Lesson Research Proposal for

second year ordinary level area and volume

For the lesson on 31/1/18

At Ardee Community School, 2nd Year Ordinary Level

Instructor: Mr. Shane Mc Gowan

Lesson plan developed by: Louis Callaghan, Louise Corrigan & Shane Mc Gowan

1. **Title of the Lesson:** Pump up the Volume
2. **Brief description of the lesson**

In this lesson students will find the volume of two irregular Rectangular Solids. They will learn that although they only have a formula for the volume of a cuboid they can using various strategies to transform the irregular solids into familiar cuboids and get the volume. They will also learn that problems have various strategies to solve them and all are equally valid and some more efficient than others.

3. Research Theme

The teaching and learning goals set for this year by our Mathematics Department is the improvement of our students' learning and our teaching are as follows:

- To select and use assessment practices that progress students' learning
- Value and engage in professional development and collaboration

On a regular basis to achieve the above as a Mathematics Department we are going to:

Focus on the family of techniques from Assessment for Learning by using more in-between desk work when students are at work to assess students learning in real time and as far as possible note student's strategies, misconceptions etc. as the class progresses. We feel that by doing this we will gain a greater understanding of the general misconceptions, strengths and weaknesses of our students and address these in the following way:

- Encouraging our students to present and explain to their peers in class their various ideas/solutions. By making time for this, students will not only present but grow familiar with presenting at the board by saying "First I did this because……", "Next I did because…."
- Encourage students to actively listen to each other and make relationships, comparisons and differences in various methods presented leading to greater collaboration
- Encourage students to make explicit preferred ways to tackle a problem and why?

- Pay particular attention to our Boardwork so that the flow of the lesson can be seen by all students
- Support students' reflections on the lesson
- Concentrate on effective questioning

As a Mathematics Dept. we value our professional development and have decided to engage in Lesson Study as a means of greater collaboration. Developing a research lesson over 6 evening meetings is the key to unlocking this collaboration. In addition, when the research lesson is conducted we will invite all Mathematics teachers and management into our classroom to engage in the classroom observation and post-lesson discussion. We hope to invite a knowledgeable other to the research lesson.

4. Background & Rationale

In primary school students in Third and fourth class learn to identify, describe, compare and classify 2-D shapes. An exploration of their properties in terms of sides, angles, parallel and non-parallel lines is carried out. Students learn to construct and draw 2-D shapes using a ruler and set square. They also engage in solving problems involving 2-D shapes. These shapes include: square rectangle parallelogram rhombus, pentagon, octagon, triangle (equilateral, isosceles and scalene) hexagon circle semicircle oval and irregular shapes. 3-D shapes that are identified are cube, cuboid, cylinder, cone, sphere, triangular prism, pyramid through an exploration of number of faces, number of edges and corners, ability to roll, slide or stack. The exploration and relationships between 2-D and 3-D shapes is described through exploration. Students construct 3-D shapes from 2-D shapes.

In Fourth & Fifth class students estimate and compare the area of regular and irregular shapes using standard square units. They also estimate, compare and measure using metric units In Fifth & Sixth class students have learned to classify 2-D shapes such as triangles and quadrilaterals in terms of angles and line properties. They also discover how to get the area of rectangles, irregular shapes and measure surface area. They continue to examine and identify relationships between 2-D and 3-D shapes and construct and deconstruct nets of simple shapes. In addition. they explore and compare circles and identify with different sizes of circles the relationship between the diameter and the circumference. They also examine the area of a circle by counting squares and construct a circle of given radius or diameter.

Given this extensive syllabus and obvious relationships to the Van Hiele Theory of Geometric thought we are surprised that when we teach "Area & Volume" in Second year of Post-Primary school students struggle and lack confidence with area and volume especially in the following areas: Different orientations of 2-D shapes, Area of Compound and irregular shapes, Area of triangles when the perpendicular height of non-right-angled triangles, Nets of 3-D shapes and Volume of Compound and irregular 3-D shapes. Students can use the formulas for a familiar shape but if applications and connections of various formulas are required they struggle beyond the obvious one or two step applications of them. We want our students to have more confidence and a positive disposition towards the unfamiliar. This is the problem we wish to solve through our Lesson Study.

Research Findings

From our study of the Primary Curriculum and recommended approaches in the Primary teacher's guide we feel that the Van Hiele Model of Geometric thought has been used in

Primary School without naming it per se. Given this insight we realize that perhaps we should adopt more hands -on activities and exploration for our students in our school. This research lesson aims to build upon our work with

2-D shapes in this topic and see if a problem is properly devised and a structured problem-solving approach is adopted can and will students gain in their confidence and have a more positive disposition towards tackling unfamiliar problems.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes from Primary School	Learning outcomes for this unit	Related later learning outcomes for Leaving Cert
3rd & 4th Class: 2-D shapes include: square rectangle parallelogram rhombus, pentagon, octagon, triangle (equilateral, isosceles and scalene) hexagon circle semicircle oval and irregular shapes. 3-D shapes that are identified are cube, cuboid, cylinder, cone, sphere, triangular prism, pyramid. Identify, describe, compare and classify 2-D shapes. Explore properties in terms of sides, angles, parallel and non-parallel lines Construct and draw 2-D shapes using a ruler and set square. Solve problems involving 2-D shapes Explore relationships between 2-D and 3-D shapes is. Construct 3-D shapes from 2-D shapes. 4Th & 5th Class Estimate and compare the area of regular and irregular shapes using standard square units.	In this Unit students will solve problems that involve 2-D and 3-D shapes including nets of solids and two-dimensional representations of three dimensional objects. Use nets to analyse figures and to distinguish between surface area and volume. Solve problems involving perimeter, surface area of rectangular solids. Find the volume of rectangular solids and cylinders. Model and solve problems including multi-step problems involving surface areas and volumes of cylinders and prisms. Develop an understanding of the relationship between circumference, diameter and π.	investigate the nets of rectangular solids and cylinders select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, and figures made from combinations of these select and use suitable strategies to estimate the area of a combination of regular and irregular shapes select and use suitable strategies to find the volume and surface area of rectangular solids, cylinders and spheres draw and interpret scaled diagrams

Estimate, compare and measure using metric units 5^{th} & 6^{th} & Classify 2-D shapes such as triangles and quadrilaterals in terms of angles and line properties. Discover how to get the area of rectangles, irregular shapes and measure surface area. Examine and identify relationships between 2-D and 3-D shapes Construct and deconstruct nets of simple shapes. Explore and compare circles and identify with different sizes of circles the relationship between the diameter and the circumference. Examine the area of a circle by counting squares and construct a circle of given radius or diameter.		

6. Goals of the Unit

This Unit aims for students to firstly find the area of a simple L-Shape in as many ways as they can. The goals for this particular lesson is for students to observe that the most efficient way to get the area of an L-Shape is by using the following 2 strategies:

I. Cut and Paste the L-Shape to form a familiar shape (a rectangle) and recognise that this preserves the area of the irregular shape

II. Duplicate the L-Shape to form a familiar shape (a rectangle) and recognise that this doubles the area so they must divide by 2.

From these strategies above students will then think, explain and develop formulas for finding the areas of parallelograms, triangles, trapezoids, rhombuses. In addition, when students need to think about formulas for new geometrical figures, students can go back to the area formulas they previously learned easily and have the confidence to make use of those for their thinking. Worksheets will be utilised so that students can write down their diverse ideas and share with their peers.

Students will then apply their knowledge to further area problems on compound shapes. Perimeter will have been revised in Lesson 1 and now students will meet problems with area and perimeter in “Application” type problems. Throughout, particular attention will be considered regarding the orientation of figures. With the use of manipulatives students will explore and develop an appreciation and understanding of the importance of nets in terms of their understanding of 3D shapes in the context of understanding that the size/volume of solids are determined by the lengths of their edges. We hope to develop students' understanding of the units of area and volume as well as the formulae for calculating the volume of cubes, rectangular prisms (see research lesson for details on goals) and cylinders. Throughout we want to extend students' ability to solve problems independently by making use of their prior learning and enrich their quantity-sense with respect to area, perimeter and volume. It is intended that students will recognise the merits of generalisation and derivation of the formula on their own.

When they do, their understanding of the meaning of the calculation of volume will be solidified. We will also incorporate activities to empirically determine the volume of solids by filling the figures with unit cubes, students' quantity-sense with volume will be enriched. They will understand the unit of volume, "cubic meter (m3)" and the relationship, 1 m3 = 1000000 cm3. Understand the meaning of and ways to determine "inside measurement" and "capacity." Understand the relationships, 1 L = 1000 cm3 and 1 mL = 1 cm3.

Further activities will be used for students to explore the relationship between the diameter of the circle and its circumference to arrive at an understanding of π and the formula for the circumference of a circle. An exploration of the area of a circle will be carried out by so that students’ interest will be developed and an understanding of the formula arrived at. Skills related to area, volume and perimeter of geometrical figures will be focused on so that students can calculate the latter.

7. Unit Plan

Lesson	**Learning goal(s) and tasks**
1	Revision of Primary Work to include Area & Perimeter
2	Area problem on L-Shape
3	Area problems on squares, rectangles, parallelograms, triangles, trapezoids and rhombuses
4	Further area problems of compound shapes
5	Application of Area & Perimeter
6	Nets of 3-D Shapes

7	Volume of Cuboids (Rectangular Solids)
8 Research Lesson	Volume of Irregular Rectangular Shapes
9	Further problems
10	Investigate Pi
11	Circumference of Circle
12	Area of Circle
13	Volume of Cylinders
14	Further problems

8. Goals of the Research Lesson:

Students will:

- Extend their ability to solve problems independently by making use of their prior learning
- Think about diverse ways to determine the volume of figures composed of rectangular solids
- Share and discuss their solution strategies

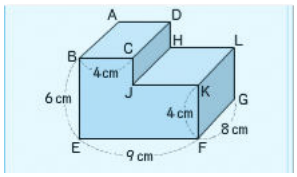
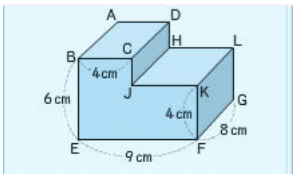
Key Skills

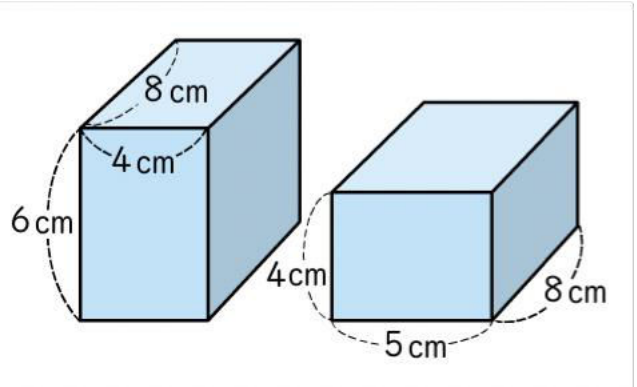
- ✓ Being Numerate: Students are interested in the volume of objects. They think about ways to determine the volume and try to calculate it.
- ✓ Managing Information and thinking: Mathematical Way of Thinking Based on their prior learning about area, students can think about expressing the volume in terms of the number of units. They can think about ways to calculate the volume using the length of edges. Students can calculate the volume of cubes and rectangular
- ✓ Being Literate & Communicative: Students will express and share their ideas clearly and listen to their friends
- ✓ Learning with others: During presentations at the board students will explore the most efficient way to calculate the volume
- ✓ Managing myself: At the end of the lesson students will be required to reflect on their knowledge and understanding about the meaning of units and volume measurements as well as the fact that the volume of rectangular prisms can be calculated.

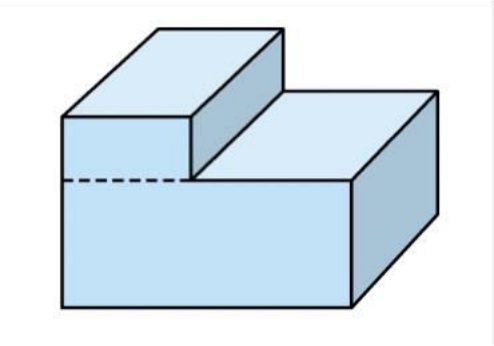
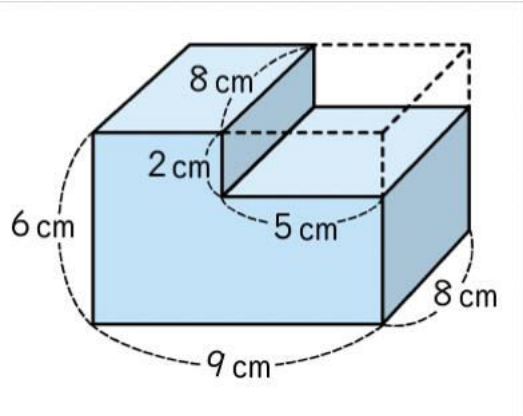
Junior Cycle: Statements of Learning
In this lesson students will: recognise the use of mathematics in finding the volume of a compound rectangular solid.

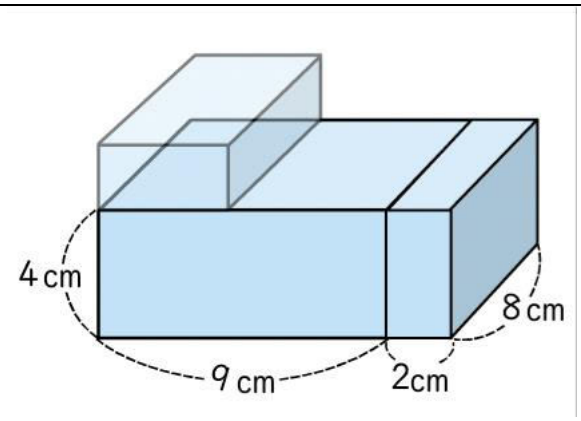
They will illustrate their approaches and explain their solutions
Throughput students will be involved in solving a problem and use their mathematical knowledge and skills to get the volume of 2 compound rectangular solids

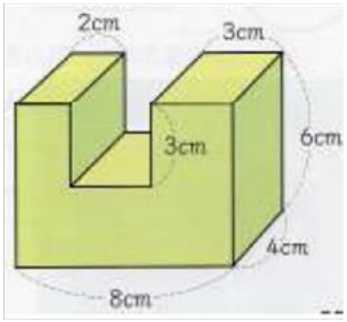
9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction **Review the ways to calculate the volume of cubes and rectangular solids.**		
Posing the Task **♦Teacher (T): I want you to think about ways to calculate the volume of the L-Shaped solid.** **Problem: calculate the volume of the solid in as many ways as you can.**  **What is the difference between this and what we have been doing yesterday?** **♦Students' Responses (SR)** • It's like steps. • It's not a cube or a rectangular prism. • We can't use the formulae. ♦T: Think about ways to calculate the volume using diagrams and mathematical expressions. Summarize your own ideas by marking the figures from the worksheet that you have pasted in their copybooks. You may use the building blocks on your tables if you find they	Post the figure and help students grasp the task by having a prepared figure with plastic building blocks. Place the Problem on the Board  	If students have difficulty imaging the figure, remind them about the ways they figured out the area of composite 2-D figures as a hint. (L-Shape) Distribute copies of the figures to students so that they can paste them in their notebooks. Distribute building

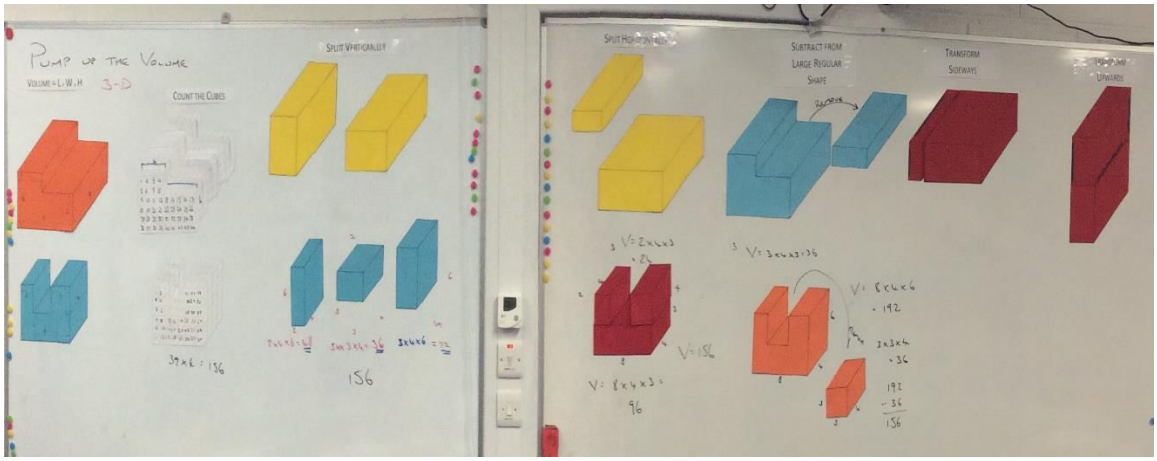
will help.		blocks to students
Student Individual Work Response 1 ♦Subdivide (1) Partition vertically to create 2 rectangular prisms and find the sum of their volumes.  8×4×6+8×5×4=352 Response 2 ♦Subdivide (2) Partition horizontally to create 2 rectangular prisms and find the sum of their volumes.	If students find the volume, encourage them to think about other ways to find the volume.	For those students who cannot get started, give them the prepared model so that it can be taken apart. [Thinking] Students are thinking about ways to calculate the volume of the L-shaped solid by partitioning and adding on to the given solid, using the diagram and mathematical expressions. They may remember strategies of "cut & paste into a familiar shape to preserve area" and/or "duplicate the shape to form a familiar shape and divide by 2" Will students notice that the duplicate strategy will not generalize here? If a student is struggling: Ask them is there another way to put the blocks to get the same shape? s

8×4×6+8×9×4=352  Response 3 ♦Subtract From the large rectangular prism, subtract the rectangular prism that is not actually there. 8×9×6−8×5×2=352  Response 4 ♦Move Transform Cut the solid horizontally and move the top part to the side of the bottom part. 8×(9+2)×4=352		If students are allowed iPads in class allow to photograph and record their work.

		
Sharing Solutions Students share their ideas on the board and learn their peers' ideas. Remind students to think as they listen to other students' ideas and think about the mathematical expressions for various approaches. From the school's research theme concentrate on the following questions: "What do you think"? (ask another student(s) other than the presenter) "Why is that"? "Did anyone else solve it the same way? Can you explain this method"? If students omit any method post the mathematical expression e.g. 8×(9+2)×4=352 and have students figure out the method. Combine multiple students' voices to complete each undiscovered strategy/idea. **Ceardaíocht:** ♦Teacher: What have the strategies in common?	Students present their solutions in the order shown in student responses : column 1. Possible misconceptions: Mix up perimeter and volume and multiply all the numbers. Missing dimension may not use numbers given to figure them out and use any number. Just get L x B x H and stop Just get the area of the face. To handle these misconceptions ask a student(s) who don't agree with these answers "why not"?. Create the space for students to teach each other. Because all students' ideas will be posted, students can autonomously engage in	Can students paraphrase each idea so that all students understand each idea? Do students realise that each idea is based on the shapes they have learned previously, and do

How are the strategies different? Can you discover what all the strategies have common idea.? ♦Students' Response: All the ideas are making use of cubes and rectangular prisms. Even shapes like the L-shape, we can find the volume by making use of cubes and rectangular prisms. , I want to try another shape. Problem: Using all of the methods used today calculate the volume. of this shape: Have students share which options they picked and the mathematical expressions the following day..	the task. Place poster on the board and students have a copy to paste into their copybooks.  	they recognise the merit of using prior learning from their presentations and explanations?
Summing up & Reflection ♦Teacher: Using the board work summarise students' ideas for the L-Shape problem. Also address what they may have had difficulty with. ♦ Students' write a reflection based on the following: Today I understood …. Today I noticed …. Today the questions I have … Today I learned from my friend's ideas…..		

10. Board Plan



Evaluation

The lesson opened with the song “Pump up the Volume” playing.

The teacher revised relevant previous learning and then posed the problem to the class. Some students immediately worked out what to do and set to work completing the task. Others worked with three dimensional models supplied for this purpose. During the board-work many students deepened their learning and were then all eager and excited to move on to the next question. As the teacher guided the students through their solutions you could see the light bulb moments.

Reflection

Starting the class by playing the music lightened the mood for the students in the room.

The use of the three dimensional models was very helpful for some students who picked them up and used a ruler to measure the dimensions.

Probably the most effective tool in this lesson was the moveable shapes made especially for the class. Although it is time consuming, this is what deepened the students understanding. Students moving each piece, showing how they separated into smaller regular shapes showed others exactly how many different ways there were of solving the problem.

The students’ sense of confidence and understanding was visibly increased during this class. By the end they were extremely eager to come to the board, and explained their understanding well.