

Lesson Research Proposal for 2nd Year - Algebraic Fractions

For the lesson on Tuesday 30th January at 12:00 -1:20

At Ardscoil Na Mara, Ivan Cliffe's class

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1. Title of the Lesson: Sharing The Unknown (How to form and add algebraic fractions)

2. Brief description of the lesson

Students will build on their understanding of numeric fractions to become more comfortable working with algebraic fractions. They will investigate how to go from a real life scenario of sharing amounts of money to expressing these shares as algebraic fractions.

3. Research Theme

In Ardscoil Na Mara we want to:

- a) Increase the percentage of students who like maths by developing a positive attitude towards maths in general and towards numeracy in all subject areas.
- b) Give students of all abilities opportunities to experience challenges and successes in maths.
- c) Encourage students to recognise maths in the world around them.

As maths teachers, we will actively support the achievement of these goals by paying attention to the following entry points in our every-day classes:

- a) Promotion of a "growth mindset"

Teachers should be positive and encouraging and model a "can do" attitude. Teachers should praise and value effort and attempt, instead of just the "right answer".

- b) Provide a learning environment where students feel comfortable to try their best.

Problems should be designed that allow students to find a solving method according to his / her abilities. The classroom should be "safe for mistakes". Students should be encouraged to not worry about mistakes, rather to learn from them when they happen.

MATH: Mistakes Allow Thinking to Happen

4. Background & Rationale

1. Why we chose this topic:

Students regularly struggle with the addition and subtraction of algebraic fractions. Even when they have a good grounding in handling numerical fractions they often fail to understand what algebraic fractions represent and how to handle them. This came up again in a recent higher level second year test. We would like to analyse how, why and where students are making errors. A sample of students' answers were as follows:

Write the following as a single fraction in its simplest form:

$$\frac{x+2}{3} - \frac{x-3}{4}$$

The student has written the fractions $\frac{3x}{3}$ and $\frac{-3x}{4}$ on the top line, with a minus sign between them. Below these, they have written $\frac{17}{12}$. There is a large 'X' over the original fractions, indicating this approach is incorrect.

This student added the x value with the constant on the top line of both fractions.

Write the following as a single fraction in its simplest form:

$$\frac{x+2}{3} - \frac{x-3}{4}$$

The student has correctly multiplied each fraction to get a common denominator of 12. They have written $\frac{4x+8}{12} - \frac{3x-9}{12}$. A circled '2' is next to the second fraction, indicating the multiplier used. Below this, they have written $\frac{1x-1}{12}$, which is the result of combining the terms.

This student correctly multiplied each fraction to get the common denominator. However, the difficulty arose with the addition of the fractions.

Write the following as a single fraction in its simplest form:

$$\frac{x+2}{3} - \frac{x-3}{4}$$

The student has correctly multiplied each fraction to get a common denominator of 12. They have written $\frac{4x+8}{12} - \frac{3x-9}{12}$. A circled '3' is next to the second fraction, indicating the multiplier used. Below this, they have written $\frac{17}{12}$, which is the result of combining the terms.

This student understood how to get the common denominator for the two fractions but struggled with combining the terms.

Write the following as a single fraction in its simplest form:

$$\frac{x+2}{3} - \frac{x-3}{4}$$

The student has correctly multiplied each fraction to get a common denominator of 12. They have written $\frac{4x+8}{12} - \frac{3x-9}{12}$. A circled '2' is next to the second fraction, indicating the multiplier used. Below this, they have written $\frac{1x-1}{12}$, which is the result of combining the terms.

This student understood they needed to get a common denominator but multiplied both fractions by 3. From talking amongst ourselves we realise that there are different approaches used across the school (and different to primary school). We would like to develop a common approach for maths teachers in

our school. We have asked the maths department in Ardscoil Na Mara how they teach algebraic fractions.

SEC reports regularly state that handling fractions is an area of weakness. The question assessed in the recent Christmas test was from the 2015 Junior Cert. paper. The 2015 SEC report outlined that candidates significantly struggled when asked to add two algebraic fractions (Question 7(c)). The examiners reported that many candidates were unable to carry out the procedures accurately, with most having errors (sign, distribution, transposition, rounding, etc.) of some kind. Many candidates also appeared to conflate methods related to simplifying expressions with those related to solving equations, as evidenced by the fact that they multiplied each of the algebraic fractions in Question 7(c) by 12 (the common denominator) rather than expressing each one as a fraction with 12 as the denominator.

We would like to have more of our incoming 5th year students proficient in working with algebraic fractions so they are better prepared for leaving cert. material including limits, calculus etc.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<p>5th class Add and subtract simple fractions</p> <p>6th class Add and subtract simple fractions and mixed numbers.</p> <p>Explore the concept of a variable and translate word problems with a variable into number sentences</p> <p>1st year (CIC) We aim to consolidate, investigate and explain operations on rational numbers. “Students learn strategies for computation that can be applied to any numbers”</p> <p>Students also build on their primary school work on patterns, variables and basic algebraic operations.</p>	<p>2nd year (higher level)</p> <p>– add and subtract simple algebraic expressions of forms such as.....</p> $\bullet \frac{ax+b}{c} \pm \frac{dx+e}{f}$ $\bullet \frac{a}{bx+c} \pm \frac{p}{qx+r}$ <p>where $a, b, c, d, e, f, p, q, r \in \mathbb{Z}$</p> <p>• use the associative and distributive property to simplify expressions such as:</p> $\bullet a(bx+cy+d)+\dots + e(fx+gy+h)$ <p>where $a, b, c, d, e, f, g, h \in \mathbb{Z}$</p>	<p>Leaving Certificate</p> <p>At higher level students should be able to ...</p> <p>– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form:</p> $f(x) = g(x)$ <p>with $f(x) = \frac{ax+b}{ex+f} \pm \frac{cx+d}{qx+r}$; $g(x) = k$</p> <p>where $a, b, c, d, e, f, q, r \in \mathbb{Z}$</p>

6. Goals of the Unit

- Students will recall and revise operations involving numeric fractions
- Students will understand how to visualise and form algebraic fractions using fraction strips
- Students will recognise how algebraic fractions arise naturally in problems
- Students will understand how to add and subtract simple algebraic fractions e.g. $\frac{x}{2} + \frac{x}{3}$
- Students will understand how to add and subtract more complex algebraic fractions e.g. $\frac{x+2}{2} + \frac{x-1}{3}$
- Students will understand how to solve equations involving algebraic fractions

7. Unit Plan

Lesson	Learning goal(s) and tasks				
1	Distributive law $3(x-1) = 3x - 3$				
	Distribute (X Multiply)	$x+1$	$x+2$	Addition	Subtraction
	3	$3x+3$	$3x+6$	$6x+9$	-3
	2	$2x+2$	$2x+4$		
	1	$x+1$	$x+2$		
	$1/2$	$\frac{x}{2}+\frac{1}{2}$	$\frac{x}{2}+\frac{2}{2}$		
	$1/3$	$\frac{x}{3}+\frac{1}{3}$	$\frac{x}{3}+\frac{2}{3}$		
2	Combining terms: $7x - 1$ NOT= $6x$ (Using algebra tiles)				
3	Signs: $4(2x+1) - 3(3x+2)$				
4	Common Misconceptions: students will identify and explain some relevant algebraic errors				
The Research Lesson	Introduce forming algebraic fractions in a Problem-Solving context Understand how to correctly add algebraic fractions Leading to the expression: $\frac{V+2}{3}+\frac{V-3}{4}$ and asking the students to express this as a single fraction.				
6	Identifying, explaining and correcting errors from earlier assessment questions. also making sense of the result: e.g. $\frac{7x-1}{12}$ from research lesson. Practice adding and subtracting algebraic fractions.				
7	Forming and solving equations with algebraic fractions. From research lesson: What is the voucher's value if Ann and Barry have €11worth between them (i.e. solve $\frac{V+2}{3}+\frac{V-3}{4} = 11$				
8	Forming and solving equations with algebraic fractions				

8. Goals of the Research Lesson:

a) Mathematical Goals

Students will:

- be able to form algebraic fractions from word problems.
- recognise the need for algebraic fractions.
- be proficient with the distributive property including fractions.
- be able to add algebraic fractions.

b) Key Skills and Statements of Learning

We have considered many of the new Junior Cycle Key Skills and Statements of Learning in the planning of this lesson. In this lesson we will encourage and implement the Junior Cycle Key Skills as follows:

- **Managing Myself:** Students will have the opportunity to take a step back and reflect on their learning. By doing this, it allows the students to develop their critical thinking skills and improve future performance.
- **Managing Information and Thinking:** By instilling curiosity into our students they will begin to think creatively and critically. Students will be encouraged to question ideas and to adjust their thinking in light of new information.
- **Working with Others:** Students will develop key skills when working in groups and respecting differences of opinion.
- **Communicating:** Students will present, interpret and compare information.
- **Being Creative:** Students will evaluate different ideas and actions as they take risks and learn from mistakes and failures. They will have opportunities to present their mathematical thinking in different forms.
- **Staying Well:** Students will be encouraged to be confident and contribute to decision-making within the group and class.

Our lesson is also designed to meet the following Junior Cycle Statements of Learning:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The student describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
<p>Introduction and Posing Task 1 (5 mins)</p> <p>Students will be shown a picture of a Vodafone voucher of unknown value and two €1 notes on the board. Students will be asked how the voucher and the two €1 notes can be divided equally between three people.</p> <p>In order to aid students with the task and to cater for different abilities a paper voucher, two “€1 notes”, a scissors, blu-tak and a ruler will be handed to each group.</p>	<p>Four Goals of the Lesson will be on display in the room.</p> <p>Students will be encouraged to be as involved as they can in the lesson and to keep trying to think of different ways to approach the problems.</p> <p>The problem will initially be posted on the board and will be on the desk for the students</p> <p>Students will be asked to use their placemats to represent their answer in different ways.</p>	<p>Are students clear on what they are being asked to do?</p>
<p>Student Work (15 minutes group work)</p> <p><u>Student Response 1:</u> Paper voucher and notes cut into thirds. One third of voucher and two thirds of a note in each share. (Could also be drawn)</p> <p><u>Student Response 2:</u> “One third of the voucher and 67 cents” (or 66 cents)</p> <p><u>Student Response 3:</u> “One third of the voucher and 66 and two third cents”</p> <p><u>Student Response 4:</u> “One third of the voucher and two thirds of a Euro”</p> <p><u>Student Response 5:</u> “One third of (the voucher plus €2)”</p> <p><u>Student Response 6:</u> $\frac{V}{3} + \frac{2}{3}$</p> <p><u>Student Response 7:</u> $\frac{V+2}{3}$</p>	<p>If no group has response 7 but group(s) have response 6 teacher may discretely nudge them to express their response as a single term.</p>	<p>Teacher with seating plan looking for responses 1 to 7 during <i>kikkanshidō</i></p> <p>Do the manipulatives help students gain insight into the problem? (Or are they a time consuming distraction?)</p> <p>Can students convert the word problem into a mathematical statement and think about it?</p>
<p>Ceardaíocht /Comparing and Discussing Task 1 (15 minutes)</p> <p>Teacher to guide students towards response 7 as the “most sophisticated” and succinct response (thinking ahead to adding fractions and solving equations)</p>	<p>Correct rounding errors if seen (66 or 67 cents are not exactly two thirds of €1).</p> <p>If seen, emphasise accuracy of fractions compared to rounded decimals.</p> <p>Will anyone respond:</p> <p>$\frac{V}{3} + 2$ or $V + \frac{2}{3}$?</p> <p>Why do these NOT make sense?</p>	<p>What would/should you get if you added the three shares together?</p> <p>Alan can you explain Bella's response to me?</p>

	<p>In Christmas test some students wrote equivalent of:</p> $\frac{V}{3} + \frac{2}{3} = \frac{V+2}{6}$ <p>Why does this NOT make sense?</p> <p>Response 7 is “most sophisticated”</p>	
<p>Posing Task 2 (3 minutes) One voucher and three €1 IOUs will be shared between 4 people. How much will each person get?</p>	<p>Task 2 will be posted up on display.</p> <p>Make sure that there are no misconceptions with the term IOU.</p> <p>How will students represent the IOUs? Agree that three €1 IOUs should not be represented the same way as three €1 notes.</p>	<p>Teacher to ask “Can we represent three €1 IOUs the same way as we would three €1 notes?”</p>
<p>Task 2 (10 minutes individual work)</p> <p><u>Student Response 1:</u> Paper voucher and IOUs cut into quarters. One quarter of voucher and three quarters of an IOU in each share. (Could also be drawn)</p> <p><u>Student Response 2:</u> “One quarter of the voucher and 75 cents IOU”.</p> <p><u>Student Response 3:</u> “One quarter of (the voucher plus three €1 IOUs)”</p> <p><u>Student Response 4:</u> $\frac{V}{4} - \frac{3}{4} \quad \left(\text{or } \frac{V}{4} + \left(\frac{-3}{4}\right)\right)$ </p> <p><u>Student Response 5:</u> $\frac{V-3}{4}$ (or $\frac{V+(-3)}{4}$)</p>		<p>Why does</p> $\frac{V}{4} + \frac{3}{4}$ <p>NOT make sense?</p>
<p>Ceardaíocht /Comparing and Discussing Task 2 (10 minutes)</p>	<p>Which response was the quickest to form? Which looks most like response 7 from task 1?</p>	<p>Why does</p> $\frac{V}{4} + \frac{3}{4}$ <p>NOT make sense?</p>
<p>Posing Task 3 (3 minutes) Ann gets a share from task one, Barry gets a share from task 2, Can you represent how much the couple Ann and Barry get between them in a single term/fraction?</p>	<p>Task 3 will be posted up on display.</p>	
<p>Task 3 (7 minutes individual work)</p>		<p>Did anyone go back to the picture representations? Do any students work with 4 separate fractions i.e.</p>

		$\frac{V}{3} + \frac{2}{3} + \frac{V}{4} - \frac{3}{4}$ <p>Are errors from Christmas test (sign, distribution..) still as prevalent?</p>
Ceardaíocht /Comparing and Discussing Task 3 (8 minutes) $\frac{V+2}{3} + \frac{V-3}{4} = \frac{4(V+2)}{4(3)} + \frac{3(V-3)}{3(4)}$ $= \frac{4V+8}{12} + \frac{3V-9}{12} = \frac{7V-1}{12}$	<p>Emphasise:</p> <ul style="list-style-type: none"> • Similarity with numeric fractions (LCD) • Appropriate use of brackets to apply distribution correctly • Correct handling of signs <p>Tomorrow we will try to make more sense of this result and you will get more information that will enable you to find the value of the voucher.</p>	
Summing up & Reflection (5 minutes) <p>We learned:</p> <ul style="list-style-type: none"> • we can form algebraic fractions from word problems. • we need algebraic fractions to represent real life scenarios. • how to correctly distribute multiplication in algebraic fractions. • how to add algebraic fractions. 		<p>Students will be asked to quickly fill out the reflection sheet (see appendix):</p> <ol style="list-style-type: none"> 1. What did you learn in today's lesson? 2. Is there anything that you don't fully understand? 3. Did you enjoy today's lesson? Why or why not?

10. Board Plan

Tasks

Lesson Goals

- Be able to form algebraic fractions from word problems
- Recognise the need for algebraic fractions
- Be able to apply distribution properly (when fractions involved)
- Be able to add algebraic fractions

Task 1

IF THE VOUCHER AND THE TWO €1 NOTES ARE SHARED EQUALLY BETWEEN THREE PEOPLE, HOW MUCH DOES EACH PERSON GET?

Task 2

IF THE VOUCHERS AND THREE €1 IOU'S ARE SHARED EQUALLY BETWEEN FOUR PEOPLE, HOW MUCH DOES EACH PERSON GET?

Task 3

If you take one student from Task 1 and one person from Task 2, how much do they have in total?

Anne and Barry

Tasks 1 and 2

Task 1 Solutions:

- Ciora: $\frac{1}{3} \text{ OF (VOUCHER + €2)}$
- Simon: $\frac{1}{3} (x + 2)$
- Kate: $\frac{1}{3} \text{ OF VOUCHER AND } 66\frac{2}{3} \text{c.}$
- Michael: $\frac{1}{3} \text{ OF THE VOUCHER AND } \frac{2}{3} \text{ OF A EURO}$
- Alex: $\frac{1}{3} \times \frac{(x+2)}{1}$
- Fiona: $\frac{1}{3}x + \frac{2}{3}$
- Barry: $\frac{V+2}{3}$

Task 2 Solutions:

- Leah: $\frac{V+3}{4} = \frac{V-3}{4}$
- Rhian: $\frac{V+3}{4} ?$
- Matthew: $\frac{1}{4} \text{ (VOUCHER + 3 IOUs)}$
- Caoimhin: $\frac{1}{4}x - \frac{3}{4}$

Other Work:

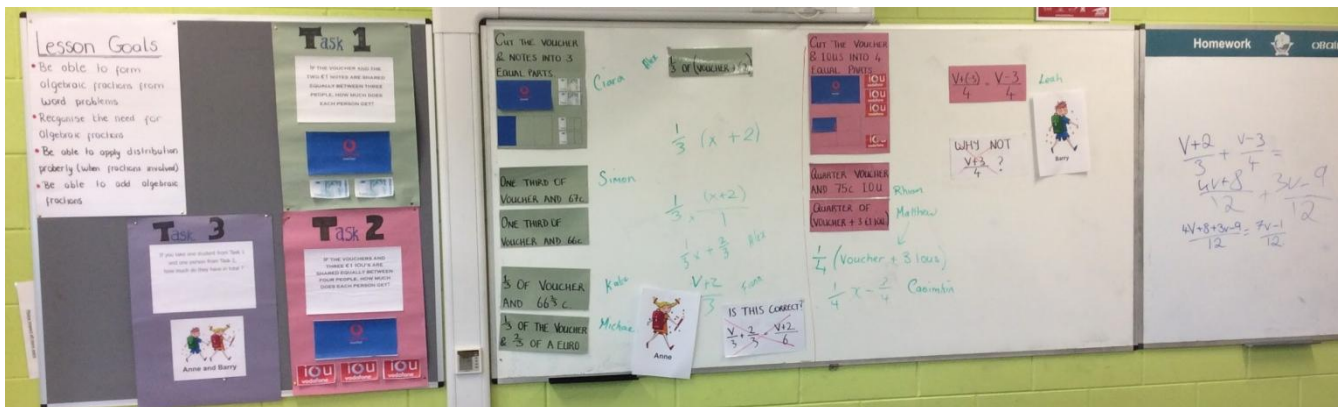
- IS THIS CORRECT? $\frac{V}{3} + \frac{2}{3} = \frac{V+2}{6}$

Task 3

Kezia

$$\frac{V+2}{3} + \frac{V-3}{4} = \frac{4(V+2)}{4(3)} + \frac{3(V-3)}{4}$$

$$= \frac{4V+8}{12} + \frac{3V-9}{12} = \frac{7V-1}{12}$$



11. Evaluation

Plan for observing students is as follows:

- Teacher to supply classroom layout in advance of research lesson
- Three teachers and one knowledgeable other will observe the lesson. They will use the “lesson observation” form to track student responses and record their observations.
- Observers may take photographs of some students work (one teacher may circulate, others will be stationary) and students will leave their placemats and other work in the room when they leave.
- Observers will record questions asked by students.

For the post lesson discussion, the group are interested in whether the lesson:

- Gave students of all abilities opportunities to experience challenges and successes in maths.
- Encouraged students to recognise maths in the world around them.

And to what extent the mathematical goals of the research lesson were met i.e. students will:

- be proficient with the distributive property including fractions.
- be able to form algebraic fraction from word problems.
- recognise the need for algebraic fractions.
- be able to add algebraic fractions.

Other points of interest include:

- Whether the manipulatives worked or were a distraction.
- Whether the tasks were clear to the students.

12. Reflection

The major patterns and tendencies in the evidence - key observations of student learning and thinking

Reflecting on the lesson, we felt that it was pitched to the correct level and that the students benefited from it and appeared to enjoy it. We were particularly pleased to observe active participation by all students during the task for what was effectively a double maths class.

At the beginning, the students were slightly reluctant to use the physical vouchers which were there to help them. Once students realised that cutting up the images was a tool to help them progress with the task all groups began cutting up the vouchers which really helped the students to visualize the task set. Out of the two groups that each teacher observed, using the pictures section on the placemat really helped students to progress to expressing the task as words and as a mathematical expression.

Picture	Words	Table/Other
<p>€1 notes</p> <p>for person: $\frac{1}{3}$ of voucher $\frac{1}{3}$ of money</p> <p>$1 \div 3 = \frac{1}{3}$ $2 \div 6 = \frac{1}{3}$</p>	<p>We cut the voucher into 3 pieces to evenly distribute it and did the same with the one euro note. Each person would get $\frac{1}{3}$ of the voucher and $\frac{1}{3}$ of the money.</p> <p>Task 1</p> <p>If the voucher and the two €1 are shared equally between three people, how much does each person get?</p> <p>$1 \div 3 = \frac{1}{3}$</p>	<p>(Each, Each)</p> <p>Each person would get $\frac{1}{3}$ of the voucher and $\frac{2}{6}$ of the money (€2)</p> <p>$\frac{2}{6} \times 2 = €0.67$</p>
Mathematical Expression	Single Mathematical Expression	Preferred Answer

The students were confident in expressing the shares in words form on their placemat. They also greatly benefitted from the boardwork of the other students. It was good to see that many of them were

really engaging during the first task and that several of the different responses were used. Many students struggled with expressing the share for Task 1 as a mathematical expression and needed guidance from the teacher when doing the boardwork to reach the “most sophisticated” response.

Students were extremely engaged and were self-motivated to carry on from task to task. Some students were slow to link task 1 to task 2. After a small bit of probing and explanation during ceardaiocht, students quickly made the link and were very quick linking Task 2 to Task 3.

It was very clear that all students learned from this task and remained engaged throughout the whole lesson. It was also nice to incorporate some structured problem-solving into this lesson. The following are some samples of the students reflections at the end of the lesson:

What did you learn in today's lesson?
How to form fractions from word problems.

Is there anything that you don't fully understand?
No.

Did you enjoy today's lesson? Why or why not?
Yes - it was easier to understand because of the pictures and going through everything step by step slowly.

What did you learn in today's lesson?
 I learned how to form fractions out of word problems.
 I have a greater understanding of fractions

Is there anything that you don't fully understand?
 I feel like if I reuse the terms more I could get full marks in everything or fully understand what's going on.

Did you enjoy today's lesson? Why or why not?
 yes. At the start I felt overwhelmed but overall it was really fun & interesting.

What does the evidence suggest about the student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

During Task 1 we noticed that students were very slow in forming the algebraic expressions. When students were coming up to the board it was positive to see that many of the partial responses were used. Students struggled to move from the verbal sentence of "One third of (the voucher plus €2)" to the mathematical expression of $\frac{v}{3} + \frac{2}{3}$ and $\frac{v+2}{3}$.

It was encouraging to see that many of the students moved away from the idea of expressing the two €1's as 0.67c for each person and instead knew it was more accurate to express the €1 as $\frac{2}{3}$ of €1. See below:

$$\frac{2}{3} \times 1 = .67c$$

$$\frac{1}{3} \times x =$$

$$.67c + \frac{1}{3} \text{ of the Voucher}$$

Mathematical Expression

~~$$x + \frac{2}{3}$$~~

$$x + 2y$$

$$\frac{1}{3} \text{ of } x + \frac{2}{3} \text{ of euro.}$$

For Task 2 some students were not able to make the connection between the expression from Task 1 and the desired expression for Task 2. We believe this may have been due to some misconceptions regarding the IOU as took the students a lot longer to recognize the IOU's as negative values. In future we would make this more obvious in advance.

Other	Words Each person will get $\frac{1}{4}$ of the voucher and $3\frac{1}{2}$
<p align="center">Task 2</p> <p align="center">If the voucher and three €1 IOU's are shared equally between four people how much does each person get ?</p>	
Mathematical Expression $\frac{1}{4}(x+3)$ $\frac{1}{4} + \frac{x+3}{1}$	Single Fraction $\frac{x-€3}{4}$

This student was unsure of the need to express an IOU as a negative value but self-corrected, made the connection and expressed it correctly as a single fraction.

From analyzing their recent test responses (see section 4. Background & Rationale) we thought that students would struggle with Task 3 but we were pleasantly surprised at how quickly and accurately students were able to express their answers from Task 1 and Task 2 as a single fraction with minimal distribution errors.

Task 3

If you take one student from Task 1 and one person from Task 2, how much do they have in total ?

Express your answer as a single fraction.

TASK 1:
ANNE has $\frac{1}{3}(\text{voucher} + €2)$

TASK 2:
BARRY has $\frac{1}{4}(\text{voucher} + 3 \text{ €1 IOUs})$

$$\Rightarrow \left(\frac{V+2}{3}\right) \times \frac{1}{3} + \left(\frac{V-3}{4}\right) \times \frac{1}{4} \rightarrow \frac{V+2(4)}{3(4)} + \frac{V-3(3)}{4(3)}$$

$$\frac{(4V+8)+(3V-9)}{12} = \frac{7V-1}{12}$$

In what ways did students achieve or not achieve the learning goals?

The vast majority of students achieved the four learning goals that were set out at the beginning of the lesson. As evident from the work of the students in Task 1 and Task 2 they were successfully able to form an algebraic expression from a word problem. Students were very confident in adding algebraic fractions during Task 3 and were able to apply distribution correctly with only a small minority of students not fully achieving this learning outcome. At the end of the lesson during the class discussion we feel that a lot of the students began to recognize the importance of algebraic fractions which will be beneficial to them in future problem solving questions.

Based on your analysis, how would you change or revise the lesson?

Students needed more time when tackling Task 1 as many of them felt overwhelmed and out of their comfort zone. Students need to be clear on what is meant by an “algebraic expression”. Students would need to have more prior knowledge of the concept of IOU’s and that they should be expressed as a negative value. This would clear up that misconception and more students should be better able to transition from Task 1 to Task 2.

Students should be encouraged to regularly check their answer to see “does it make sense”.

What are the implications for teaching in your field?

We plan to share our work with the rest of our school maths department as we feel this is an effective approach towards working with algebraic fractions and can really help our students.

As teachers we see that we should:

- Encourage students to verbalise their answers more in class.
- Encourage more collaborative learning where students have to discuss problems with each other.
- Hold back as a teacher with the answer and allow the students the time and space to struggle and make progress themselves.

As a team we’re very happy we chose this particular research topic as it gave us a great opportunity to properly investigate how and why our students are struggling with algebraic fractions. Lesson study then gave us the format to collaborate on addressing these issues in new ways and to assess the effectiveness of our designed approaches.

Appendix A: Vouchers, “money” and IOUs that students can manipulate for lesson



Appendix B: placemats for 3 tasks

Task 1

Picture	Words	Table/Other
<p>Task 1</p> <p>If the voucher and the two €1 are shared equally between three people, how much does each person get ?</p>		
Mathematical Expression	Single Mathematical Expression	Preferred Answer

Task 2

Picture	Words	Table
<p>Task 2</p> <p>If the voucher and three €1 IOU's are shared equally between four people how much does each person get ?</p>		
Mathematical Expression	Single Fraction	Preferred Answer

Task 3

<p>Task 3</p> <p>If you take one student from Task 1 and another from Task 2 how much do they have altogether ?</p>
--

Express your answer as a single fraction.

Appendix C: Student Reflection Sheet

Reflecting on my Learning

Student:

What did you learn in today's lesson?

Is there anything that you don't fully understand?

Did you enjoy today's lesson? Why or why not?