Lesson Research Proposal for 5^{th} Year Coordinate Geometry

For the lesson on 01/02/18
At Coolmine Community School, Eimear D'Arcy's class
Instructor: Eimear D'Arcy
Lesson plan developed by: Patricia Stafford, Amy Doolan, Phil O'Donnell and Eimear D'Arcy

1. **Title of the Lesson:** Run Circles around Pythagoras.

2. **Brief description of the lesson:** The lesson will begin with a reminder of Pythagoras' Theorem; $a^2 + b^2 = c^2$. Students will be given the task of finding as many a's and b's as they can when c is 5. They will then be prompted to plot their solutions on the coordinate plane, leading to the discovery that they form a circle, and introducing the circle as a locus of points. The class discussion will lead towards the formation of the equation of a circle centred at (0, 0); $x^2 + y^2 = r^2$.

3. **Research Theme**

   At Coolmine Community School, we want students to;

   a) Engage purposefully in meaningful activities
   b) Grow as learners through respectful interactions and experiences that are challenging and supportive.

   As mathematics teachers, we will actively support the achievement of these goals by paying attention to the following entry points in every day classes:

   a) Collaborative lesson planning
   We will endeavour to create a problem-solving lesson, where the problem chosen is meaningful to the students and will encourage them to engage in the problem-solving process to the best of their ability.
   b) Use a variety of methodologies that will give students the opportunity to engage with the work individually and as a group.

Background & Rationale

a) Why we chose the topic;
We want to focus on Leaving Cert ordinary level, with a greater uptake at higher level the cohort studying at ordinary level is less able. From previous teaching experience of co-ordinate geometry of the circle, students earn very poor marks or avoid the question. Creating more visual and engaging tasks will improve their understanding of the topic.

b) Our research findings;
Through discussions, meetings and reflecting upon results our group found teaching was heavily geared towards procedural methods, giving them the formula before understanding what the formula represents. We have chosen this topic to see if we can reverse this by giving them the understanding and allowing them to develop the formula.

4. Relationship of the Unit to the Syllabus

<table>
<tr><th>Related prior learning Outcomes</th><th>Learning outcomes for this unit</th><th>Related later learning outcomes</th></tr>
<tr><td>In Junior Cert, students learn Pythagoras Theorem, co-ordinate geometry of the line, trigonometry, area and volume, synthetic geometry.

From the Junior Cert Syllabus:

2.1 <u>Synthetic geometry</u>

Theorem 3: Each angle in a semicircle is a right angle

Theorem 14: (Theorem of Pythagoras) In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides

2.2 <u>Co-ordinate geometry</u>

Coordinating the plane: Properties of lines and line segments including the slope, distance and the equation of a line in the form

$y - y_1 = m(x - x_1)$

$y = mx + c$

2.3 <u>Trigonometry</u>

Right- angled triangles

Trigonometric ratios

In addition to the above, from the Leaving Cert syllabus, they will have prior knowledge of:

2.1 <u>Synthetic geometry</u>

Corollary 6: if two circles share a common tangent line at one point, then the two centres and that point are collinear</td><td>From the Ordinary Level syllabus;

2.2 <u>Co-ordinate geometry</u>

Recognize that $(x - h)^2 + (y - k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle with (h , k) and radius r

Solve problems involving a line and a circle with centre (0, 0)</td><td>Transformations on the complex number plane.

Representing data graphically in statistics.

Area and volume, specifically the perimeter and area of a disc.</td></tr>
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2.2 <u>Co-ordinate geometry</u> Use slopes to show that two lines are • parallel • perpendicular Recognise the fact that the relationship $ax + by + c = 0$ is linear Solve problems involving slopes of lines. Calculate the area of a triangle.		

5. Goals of the Unit

- Students will develop and understand the connection between the centre, the radius and all the points on the circumference of the circle.
- Students will gain an understanding of the circle as a locus of points rather than a planar object.
- Students will be confident in constructing an accurate circle on the co-ordinate plane given the centre and radius or centre and a point on the circle.
- Students gain skill in formulating the equation of the circle.
- Students will be given ample opportunities to develop skills in solving problems involving lines and circle centred at (0, 0).

6. Unit Plan

Lesson	Learning goal(s) and tasks
1 The Research Lesson	At the end of the lesson the goal is the students will be able to formulate the equation of the circle $x^2+y^2=r^2$.
2	Students will perform a task similar to the research lesson for circles not centred at the origin.
3	Students will practice formulating equations and finding the centre and radius from given equations.
4	Students will investigate whether points are inside, outside or on the circle.
5	Students are given problems involving the learning they have acquired thus far in the unit.
6	Make links with co-ordinate geometry of the line and algebra.
7	Practice day for problems involving intersection of a line and a circle, including axes.

7. Goals of the Research Lesson:

a) Mathematical Goals

Recognise the circle as a locus of points, equidistant from the centre.

Recognise $x^2 + y^2 = r^2$ as the equation of this circle centred at (0, 0) with radius r.

b) Key Skills and Statements of Learning

In the planning and design of this lesson the Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will implement and promote JC Key Skills in the following ways:

1. Being Numerate: It will develop a positive disposition towards problem solving.
2. Managing Myself: students will have the opportunity to reflect on their own learning.
3. Staying Well: Students will feel positive about their learning and build confidence.
4. Communicating: Students will present, discuss and debate their mathematical thinking.
5. Being Creative: students will explore options and alternatives as they try to creatively solve a problem.
6. Working with others: Students will learn with and from each other.
7. Managing Information and thinking: Students will be encouraged to think creatively and critically.

This lesson is also designed to meet the following JC Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The student describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

8. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction In today's lesson, we will use our mathematical knowledge to solve a problem. The students will work individually and then they will collaborate and use their knowledge to come up with something new. **Prior Knowledge:** Pythagoras theorem. "Can anyone remind me of what our good friend Pythagoras is famous for?"		Are the students engaged?

Posing the Task How many different a's and b's can you find that work when c is 5. **Clarifying the problem:** Get a student to repeat the problem in their own words.	The teacher will have written Pythagoras on the board in the form $a^2 + b^2 = c^2$ during prior knowledge	Do they understand the problem? Can be assessed by asking students to repeat problem in their own words.
Student Individual Work <u>Student Response 1</u>: We expect that the first solution the students will come up with will be (3, 4) or (4, 3). <u>Student Response 2</u>: We expect that the second solution the students will come up with will be (0, 5) or (5, 0). <u>Student Response 3</u>: We expect the students to come up with negative values such as (4,-3), (-4, 3), (-4,-3), (-3, 4), (-3,-4), (3,-4), (0,-5) and (-5, 0). <u>Student Response 4</u>: We expect the students to come up with points involving surds for example $(\sqrt{5},\sqrt{20})$. There are an infinite number of solutions they could come up with in this section, we will be looking for students to come up with one. **Misconception:** Student may only come up with numbers that add to 5.	Teacher may prompt students individually "Do they always have to be positive?" Teacher may prompt students individually "Do they always have to be integers?" This will be addressed with individual students.	Teacher will assess understanding by observing students during the ten minutes they are working on the problem.
Ceardaíocht /Comparing and Discussing We anticipate that we will share the responses on the board in the order we have listed them above. We will push students to present their solution as a paired couple, for example $4^2 + 3^2 = 5^2$ $16 + 9 = 25$ $25 = 25$ $(4, 3)$	Teacher may need to use the following prompts; Will you put your solution on the board please? What is your a and b? Can you write them together underneath?	

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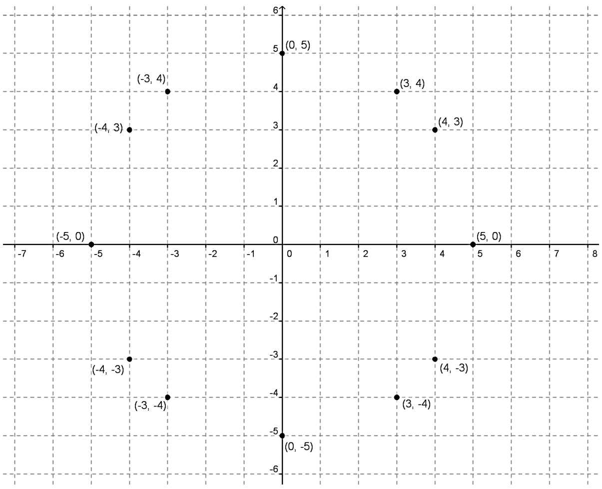
Because of the large number of points the students will come up with, it might be more efficient to ask students to present one solution from each of the 4 quadrants and one of the solutions on the axes. We can then ask the class for suggestions of similar solutions to those and list them. If a student is successful with finding a non-integer solution we will ask them to present this solution.
Ask students can they see any similar solutions.
We then prompt the students towards using the co-ordinate plane as a method of displaying these solutions. Once a student has come up with the idea we will then ask other students to come forward and plot their solution on the co-ordinate plane prepared.


We expect that at least one student in the class will observe that the points will join to form a circle.
Once a student has suggested the circle we will generate the circle on the GeoGebra file already on the board.
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What do those solutions remind you of?
What could we do with them?
If the students do not come up with the suggestion as writing them as a couple then the teacher will put brackets around one solution on the board as a prompt.
How can we display these?
What do all these points have in common?
What do you notice?
How far are they from the origin?
What is this distance called?
What is the origin in this circle?
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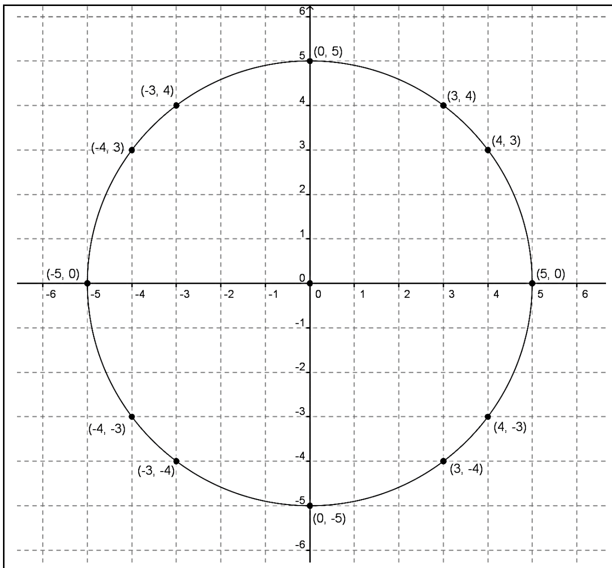
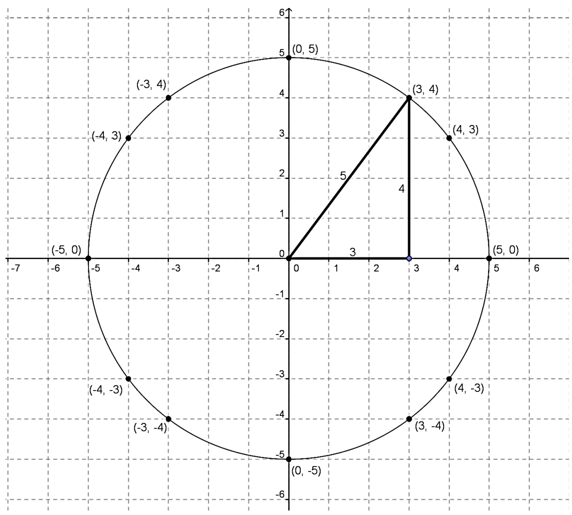


“Let’s take a closer look at one of our solutions”

Starting at the centre (0, 0) the teacher will draw a and b completing a right-angled triangle for a particular point, labelling the sides a, b and c.



After the teacher prompt some student will say 5 is the radius.

The teacher will choose another point, not one of the solutions given by the students, and connect the point to the origin. For this point, we will draw in the right-angled triangle and prompt the students to label them x, y and 5.
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A point in the circle e.g. (1, 2), is this on the circle?

In this shape on the board what is the length 5 called?

What is it called in relation to the circle?

What is the point (0, 0) representing in this circle?
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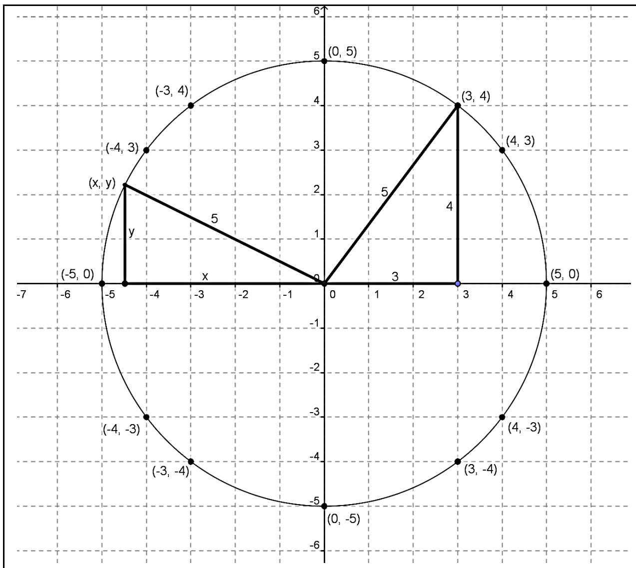 For this example, using Pythagoras Theorem, we can then say that $x^2 + y^2 = 5^2$. Teacher will ask is this true for other points? Why is it always 5^2? We expect that the students will state that it's always 5^2 because 5 is the radius. At this stage, the students will be asked for a relationship which describes every point on the circle, thereby generating the equation of a circle.	What will we call this unknown length along the x-axis and similarly along the y-axis? What equation can we write for these lengths? What if the radius was 4? What if the radius was 3? How will we always write it down?	
Summing up & Reflection The teacher will summarise the main ideas of the lesson. At this point we will also give them another radius length and see if they can write the equation of that circle. The students will be asked to write a short reflection on the lesson.	The teacher may use the layout of the board work to help provide students with a summary of the progression in their learning.	

9. Board Plan

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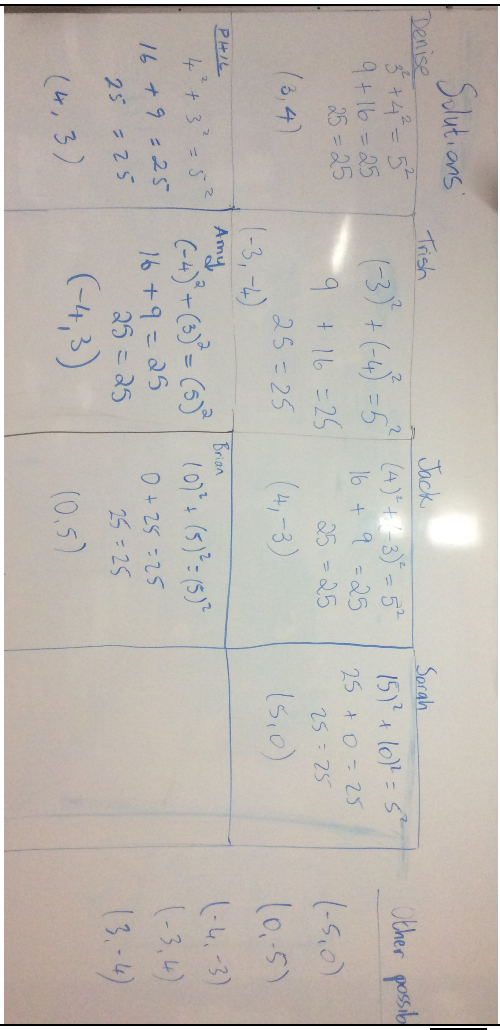

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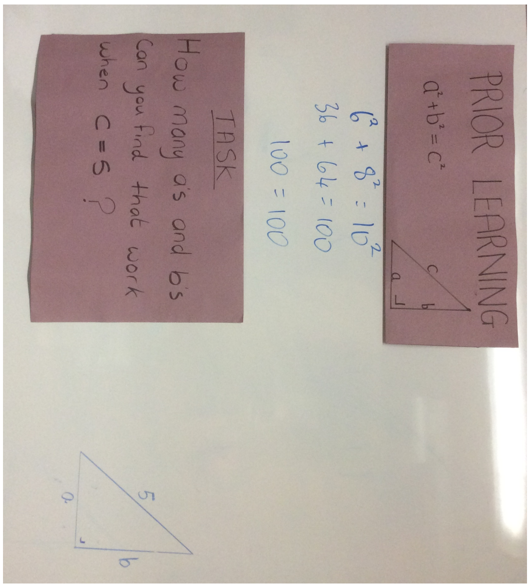

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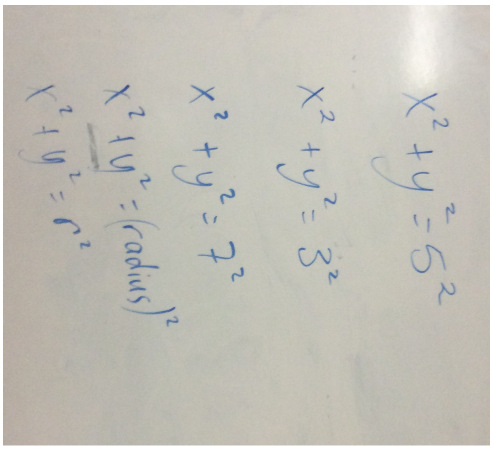

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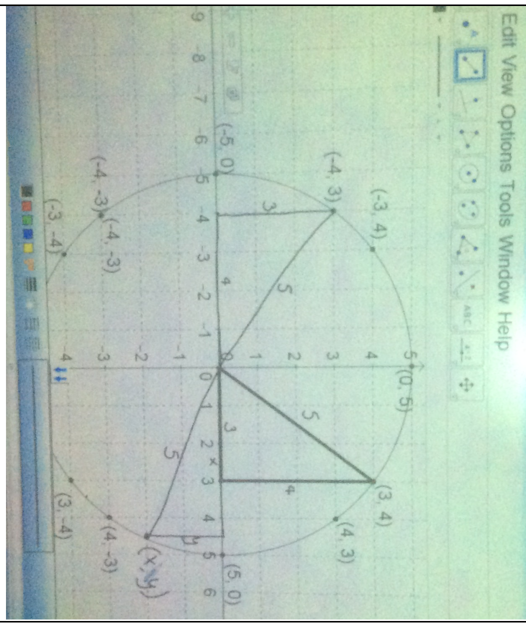


10. Evaluation

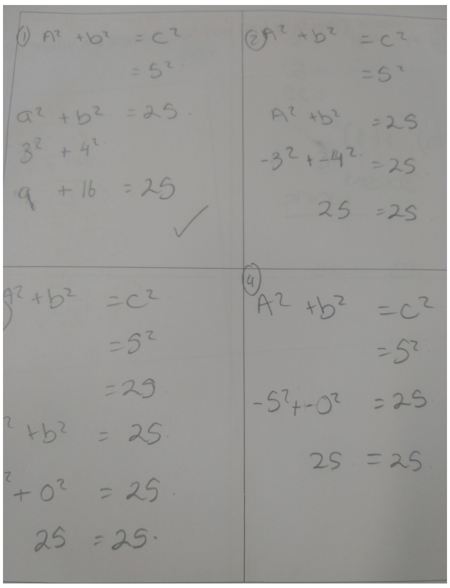
The consensus was that the lesson was successful, and the goals stated by the group at the outset had been met. It was clear to the team that the students were engaged in the task from the beginning. The students responded well to the prior knowledge and once the task was set they engaged immediately in trying to find solutions. All students were successful in finding at least one solution, with many students finding multiple solutions.

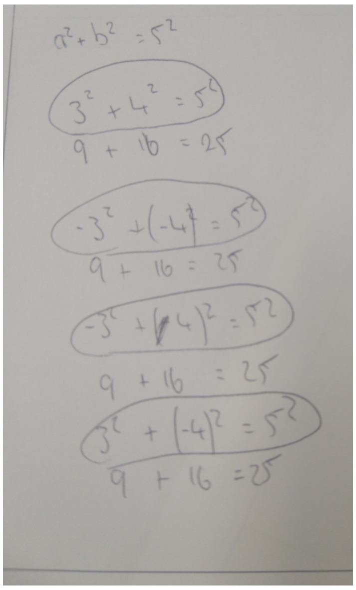
Once the Ceardaíocht began, and the students saw their classmates work, they were confident in suggesting further solutions not on their worksheet. When the solutions had been plotted on the coordinate plane, the students immediately noticed that they formed a circle. On further discussion, handled very adroitly by the teacher, they understood that there were many more points on the circle than they had plotted, hence showing understanding of the circle as a locus of points.

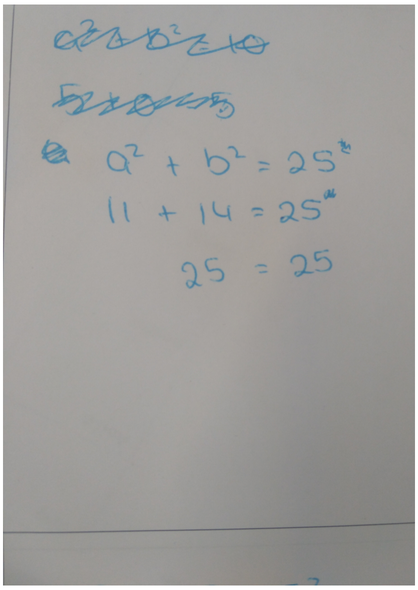
The next stage of the Ceardaíocht went very well, more smoothly than the instructor herself anticipated. The class examined a small number of solutions in more detail, as described above in the flow of the lesson, and they quickly advanced to the generalized formula $x^2 + y^2 = r^2$, thereby achieving the second goal of the lesson.

11. Reflection

It was agreed by the team that the straightforward task was a factor in the success of the lesson. It allowed all students to engage and find at least one solution. It was based on knowledge that they were very familiar with. There was, however, a misconception that arose, which the teacher dealt with individually while the students were working. A small number of students (less than 5) found two numbers which added to 5 i.e. $1^2 + 4^2 = 5^2$. Once the teacher corrected this, they went on to find at least one solution.







The prompt questions indicated in the proposal were actually used by the students themselves. After working for a number of minutes one student asked “Do they have to be positive?” This gave the required prompt to the whole class with no input from the teacher. Another student asked the teacher

“Can they be decimals?” allowing the teacher to answer that they did not have to be whole numbers. The student did not successfully find a non-integer solution however. One student was attempting to do so. Having written 11 + 14 = 25 on her worksheet, she was attempting to find numbers, by trial and improvement, whose squares were 11 and 14. In discussion during the development of the lesson proposal, it was acknowledge by the team that ordinary level students would be unlikely to think of using surds.

While reflecting on the lesson the team were asked “Did students have enough to do?” Although it was suggested during the design process that students would plot the solutions themselves as a secondary task, we felt that this might be more valuable if done as a group during the Ceardaíocht. After the lesson had been taught we feel, in hindsight, that giving the students this secondary task on pre-prepared graph paper should be included in the lesson. It would allow for more student involvement as the Ceardaíocht felt long. It would also give students a valuable record of the Ceardaíocht to take with them. The lesson was designed to be an hour long lesson, but fell short of the hour. The students were given some practice work on what had been learned for the remainder of the hour. With the students plotting the points themselves as suggested, we feel that this might be more suitable to a 40 minute lesson. The lesson could be easily adapted to a higher level class, but would need to be part of a longer lesson where they were given a similar task for circles centred at (h, k).

The team felt that exposure to these problem solving lessons was essential for our senior cycle students. The effect on their mathematical learning was discussed and would be a very important reason for continuing to include these lessons in our class planning. We did note during the discussion as well, that many of the students found the process of working alone for the ten minutes uncomfortable. They seemed to want validation from the teacher for each step or solution before they continued. In order to give them confidence, we need to provide them with opportunities in which their confidence can grow. On reading their reflections it also became apparent that many of the students found the presence of the other teachers hugely intimidating, and may have inhibited them during the class. We feel that should we take part in Lesson Study again, videoing the lesson might be an option to avoid this.

This was the team’s first time to participate in Lesson Study. We found it be hugely enjoyable and of huge benefit, both for ourselves and for our students. It allowed us to engage with each other in a way that has become less and less a part of our school planning process. To explore a topic in such detail was very enjoyable, and allowed us to see the potential for links right across the curriculum. Using a similar lesson to introduce the equation of a circle centred at (h, k), and the unit circle was discussed as a possible topic for further study.

<u>Some extracts from students’ reflection on the lesson:</u>

“It was interesting, but it was a little difficult to understand at the start. I ended up enjoying it”

“I liked the lesson. I thought it was very interactive which made me enjoy it more”

“I enjoyed the lesson. It was very helpful and it was easy to understand because of the way it was taught. I learned something new going out of the class”

“It was an interesting way of learning something new. I’d like to have more classes where, as students, we get to engage more. It was nerve wracking having other teachers watching us. Overall I did enjoy the lesson.”

“It was fine. I got tea and cake out of it. Wouldn’t mind a few biscuits.”

Name:

How many a's and b's can you find that work when $c=5$?

Anna	Sam	Spiro			Vio	Kieran	Hang
Adam B.							Aoife
Patrick							Sophie
Oana	Lauren	Fatima			Jack	Cian	Sarah
Victoria							Amy
Lovelyn					Pascalina	Vanessa	Jade
Ore							Rebecca
Leah			Faith	Beatrice	Oti	Adam W.	Adam O.
			Teacher				