

# Lesson Research Proposal for 2018: Simultaneous Linear Relationships

For the lesson on 31/01/2018

At Davis College Mallow, Ross Durity, 2nd Year, Higher Level

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## 1. Title of the Lesson: Power of Algebra through “Simultaneous Games”

## 2. Brief description of the lesson

Students will be presented with a simultaneous linear functions problem. They will be encouraged to solve the problem in a variety of ways, which should help them observe a relationship that is linear in nature and that can be represented in a variety of ways. In doing so we hope to show the pupils the links between different strands and how the world of mathematics and the skills and concepts that they learn are all inter-connected.

## 3. Research Theme

In Davis College Mallow

- (a) We want students to develop a positive disposition towards investigating, reasoning and problem solving.
- (b) We want students to be more motivated, self-directed and self-regulated.
- (c) As mathematics teachers, we will actively support the achievement of these goals by paying attention to the entry points in our every day classes.
- (d) Students are regularly given opportunities to use prior learning to investigate problems.

## 4. Background & Rationale

- a) We have observed that students lack the skills needed to apply algebra to real life problems and hence become demotivated when studying it. This problem and our responses should help the students who struggle to model relations by using linear equations. We also want to highlight the need for algebra in quickly and efficiently making predictions by comparing its efficacy to other methods.
- b) We believe that a great number of our students are visual learners. Hence we will be linking all other representations of the problem with the graphical solution and highlighting the links between them.
- c) We are also aware of the on-going need to present students with problems that allow them to see the links between various strands hence improving their conceptual understanding of the course in its entirety. We want to present a problem that allows them to create links between strands such as algebra, patterns, functions and co-ordinate geometry. We want to try to empower students to develop these links by creating a practical problem that can be interpreted in various ways which include tables, diagrams and graphs.

## 5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<p><b>In Primary School the child should be enabled to:</b></p> <ul style="list-style-type: none"> <li>● explore the concept of a variable in the context of simple patterns, tables and simple formulae and substitute values for variables.</li> <li>● identify and discuss simple formulae from other strands.</li> <li>● substitute values into formulae and into symbolic rules developed from number patterns.</li> </ul> <p><b>In other subjects they will have:</b></p> <ul style="list-style-type: none"> <li>● used tables, diagrams and graphs as a tool for analysing relations.</li> <li>● developed and used their own mathematical strategies and ideas and considered those of others.</li> <li>● presented and interpreted solutions, explained and justified methods, inferences and reasoning.</li> </ul>	<p><b>Students should be able to:</b></p> <ul style="list-style-type: none"> <li>● use tables to represent a repeating-pattern situation.</li> <li>● generalise and explain patterns and relationships in words and numbers.</li> <li>● write arithmetic expressions for particular terms in a sequence</li> <li>● use tables, diagrams and graphs as tools for representing and analysing linear relations.</li> <li>● develop and use their own generalising strategies and ideas and consider those of others.</li> <li>● present and interpret solutions, explaining and justifying methods, inferences and reasoning</li> <li>● use representations to reason about the situation from which the relationship is derived and communicate their thinking to others.</li> </ul>	<p><b>Junior Cert</b></p> <p><b>Co-ordinate Geometry:</b></p> <ul style="list-style-type: none"> <li>● Finding the equation of the line using - <math>y=mx+c</math></li> <li>● Finding the y-intercept and slope of a line using <math>y=mx+c</math></li> <li>● slope of a line</li> <li>● Intersection of two lines</li> </ul> <p><b>Patterns:</b></p> <ul style="list-style-type: none"> <li>● Identifying and representing linear patterns and sequences.</li> </ul> <p><b>Interpreting real-life graphs</b></p> <ul style="list-style-type: none"> <li>● Exploring proportionality</li> <li>● Making predictions</li> <li>● Slopes and rates of change</li> </ul> <p><b>Leaving Cert</b></p> <p><b>Algebra</b></p> <ul style="list-style-type: none"> <li>● Solving simultaneous equations (linear and linear, linear &amp; non-linear)</li> </ul> <p><b>Co-ordinate Geometry:</b></p> <ul style="list-style-type: none"> <li>● Finding the equation of the line using <math>y=mx+c</math></li> <li>● Finding the y-intercept and slope of a line using <math>y=mx+c</math></li> <li>● slope of a line</li> <li>● Intersection of two lines</li> </ul>

		<p><b>Sequences and Series</b></p> <ul style="list-style-type: none"> <li>Identifying and representing linear, quadratic and exponential patterns and sequences</li> </ul> <p><b>Calculus:</b></p> <ul style="list-style-type: none"> <li>Rates of change</li> <li>First differentials</li> <li>Second differentials</li> <li>Interpreting graphs</li> </ul>
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## 6. Goals of the Unit

In the course of studying this strand the learner will;

- engage with the concept of a function (that which involves a set of inputs, a set of possible outputs and a rule that assigns one output to each input).
- emphasise the relationship between functions and algebra.
- connect graphical and symbolic representations of functions.
- use real life problems as motivation for the study and application of functions.
- use appropriate graphing technologies.

## 7. Unit Plan

Lesson	Learning goal(s) and tasks
1 The Research Lesson 1×50 minutes	Introduce relations through a real life problem Use a suitable problem to <ol style="list-style-type: none"> <li>Understand that relations can be expressed and solved numerically.</li> <li>Understand that relations can be expressed and solved graphically.</li> <li>Understand that relations can be expressed and solved algebraically.</li> </ol>

<p>2 1×40 minutes</p>	<p><b>Proportional Relations</b> The students will explore and hence define what relations are. Students will look at relations as a relationship between two sets, a set of inputs and outputs using a table and will link this also to the graph. They will specifically explore proportional linear relations. The students will represent these relations using suitable tables and graphs. Students will be able to define what a linear pattern looks like by linking the numerical patterns that emerge from the problem to the graphical representation of the problem. Links can be made to co-ordinate geometry here. (i.e. representing the equation of a line in the form of <math>y=mx+c</math>) <i>Relate to research lesson</i></p>
<p>3 1×40 minutes</p>	<p><b>Non proportional relations</b> Explore non-proportional linear relations. The students will represent these relations using suitable tables and graphs. Link the numerical pattern that emerges to the graphical representation of these relations. Reinforce the concept of looking at relations as a relationship between two sets, a set of inputs and outputs using a table and link this also to the graph. Links can be made to co-ordinate geometry here. (i.e. representing the equation of a line in the form of <math>y=mx+c</math>) <i>Relate to the relations explored in the research lesson.</i></p>
<p>4 2×40 minutes</p>	<p><b>Linear relations</b> Represent linear relations using graphs. Reinforce the concept of looking at relations as a relationship between two sets, a set of inputs and outputs using a table and link this also to the graph. <i>Relate to the relations explored in the research lesson</i></p>
<p>5 2×40 minutes</p>	<p><b>Quadratic relations</b> Represent quadratic relations using graphs. Reinforce the concept of looking at relations as a relationship between two sets, a set of inputs and outputs using a table and link this also to the graph.</p>
<p>6 1×40 minutes</p>	<p><b>Exponential relations</b> Represent exponential relations using graphs. Reinforce the concept of looking at relations as a relationship between two sets, a set of inputs and outputs using a table and link this also to the graph.</p>
<p>7 2×40 minutes</p>	<p><b>Introduction to functions</b> Introduce the students to functions. Build on the concepts of inputs and outputs. Represent relations using mapping diagrams and couples. Students will start to define the <math>2\times 40</math> minutes of a function from a mapping diagram. Link the mapping diagrams to the graphical representations of the problem. <i>Create mapping diagrams for the relations explored in the research lesson. Define the domain, co-domain and range of such relations.</i> Explore how to define if a relation is or is not a function using a variety of different representations.</p>
<p>8 3×40 minutes</p>	<p><b>Function Notation</b> Explore different notation using different mathematical relationships (linear, quadratic and exponential). Explore different functions in context and link the function notation and algebraic notation to the different relationships being explored. Answer questions both graphically and algebraically. <i>The lesson study problem can be used here also to contextualize the skills being practiced.</i> Students will begin to graph basic functions.</p>

## 8. Goals of the Research Lesson:

- Students will understand that there is more than one way of representing linear relationships, i.e. problems involving such relationships can be solved numerically, by using a table, algebraically (and by using function notation) or graphically.
- Students will experience how and why algebra is used specifically in relation to representing relationships between variables.
- We want students to recognize that all strands are interconnected and that it is important and that real life problems can be solved in a variety of ways.
- Key Skills & Statements of Learning**  
In the planning and design of this lesson the Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will implement and promote JC Key Skills in the following ways:

- Being Literate: Students will have the opportunity to express their ideas clearly and accurately
- Being numerate: it will develop a positive disposition towards problem solving
- Managing myself: Students will have the opportunity to reflect on their own learning
- Staying well: Students confidence and positive disposition to learning will be promoted
- Communicating: Students will present and discuss their mathematical thinking
- Being creative: students will explore options and alternatives as they actively participate in the construction of knowledge.
- Working with others: Students will learn with and from each other.

### JC Statements of learning:

- The student communicates effectively using a variety of means in a range of contexts
- The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning
- The student describes, illustrates, interprets, predicts and explains patterns and relationships.
- The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

## 9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
<p><b>Introduction (2 mins)</b> Welcome students into the class and make them aware of the role of the teachers in the class to help them feel comfortable about the process. Ensure that they know that the teachers are not here to judge and that today's class is all about participation and enjoyment.</p>	Does anyone notice anything different about today's class?	Students will be made to feel comfortable in order that they are willing to explore the problem without any stress.
<p><b>Posing the Task (5mins)</b> There are two companies that can supply a new video game. Game Stopper tells us that it has 40 units in stock but is selling more than it is producing resulting in its warehouse losing 2 units each week. Crazy Games Inc. informs us that it has 20 units in stock but is producing more than it is selling resulting in its warehouse gaining 2 units each week. After how many weeks will CrazyGames Inc. have the</p>	At the start after week "0" how much does each supplier have? What does "losing" and "gaining" mean? What mathematical operations do these apply to?	The first three questions are for the students who might struggle to comprehend the nature of the problem?  Students need to

same number of units as GameStopper? Can you solve this problem in as many ways as possible?

**Game Stopper:**



**Crazy Games:**



After week 1 and 2 how many units do each company have in stock?

How many ways can we solve this problem? How can we solve this problem quickly and efficiently?

comprehend the problem fully to spot the patterns involved and hence, be able to represent the problem using a table and graph. In order to do so we can look at what happens to both companies' stock over the first few weeks.

The 4<sup>th</sup> question is to ensure that students approach this problem in diverse ways using multiple methods.

We want to show the connections between different strands by allowing the pupils to represent and solve the problem in a variety of ways.

**Student Individual Work**

**Response 1:**

*Numerical Method (Number crunching)*

Crazy Games:                      Game Stopper:  
 20,22,24,26,28,**30**,32,34    40,38,36,34,32,**30**,28,26  
   1  2  3  4  **5**  6  7            1  2  3  4  **5**  6  7  
 After week 5 they are the same.

**Response 2:**

*Creating a table to help explore the relationship*

One of many possible responses in table form

Week	Number of units
0	20
1	22
2	24
3	26
4	28
5	30

If a student is struggling to comprehend the problem and approach it numerically this will be the only time that assistance will be provided. Otherwise the teacher will monitor the progress of the class and gather appropriate responses that they will use to show the flow of logical responses.

Response 1  
 If a group is struggling hopefully they will be able to use the numerical method to solve the problem.

Response 2  
 We will try to use an example of a table that shows the relationship between time and the number of units. Ideally we would like a highlight a table that shows the pattern emerging through the use of multiplication.

Week	Number of units
0	40
1	38
2	36
3	34
4	32
5	30

**Response 3:**

*Using Arithmetic:*

GameStopper starts with 40 and CrazyGames with 20. Therefore, GameStopper has 20 more than CrazyGames. Each week GameStopper reduces by 2 and CrazyGames increases by 2 so the difference is 4 units.  
 $20/4=5$  weeks

**Response 4:**

*Creating algebraic representations of the relationship.*

Number of units =  $40 - 2 \times$  number of weeks  
 number of weeks =  $x$  (or another variable possibly)  
 number of units =  $y$  (or another variable possibly)  
 $y = 40 - 2x$

Number of units =  $20 + 2 \times$  number of weeks  
 number of weeks =  $x$  (or another variable possibly)  
 number of units =  $y$  (or another variable possibly)  
 $y = 20 + 2x$

Students may use the substitution method or solve the two equations simultaneously.

$$40 - 2x = 20 + 2x$$

$$20 = 4x$$

$$5 = x = \text{number of weeks}$$

Or

$$y = 40 - 2x$$

$$y = 20 + 2x$$

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$$0 = 20 - 4x$$

$$4x = 20$$

$$x = 5$$

Response 3

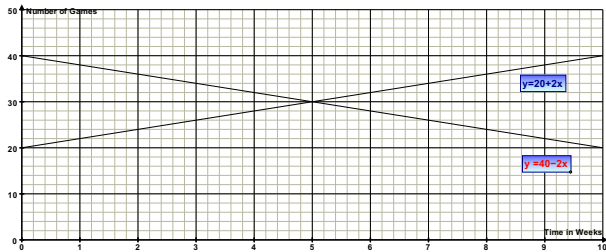
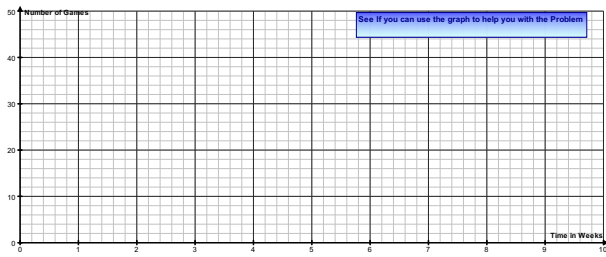
If a student has started to use this approach and not succeeded, we may look at using that approach in a further class

Response 4 can be written in different variables and this must be taking into account. The equations can also be solved simultaneously or by using substitution. If possible we will try to find an example of each method.

**Addressing common errors:**

We can also use an example that may have failed to change all the signs when attempting to cancel one of the variables.

**Response 5:**  
**Graphical Method:**



Response 5  
We will try to find a graph that has both relationships graphed on it. If we cannot, we will use a combination of student responses to try to form the anticipated graphical response that will be pre-prepared.

**Addressing common errors:**  
If students have mixed up their x and y values and have consequently made mistakes plotting points we can highlight this common mistake by showing some of those graphs.

**Ceardaíocht /Comparing and Discussing**

**Response 1: (2 mins)**  
**Numerical Method (Number crunching)**

If a group is struggling they should be able to approach the problem numerically. If this is the case their numerical approach will be chosen. The teacher will test the speed and efficiency of this approach to predict the future?

A response that clearly shows the linear pattern that's created by each companies growing or falling number of units can be used later to show why a pattern of such is linear in nature.

We can make reference to the pattern that emerges from the increasing and decreasing number of units of each company.

**Response 2: (8 mins)**  
**Creating a table to help explore the relationship**

We will use an example that attempts to identify the relationship that the problem is creating between time and the number of units of each company. If no student has done so we will verbally ensure that the students are aware of the relationship that they are exploring. We will have a table like the one below that can be filled out in part by students who spot the pattern that emerges between the number of weeks and number of units.

Hands up who used this approach? Do you think this is a good approach? Why? When is this method suitable? If the numbers were greater in magnitude or more awkward in nature (i.e dealing with thousands and fractions) would this be a suitable method?

How many people used a table of some sort to find a solution? Why did you use it? What relationship are you looking at and what does this table show us? or Does this table identify a relationship? How many people used addition in order to find the number of units that each

Show that everyone can access a problem by selecting a student's response who may not have found further ways of solving the problem.

**Assessment worksheet:**  
Can we make quick and efficient predictions using this method? - Ask this question before and after asking the below questions to see if their opinion has changed.  
After how many weeks will Gamestopper be out of stock? How many units will CrazyGames have after 27 weeks?

We are starting to talk about a relationship between 2 things and how we can present it in a structured manner. Through more structure we can start to define the relationship.

Use a variety of



Week		Number of units	Number of units
0	20	$20 + 2(0)$	20
1	$20+2$	$20 + 2(1)$	22
2	$20+2+2$	$20+2(2)$	24
3	$20+2+2+2$	$20+2(3)$	26
4	$20+2+2+2+2$	$20+2(4)$	28
5	$20+2+2+2+2+2$	$20+2(5)$	30

The table helps show how the relationship is formed.  
 “The number of units =  $20 + 2$  times the number of weeks”

Week			Number of units
0	40	$40-2(0)$	40
1	$40-2$	$40-2(1)$	38
2	$40-2-2$	$40-2(2)$	36
3	$40-2-2-2$	$40-2(3)$	34
4	$40-2-2-2-2$	$40-2(4)$	32
5	$40-2-2-2-2-2$	$40-2(5)$	30

Once again the table helps show how the relationship is formed.  
 “The number of units =  $40$  minus  $2$  times the number of weeks”

**Response 3: (2 mins)**  
*Using Arithmetic:*

If this response is used, we will look at it but will not go into the fine details of the method.

**Response 4: (10 mins)**

company had? How many people used multiplication? What are the benefits of using multiplication?

Are we becoming more efficient at predicting the future? Why? If we can spot the pattern is it easier to predict what happens next.

Does this approach allow us to define the relationship between weeks and number of units. Why?

Can we express this relationship in words?  
 “The number of units CrazyGamesd has is always  $20$  plus  $2$  times... The number of units Gamestopper has is always  $40$  minus  $2$  time”

*If someone has tried and succeeded to use this approach we will ask them.*  
 Why did you use this approach? Do you think it is an easier or more efficient approach than the previous approaches? Why? Can we predict the future quicker using this method? Let’s see. How many units will ... have after  $x$  weeks?

*Ask the class:*

responses here in order to include as many people as possible. Different people might show the relationship in a variety of ways so we need to be adaptable here.

*Assessment worksheet: Can we make predictions quickly and efficiently using this method. How many units will CrazyGames have after 33 weeks? By asking this and observing how many students write  $20+2(33)$  we can assess if the students understand the relationship that the table has revealed.*

This approach to problem solving will not be explored in detail as it has many limitations and is not really linked to the learning outcomes of the lesson.

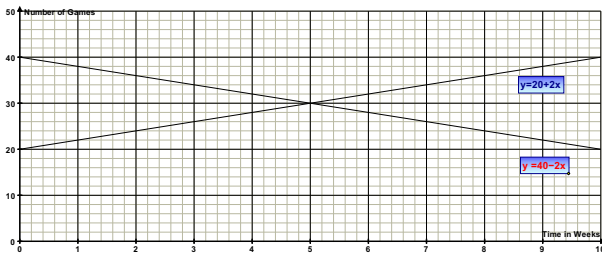
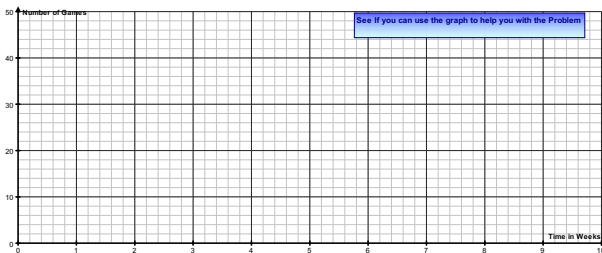
Here we are showing

<p><b>Creating and using algebraic equations to solve the problem.</b></p> <p><i>If no student has used an algebraic response, we will reason the response out in this manner</i></p> <p>What is the relationship between the number of units that each company has and the number of weeks? Where have we identified this relationship - link to response 2. – <b>VERY IMPORTANT</b></p> <p>We saw that:      “The number of units = <math>20 + 2</math> times the number of weeks”      Number of units = <math>20 + 2 \times</math> number of weeks      Let’s say we let number of weeks = <math>x</math> and let’s say that the number of units = <math>y</math>      Can we create an equation with <math>x</math> and <math>y</math> in it?</p> <p>For the other company we saw that:      Number of units = <math>40 - 2 \times</math> number of weeks      Let us say we let number of weeks = <math>x</math> and let’s say that the number of units = <math>y</math>      Can we create an equation with <math>x</math> and <math>y</math> in it?</p> <p><i>We will show the substitution and simultaneous method of solving the equations</i></p> <p><math>y=20 +2x</math>  <math>y=40-2x</math>      By substituting <math>y</math> for <math>y</math> we get:  <math>40 - 2x = 20 + 2x</math>  <math>20 = 4x</math>  <math>5 = x =</math> number of weeks</p> <p>Or</p> <p><math>y=40-2x</math>  <math>y=20+2x</math>      -----  <math>0=20-4x</math>  <math>4x=20</math>  <math>x=5</math></p> <p>We can also check if our solution is correct. How?</p> <p>We can also express this relationship in another way using functions. A function is a . . . . .</p> <p>The two functions we can create are :  <math>f(x) = 20+2x</math>      and  <math>g(x) = 40-2x</math></p> <p>If I said <math>f(1) = 22</math> what do you think that means?      If I said <math>g(1) = 38</math> what does this mean?</p>	<p>Can we use algebra by using variables to solve this problem? Who attempted to do so?</p> <p><i>If a student has solved this problem using algebra we will question them on the following:</i></p> <ul style="list-style-type: none"> <li>• How they identified the relationship?</li> <li>• Their use of variables</li> <li>• Their method of solving the equations (simultaneously or substitution method)</li> </ul> <p><i>We will compare their response to the anticipated response.</i></p> <p><i>Ask the class:</i>      How do people find this method? Would you use such a method to solve other problems this way?</p> <p>Does algebra allow us to make quick and efficient predictions? Let’s see. Use this method to answer the following:</p> <p>Assessment worksheet:      Can you use these equations to answer the following questions?      How many units will Crazy games have after 55 weeks?      After how many weeks will Crazy games have 88 units in stock?  <i>Wait for student responses and assess the amount of correct responses and the method used.</i></p> <p><i>Ask the class:</i>      What were the benefits of using algebra compared to previous methods? Was this a quick method?</p> <p>Now that we have briefly looked at function notation can you answer the following questions?</p>	<p>how a mathematical relationship can be defined using algebra. We are also showing how people can arrive at this relationship by being able to put it into words. This is important as it shows the students that there is more than one way to access this algebraic solution. We are also highlighting the power of algebra in efficiently and quickly predicting events.</p> <p>We will not spend time debating the different methods in solving the 2 equations.</p> <p>We are also introducing the concept of a function here as well as briefly exploring function notation in context.</p> <p><b>Addressing common errors:</b>      Incorrect substitution: If students have substituted the values in for the wrong variables while attempting to answer the questions on the assessment worksheet we can address this problem here.</p> <p>We are also introducing the concept of a function and function notation. We will assess their initial understanding</p>
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### Response 5: (5 mins)

#### Graphical method

We will use the most accurate graph but will also have one prepared to use on the board. If students have not included both relationships on the one graph we can use a variety of graphical responses to help create the prepared anticipated responses.



We can identify the different functions at this point by asking:

“How else can I write  $y = 20+2x$  and  $y = 40-2x$ ”

Assessment worksheet :

What is the value of  $f(2)$  ?

What is the value of  $g(3)$  ?

(Can you find  $x$  when  $f(x) = 4$ ? Optional )

*Ask the class:*

How many people used a graph to solve this problem.

How many people just plotted points? How many people joined those points? What did you see happening when you did so?

*To the person whose graph is chosen:*

How does this graph help us solve the problem? What is the point of intersection between the two graphs? Did you like to solve the problem this way? Why?

*Ask the class:*

What are the links between this solution and other responses that we have explored?

Can you link response 2 to the graph? Look at how the table creates ordered couples or points.

What does this graph show us about the relationship between the two variable quantities? Can we see why the equations are called linear equations.

Can we see why the pattern of numbers that emerges from the number of units is called a linear one. How can we tell if a pattern is linear in nature?

Here we are creating a link between the graphical representation of the relations and the other responses. This is important in showing why certain equations and patterns are called linear.

In asking them if they liked solving it this way we are exploring why graphs are used in representing different relationships in different subjects and in the real world.

We are not going to go into great detail about how to plot points but we can link response 2 to how points are created by using a table. Here we want to show them that points are plotted in the following manner (x,y)

#### Addressing common errors:

If students have mixed up their x and y values and have consequently made mistakes plotting points we can highlight this common mistake.

Assessment worksheet:

From the graph can you approximate when both companies will have 35 units of stock?

From the graph can you find  $f(6)$  and  $g(7)$ .

Which of the below pattern are linear and?

33,36,39,42

-2,-5,-8,-11

21,24,26,29

## Summing up & Reflection (5 mins)

Use the layout of the board work to show the following:

- How a problem can be solved in a variety of ways. (maths is for everyone!)
- Highlight how maths can teach us to solve problems creatively and in a variety of ways.
- Show the power of algebra in making quick and efficient predictions as opposed to other methods.
- Highlight how their algebraic skills stem from the need to solve problems such as the one being discussed.
- Show the link that we have made across different strands –highlight that more can be made including co-ordinate geometry and patterns. This example can be used in further classes to help students link different strands and to make more sense of them as they can relate what they are learning to a contextualized problem.

## 10. Board Plan

**The Problem**

GAME STOPPER HAS 40 UNITS  
AND IS LOSING TWO UNITS PER WEEK

CRAZY GAMES HAS 20 UNITS  
AND IS GAINING TWO UNITS PER WEEK.

AFTER HOW MANY WEEKS WILL CRAZY GAMES  
HAVE THE SAME NUMBER OF UNITS AS GAMESTOPPER.

**Response 1**

**Numerical Method**

40, 38, 36, 34, 32, 30, 28, 26  
0 1 2 3 4 5 6 7  
20, 22, 24, 26, 28, 30, 32, 34  
5 WEEKS

**Response 3**

**Using Arithmetic**

$40 - 20 = 20$  UNITS  
DIFFERENCE 2 and -2 = 4 units  
 $\frac{20}{4} = 5$  weeks

**Response 4**

**Using a Graph**

**Response 2**

**Using a Table**

Week	PATTERN	NUMBER OF UNITS	TOTAL UNITS
0	20	$20 + 2(0)$	20
1	$20 + 2$	$20 + 2(1)$	22
2	$20 + 2 + 2$	$20 + 2(2)$	24
3	$20 + 2 + 2 + 2$	$20 + 2(3)$	26
4	$20 + 2 + 2 + 2 + 2$	$20 + 2(4)$	28
5	$20 + 2 + 2 + 2 + 2 + 2$	$20 + 2(5)$	30

Week	PATTERN	NUMBER OF UNITS	TOTAL UNITS
0	40	$40 - 2(0)$	40
1	$40 - 2$	$40 - 2(1)$	38
2	$40 - 2 - 2$	$40 - 2(2)$	36
3	$40 - 2 - 2 - 2$	$40 - 2(3)$	34
4	$40 - 2 - 2 - 2 - 2$	$40 - 2(4)$	32
5	$40 - 2 - 2 - 2 - 2 - 2$	$40 - 2(5)$	30

**Response 5**

**Solving an Equation**

$$-20 + 2x \quad \left| \begin{array}{l} 40 - 2x = 20 + 2x \\ 20 = 4x \\ \frac{20}{4} = x \\ 5 = x \end{array} \right. \quad -20 + 2x$$

-4

-4

5

WEEK

## 11. Evaluation

- Were students more motivated, self-directed and self-regulated during this research lesson?
- Do students understand that there is more than one way of representing linear relationships, i.e. problems involving such relationships can be solved numerically, by using a table, using arithmetic, algebraically (and by using function notation) or graphically?
- Do students understand why algebra is used specifically in relation to representing relationships

between variables?

- d) Do students recognize that all strands of mathematics are interconnected and that real life problems can be solved in a variety of ways.

## 12. Reflection

- a) During the lesson we hoped to observe four key areas at work.
1. Do students connect strands in mathematics by using problem solving?
  2. Will students actually use different methods when solving a problem?
  3. During the 10 minutes, will students be more self directed and motivated?
  4. Can students use algebra to represent relationships?
- b) During the lesson we actually observed the following.
1. Students did actually connect up the different strands in mathematics. They could see first hand the different areas of mathematics being used.
  2. We found that when students are left on their own with the problem they will endeavor to find different methods from different strands. We were quite surprised at the amount of students who tackled the problem in different ways. In particular, one interesting approach which had been predicted, came up in a most unusual fashion. The students were more motivated and actually took more ownership over the learning process. It was very interesting to see this in action. There is much research to suggest that for students from lower socioeconomic backgrounds, lessons are something that the teacher does 'to them'. In fact, research by Lubienski (2000) suggests that this is a major problem facing mathematics teaching. We feel that Lesson Study is one area that will improve this situation.
  3. We did identify conceptual weakness. Not all students were capable of the next step with this research lesson. That is the idea of using algebra to represent relationships. By the end of the lesson they did understand that there was a quicker way to answer the problem. The research lesson did expose a weakness in the students learning and knowledge.
- c) Post lesson Discussion
- During the post lesson discussion there was a high air of positivity with the way the lesson had progressed. We were very impressed with the methods employed by the students. We all agreed that we as teachers can interfere in the problem solving process. Sometimes when we think we are helping we might be better leaving students to their own devices for a little longer.
- There was a weakness as regards algebra. Students did not fully understand why algebra is used specifically in relation to representing relationships between variables. However, with the flow of the lesson this "lack of knowledge" was apparent to the students even without the teacher's intervention. With the use of patterns on the board and with student responses it was quite easy to develop the knowledge needed to tackle Functions and Algebra.
- d) As the instructor I felt that the Lesson was a success overall. This was due in no small part to the level of work put into the lesson proposal and in particular the boardwork. Everything was ready, seating plans, handouts and boardplan and this made the lesson run smoothly. Each observer collected the data from his/her group of students and this led to an informed post lesson discussion. The teachers were engaged and they observed the protocols. The students displayed a very positive attitude towards the lesson. The post lesson discussion dealt with the goals and the research theme of the lesson. We went through the methods employed, misconceptions and the questions the students asked. We believe the students really gained huge benefit from the class. They got the opportunity to connect up strands of Mathematics that could not have happened in

an ordinary lesson. Overall a thoroughly rewarding experience and possibly the best CPD that we, as teachers, have done in our careers.

- e) We believe this is a great starting point for our schools in the whole process of lesson study. The knowledge we have gained is invaluable. Even as the process was unfolding over the weeks, other ideas have come to the teachers involved. Ideas for new lesson Study Research Lessons have been spoken about. Even the one Research Lesson we developed above could be taken in so many different directions, covering many different strands of Mathematics. The future then, is very bright indeed. At this stage the teachers would like to thank all those who guided us through the process, our instructor Ronan O' Sullivan and all those involved at national level with the Project Maths Development Team, (PDST).

### **STUDENT REFLECTIONS:**

- |           |  |
|-----------|--|
| Student 1 | I learned that there are various ways to do one question and that they are all linked to each other. It was more interactive and fun than a normal class. I learned a lot from it.                   |
| Student 2 | I really enjoyed this lesson as it involved student interaction and it really taught us something that we can use in the future.   |
| Student 3 | I learned that Algebra ties in with large areas of of Maths and that multiplication can simplify the work. It made me think more about connections in Maths.   |
| Student 4 | Even though I thought there would only be one or two ways to finish the problem, there were many more. It was fun to be left on your own to do the sum in as many ways as possible.                  |
| Student 5 | That there are multiple ways of solving one question, using all the different methods of maths. Also everything in Maths seem connected to each other. This lesson gave me a new insight into Maths. |

### **Worksheet for research Lesson:**

**Research Lesson Davis College 31/01/2018**

**Enjoy!**

There are two companies that can supply a new video game. Game Stopper tells us that it has 40 units in stock but is selling more than it is producing resulting in its warehouse losing 2 units each week. Crazy Games Inc. informs us that it has 20 units in stock but is producing more than it is selling resulting in its warehouse gaining 2 units each week. After how many weeks will CrazyGames Inc. have the same number of units as GameStopper? Can you solve this problem in as many ways as possible using the worksheet provided?

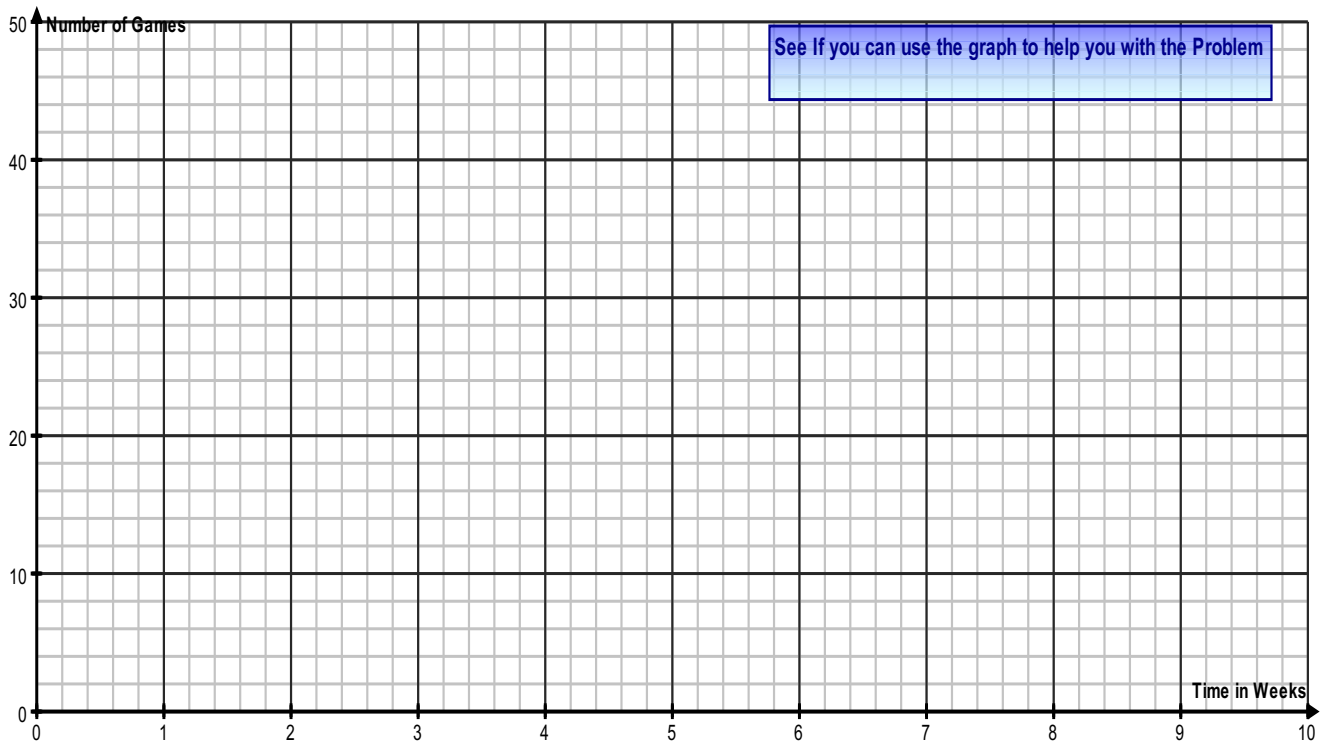
**Game Stopper:**



**Crazy Games:**



**Worksheet**



**Response 1:**

How many units will CrazyGames have after 27 weeks?

**Response 2:**

How many units will CrazyGames have after 33 weeks?

**Response 4:**

How many units will Crazy games have after 55 weeks?

**Response 4:**

After how many weeks will Crazy games have 88 units in stock?

What is the value of  $f(2)$ ?

What is the value of  $g(3)$ ?



(Can you find  $x$  when  $f(x) = 4$ ?)

**Response 5:**

From the graph can you approximate when both companies will have 35 units of stock?

(From the graph can you find  $f(6)$  and  $g(7)$ . – Only question this if we have related the graphs to functions)

Which of the below patterns are linear? State Y or N

33,36,39,42

-2,-5,-8,-11

21,24,26,29