relevant solutions

• Rotating the triangle or moving it to another place on graph for it to look more apparent that it is a right-angle triangle • Forming smaller right-angle triangles and large rectangle around ABC, calculating their areas and then subtracting the area of the triangles from the area of the rectangle Extension question: Find the area of the shaded region in the diagram below. Find this area using as many different methods as possible. 	Teacher will give each student the extension question for homework on a separate worksheet	Pupils are eager to apply their learning to the extension question

10. Board Plan



11. Evaluation

Goals of lesson study:

a) Give pupils opportunities to design, present and explain their own reasoning in coming to one solution of a problem that is of relevance

- Pupils enjoyed learning, motivated to learn and expect to achieve as learners
- Excellent discussion on accuracy of solutions
- Appreciation of accuracy using different methods
- Pupils have a better understanding of linking 2D shapes to co-ordinate geometry
- Pupils correctly picked and applied formulas
- Pupils presented their work visually and mathematically while discussing their methods

b) Students will understand that some types of problems do not have a single method to find the solution; rather they have a finite (or infinite) set of solutions methods.

- Pupils were actively engaged in coming up with solutions and requiring more paper to continue to get other methods
  Student's hands up everywhere and took loads of paper
- Students may then apply and combine their prior knowledge of properties of 2-D shapes with co-ordinate geometry. This will increase their awareness of links between topics and expose them to exam style questions earlier in 2^{nd} year

12. Reflection

a) what the team had hoped to observe during the lesson

Pupils would embrace the challenge and work hard
Pupils would come up with multiple solutions to the problem
Pupils would use their knowledge of 2-D shapes and link this with Co-ordinate geometry

b) what was actually observed during the lesson, by the team members and others;

Pupils were very relaxed and interactive and were not nervous of teachers (intimidating for pupils)
The lesson was free flowing and teacher very encouraging and was a natural sharing atmosphere and pupil knowledge was pulled out when needed
Some pupils stopped after 2 solutions
Few pupils verified triangle was right angled mathematically (most used X and Y axes or protractor)
Some tried using $\frac{1}{2} |x_1y_2 - x_2y_1|$ but applied it incorrectly
Most pupils continued to work on solutions until time was up
Some pupils were secretive about their own work

c) major points raised during the post-lesson discussion, and the team's own opinions;

Pupils slow to present methods at start of Ceardaíocht
Had to finish Ceardaíocht earlier than would have liked to ensure all methods were displayed. Did limit increased level of pupil discussion
Intimidation of many observers was discussed
Lesson was logical and structured to aid pupil problem solving
Girls came up with many of our suggested solutions