Lesson Research Proposal for Transition Year & Making Connections (Algebra, Calculus & Functions

For the lesson on January 31st 2018
At Laurel Hill Colaiste FCJ, Limerick. Transition Year
Instructor: Michael McDonagh
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1. Title of the Lesson: Shootin' Hoops

2. Brief description of the lesson

With the aid of a mathematics problem, students will be provided with a calculus based question that requires them to utilize their algebraic knowledge of roots to construct the pathway of the basketball heading towards a hoop. Next, students will be tasked with finding the quadratic function of such a curve, before lastly examining the different possible curves and their respective heights.

3. Research Theme

<u>Teaching of specific content:</u>

At this school, we want students to:

1. Enjoy their mathematics classes by engaging them in structured problem-solving questions that seeks to combine their fundamental understanding of mathematics and their able to contextualize mathematics.
2. Make connects between the different areas of mathematics and to be able to utilize all aspects of mathematics when tasked with solving a problem.

<u>Broad Teaching & Learning Goal:</u>

The broader and learning goals for this lesson stem from the *Looking at our Schools 2016 – A Quality Framework for Post-Primary Schools* document published by the Department of Education and Skills (DES, 2016) (Appendix One).

1. Students: *grow as learners through respectful interactions and experiences that are challenging and supportive*.

In selecting this standard, we assert on the importance of the interactions between students and their peers and also their teachers. These interactions and relationships are key in creating a positive learning environment of co-operative, affirmation and are conductive to well-being. Throughout this problem, we aim for our students to *risk incorrect responses and to understand the value of making mistakes, using them as learning opportunities.* In addition, we want our students to *contribute their opinions and experiences to class discussion with confidence. They are respectful of others and experiences of their classmates.* Lastly, we want our students to *demonstrate a high level of motivation, and enjoy engaging and persisting with increasingly challenging work.*

2. Teachers: *value and engage in professional development and professional collaboration*.

In selecting this standard, we identify and engage in Lesson Study as a form of Continuous Professional Development (CPD) that *develops our own practice, meets the needs of students and the school and enhances collective practice*. Furthermore, we view collaboration as a means to improve student learning and to *enhance our own professional development*.

4. Background & Rationale

Why you chose this topic:

- We find students have difficulties linking real life situations and mathematics (especially on the topics of functions and algebra), as a result we have decided to focus on a quadratic equation question that can be easily linked to realistic mathematics education.
- As we are teaching Transition Year Mathematics, we are building towards the Leaving Certificate topics of algebra, functions, calculus coordinate geometry, trigonometry and number patterns, while we seek to build connections between these different topics to highlight the interconnectedness of mathematics.
- We'd like the students to realise that there's a number of different solutions to a problem and that mathematics problems are not solely linear in their solution.
- We would like to encourage the students to take a more active role in the class and in their learning, thus improving on their ability to be self-motivation learners.
- We hope to develop the spatial awareness of our students by encouraging them to visually think about the problem.
- We would like to encourage a universal approach to teaching mathematics, by encouraging all teachers to discuss and agree upon an initial method, before expanding to different methods of solution.
- We would like the students to be less fearful of problem solving and improve their confidence to tackle a problem.
- We would like our students to show good competency in translating a problem from a paragraph of written text to an equation.

Your research findings:

The 2015 Chief Examiner's Mathematics Report highlighted that:

'most candidates demonstrated good levels of knowledge and comprehension of basic mathematical concepts and relations, which is fundamental to the successful development of mathematical proficiency. Candidates struggled at times when more involved understanding was required, or when the concepts were slightly less standard. It is recommended that candidates give due attention to understanding the mathematical terms contained in the syllabus, including being able to differentiate between them where appropriate, and being able to explain what they mean in an accurate and coherent fashion'.

From it, it is imperative that we address these recommendations within the context of our own lesson study. We aim to tackle these difficulties that students have with developing their own mathematical proficiency and their own knowledge of concepts, along with problem solving. We aim to ensure that we will utilize this lesson study session as a means of expanding on our student's knowledge of lesson study and to be able to articulate and orally explain their logic and workings.

5. Relationship of the Unit to the Syllabus

Problem solving means engaging in a task for which the solution is not immediately obvious. As teachers, we want to reinforce the concept of real life mathematics and also prepare a lesson that is fully differentiated to meet the needs of all our students.

We aim to link Strand 2 (Geometry & Trigonometry), to Strand 4 (Algebra) and Strand 5 (Functions).

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
The Primary School Curriculum for **First and Second-Class** states that student should be able to: -select appropriate materials and processes for mathematical tasks and application. -select and apply strategies for completing a task or solving a problem. -Listen to and discuss other children's mathematical descriptions and explanations. -Discuss problems and presented pictorially or orally. Furthermore, the curriculum for **fifth and sixth class** states that students will be able to: -analyze problems and plan an approach to solve them. -reflect upon and evaluate solutions to problems. -identity positive and negative numbers on the number line. -translate word problems with a variable into number	The **Suggested Ideas and Approaches for a Transition Year Programme** states that during Transition Year that students should: Develop their fluency and skills in relation to: • Factoring expressions • Common factors • Difference of two squares • Sum and difference of two cubes (Higher) • The perfect square • Functions. Furthermore, the suggested ideas and approaches recommends: *'to teach them the completion of the square for solving quadratic equations and analysing the quadratic function. Higher levels students might perhaps explore the use of completing the square in deriving the quadratic formula'*. As this lesson, will be examining multiple solutions to a quadratic problem, it is imperative to discuss the –b formula as a method of solving.	We hope to foster a deeper understanding of quadratic graphs and develop the student's ability to analyse and interpret the pathway taken by a graph. This will lead us into rates of change, stationary points, increasing/decreasing slopes and even second derivatives. Through the study of more than one pathway (into the basket) we could develop our students' knowledge of transformation geometry. We will progress to completing the square and follow on to derive the –b formula. The **Leaving Certificate Syllabus (2016)** states that students must be able to: Form quadratic equations given whole number roots (4.2) Furthermore, students must be able to use the complete square form of a quadratic function to: find the roots and turning points and sketch the function (5.1).

sentences. -explore the relationship between time, distance and average speed. The **Junior Certificate Mathematics Curriculum (2016)** states that students should be able to: -Express a general relationship arising from a pattern or context (4.3) -Find the underlying formula written in words from which the data are derived (linear, quadratic relations) (4.3) -Examining Algebraic relationships: non-constant rate of change of quadratic relationships (4.4) - Discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula (4.4). − Explore graphs of motion, make sense of quantitative graphs and draw conclusions from them, make connections between the shape of a graph and the story of a phenomenon, describe both quantity and change of quantity on a graph (4.5).	To further develop connection between the different topics of mathematics – algebra/functions/calculus. We hope to consolidate the students understanding of roots – where they are on the graph and how they can be used to form the quadratic and conversely how the roots can be found if given the equation. The **Junior Certificate Mathematics Curriculum (2016)** states that students should be able to: - Interpret and represent linear, quadratic and exponential functions in graphical form (5.2) - Find maximum and minimum values of quadratic functions from a graph (5.2).	Students must be able to find first and second derivatives of linear, quadratic and cubic functions by rule. Associate derivatives with slopes and tangent lines – apply differentiation to: • rates of change • maxima and minima • curve sketching (5.2)

6. Goals of the Unit

We want our students to engage in problem solving and appreciate that there is more than one way to solve a problem. We hope all students through group work will actively participate in the lesson and work together through varied abilities to derive a solution. On completion of lesson we hope to have instilled the self-confidence of the student in tackling unseen problems and in appreciating the importance of method over answer.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1 & 2	Recap on graphing functions, students will be tasked with graphing: • linear functions ($f(x) = ax, where\ a \in \mathbb{Z}, a \neq 0$), • quadratic functions ($g(x) = ax^2 + bx + c, where\ a, b, c \in \mathbb{Z},\ a \neq 0$) • cubic functions ($h(x) = ax^3 + bx^2 + cx + d, where\ a, b, c, d \in \mathbb{Z},\ a \neq 0$). •
3	Introduction to the concept of the Rates of Change and Differentiation. $$\frac{d}{dx}(x^n) = nx^{n-1}, where\ n \in \mathbb{R}.$$ Introduce to the 'differentiating family tree', that the order of first derivatives is Cubic → Quadratic → Linear→ Constant.
4	Introduction to the following rules of calculus: Product rule: $$when\ y = uv, then\ \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$ Quotient rule: $$when\ y = \frac{u}{v}, then\ \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$
5	Recap on the Product and Chain rules, before progressing to the Chain Rule: $$when\ y = u(v(x)), then\ \frac{dy}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$
6	Research Lesson: Making Connections: Shootin' Hoops

7	Conclude on the research lesson and then progress onto Stationary points. Analyse and interpret slopes on either side and at turning point.
8	Explore Increasing/decreasing slopes (reconnection to lesson 1).

8. Goals of the Research Lesson:

a) <u>Mathematical Goals</u>

- Students need to know where the roots are on the graph.
- Students need to know how to apply the Factor Theorem
- Students need to know how the roots/factors can be used to form the quadratic equation
- Students need to know how to draw a negative Quadratic graph
- Students need to understand that quadratic graphs are symmetrical in nature.
- Students need to be self-motivated
- Students need to bring their theories and potential solutions to their group individually
- Students need to work together to analyse the credentials of the different solutions suggested and work on finding their set of solutions
- Students need to check their answers to ensure that they are logical (ie. not a $+ax^2$ graph)

b) <u>Key Skills and Statements of Learning:</u>

Key Skills for Leaving Certificate	Goals of the Research Lesson
Communicating	Through group work the students will actively participate in the lesson and work together. They will also present some of their work to their peers
Being Personally Effective	Students are tasked to be independent at the start of the lesson, they must utilize the time in order to be as successful as possible
Working with Others	They will work together as a group with varied abilities to best present an articulated answer
Critical and Creative Thinking	Students can be as creative as possible with their answer as we will not be limiting/helping at any point
Information Processing	Students will be filtering through large volumes of information and logically differentiating between the answers

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Teacher to set the class down and to introduce the class to today's research lesson. To acknowledge the presence of the additional staff members and the external member in the classroom. To briefly recap on the material covered thus far in this unit for the students (prior knowledge). Then to pose the problem to the class, explaining the concept of this problem-solving lesson (Appendix 2). **Shootin' Hoops…** Niamh is playing basketball. She picks the ball off the ground and in one continuous motion she shoots to the basket. The ball hits the floor 4 metres from where it was thrown passing through the basket on the way. Q1. If the basketball follows a continuous quadratic path, as accurately as you can sketch a graph of the flight of the ball? Q2. Can you find an expression for the function represented in this graph? Q3. What can you work out about the greatest height reached by the ball?  Explain your answers and show your workings Introduce students to the problem and read it aloud for them.	Use of the classroom computer and projector. Teacher has a series of 'revision' sheets done to post on the board, as needed.	Questioning in relation to the prior knowledge: What do we know about graphing functions? What functions have we graphed previously? What connection is there between the functions?
Posing the Task Teacher will pose the task to the students and provide them with the details of the task. The teacher will read the problem aloud to the students and then read the three parts of the problem. The teacher will emphasis the need to attempt each part and that students will be given approximately ten minutes to solve said problem independently. Then students will be allowed to discuss their solutions with their small teams before concluded on the time.	The problem will be projected onto the white board behind the teachers' desk. The problem will also be handed out to each student via a worksheet format and then students will be asked to complete their sheets as appropriate. Teacher will have additional sheets if students	Students can ask for clarification on certain elements of the task prior to starting. Teacher will check for further understanding throughout the initial questions.

Question 4 will only be realized if necessary depending on the time available of the students. **Sh🏀🏀tin' H🏀🏀ps…** Q4. The height of the basket was 3 metres. The ball was also 3 metres from the base of the basket when Niamh started to throw it. Does this effect your answers to the other questions?  Explain your answers and show your workings.	require more paper. The release of the second question sheet will only be done if time allows.	
Student Individual Work The following results are those that we anticipate the students to prepare. These are envisaged in the correct chorological manner also. Anticipated results: 1. Positive Quadratic Solution (no scales). 2. Negative Quadratic Solution (no scales). 3. Negative Quadratic Solution (with scales). 4. Negative Quadratic Solution (with detailed scales). 5. Negative Quadratic Solution (with detailed scales). 6. Different scale Negative Solution (with scales). 7. Multiple Representation of Graphs. 8. Algebraic: Forming positive quadratic. 9. Algebraic: Forming another positive quadratic. 10. Algebraic: Forming a correct negative quadratic. 11. (10.5) –b formula solution 12. Multiple Quadratic Solutions (y-changing). 15. Solution to Q4 part of the question. Students will work independently for 13 minutes but summarizing their answers for 7 minutes.	Teacher has prepared A3 sheets of the following answers. Depending on the 'flow' of the lesson, the teacher will select the most appropriate method of anticipated results. It will also be decided on what solutions will be discussed and student-presented in the correct order.	Students will be assessed on their ability to correct articulate the question and in their ability to work through the question in a precise and well-structured manner. Teacher will examine the student's ability to make connections between algebra, functions, graphing functions and then leading on to calculus. The teacher will also be reviewing the solutions to check for common misconceptions and common mistakes among the students, in order to best order the Ceardaiocht later.

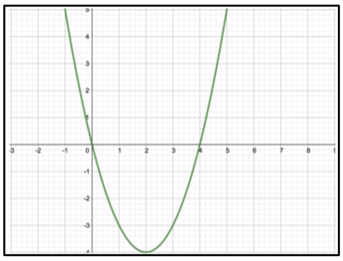
Ceardaíocht /Comparing and Discussing The Ceardaiocht will be structured in a well-constructed manner. It will address the questions in order, stating with part (i) and initially working towards part (iii). The common approach to start will be that of graphing/sketching the quadratic pathway firstly. It is envisaged that students will have difficulties in graphing the quadratic function, especially as the students will have to address the negative element to converting from roots to a function. Discussing of the incorrect roots, leading to a positive quadratic will be established and then the corrective manner will be addressed to dispel this common misconception. A student approach of the positive quadratic will be discussed and students will be questioned logically about their formulation of this answer. For the remaining parts of the lesson, students will be challenged about utilizing their correct graph to answer the following questions. It is hoped, that students will not only select the 'traditional' roots of 0 and 4, but any roots with a differ of 4 units. The use of the quadratic roots formula will be discussed as a means of solving all such roots to conclude on the same answer. Progressing on, students will be challenged at graphing/examining the possibility of different heights of the same quadratic root. Lastly, it is aimed to conclude on the fact that the geometrical transformations of graphs are wholly possible that part (iv) ties this question down to one such answer. Throughout this discussion, students will be challenged to articulate the sophistication of the answers to conclude on the precision and the robust mathematics behind each solution. Effective questioning will be utilized to deep further into the students' understanding (Why is that?, What evidence do you have for this?, How	Teacher will support the students by asking a range of probing questions depending on their possible solution method. Teacher will strive to only ask open ended questions and to assist students by asking our students on the floor.	Students will be assessed on their ability to formulate and justify each of their answers. Students will be tasked at examining other students' solutions also and striving from a logically approach to each answer. Students will be tested on their knowledge of the different areas of the curriculum and in their ability to link these to other topics of mathematics.

can we do this more effectively? etc).		
Summing up & Reflection Teacher will sum up the lesson by discussing that there are different versions of the quadratic functions that in terms of the x-axis movement will all share the same maximum height and in terms of their y-axis, this can increase continuously but that logic must prevail at some point depending on the question asked. The teacher will highlight the different functions and then lead this nicely into calculus as a means of solving.	Students will complete the reflection sheets.	Collection of the data sheets will ensure that these can be examined later on in the post-lesson discussion.

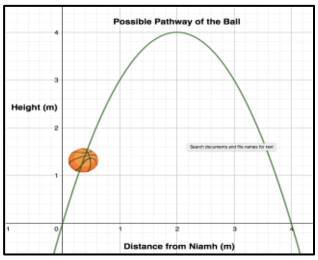
10. Board Plan

Below is the suggestive board plan for the lesson proposal. The order goes from the first column downwards, then middle column downwards and finally to the last column going downwards.

Positive Quadratic Solution (no scales).



Negative Quadratic Solution



Correct negative quadratic.

$x = 0$ and $x = 4$ [Two Roots]

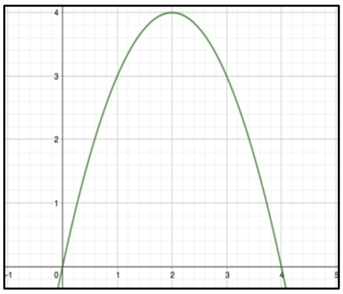
Then $f(x) = (x)(x-4)$

A minus is NEEDED

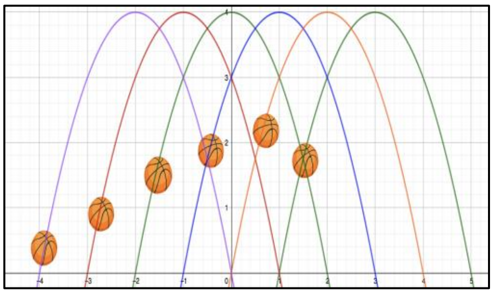
$f(x) = -[(x)(x-4)]$

$f(x) = -x^2 + 4x$

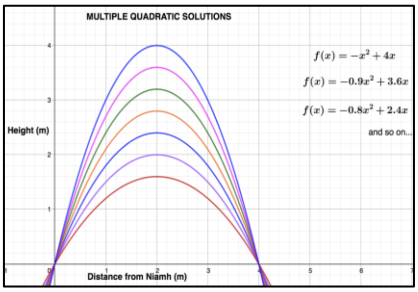
Negative Quadratic Solution (no scales).



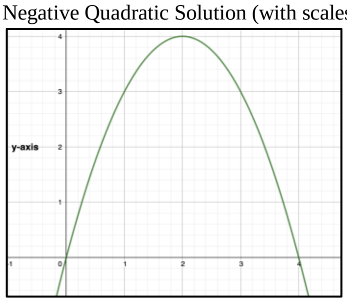
Multiple Representation of Graphs.



Multiple Quadratic Solutions



Negative Quadratic Solution (with scales).



Algebraic: Forming positive quadratic.

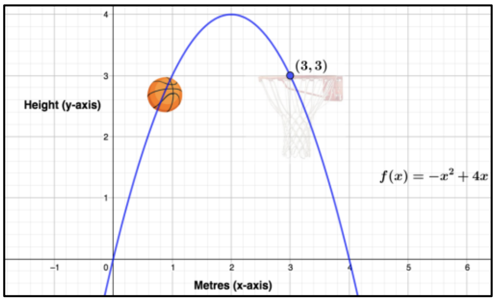
$x = 0$ or $x = 4$ [Two Roots]

Then $f(x) = (x)(x-4)$

$f(x) = x^2 - 4x$

Correct Shape?

Solution to Q4



11. Evaluation

The following questions were addressed after the research lesson on Shootin' Hoops has occurred:

Could students identify the roots?

As expected, there was some difficulty in formulating the roots at first from the information providing in the research question. However, throughout the group discussion and throughout logical trial and error, most of the students were able to articulate the roots for the function. However, it was noted that, as expected, some groups had difficulty with understanding the necessary relationship with the negative quadratic.

Could Students draw the quadratic graph and did they realise the graph was a negative graph?

Yes, all students could formulate the correct shape of the quadratic function. Initially few students had a slight difficulty in this process but overall all students did conclude on the correct shape of the quadratic function. Interestingly, most students started the problem by graphing a 'stick-man' diagram of the possible approach to the quadratic pathway.

Did students realise that initially there are many possible solutions?

Students did not initially release that there were more transformations or possible shifting of the quadratic graph, with all students deciding on graphing their solution from the origin to the point (4,0). Throughout the discussion, the idea was concreted with all students then realizing that there was a multiple of possible approaches to the solution.

When given further information about the position of the basket could the students then find the one accurate solution?

It was interesting to note that some students thought about the different possible heights of the graph, by drawing multiple approaches to the question. When provided with the additional information of question four, must student groups were then able to formulate a better understanding of the graph. Students were able to grasp that the ball would reach its maximum point and then pass through the basket before returning to the floor.

Did the students work and communicate well in their groups?

Students use of teamwork and support was fantastic to note. Students excelled at working collectively and in cooperation. This was one of the main teaching and learning aspects in planning this lesson that students would be able: *grow as learners through respectful interactions and experiences that are challenging and supportive*. Furthermore, students could meet all the required that were planned in the Key Skills section also.

Did all students in each group take some active part in the lesson?

Yes. It was evident that students were stimulate and engaged in this problem-solving lesson. They were motivated learners and both during their independent time and the group-work aspects, all students worked together to give their opinion and to share their mathematical knowledge. This is in line with the research team's hope that students would: *contribute their opinions and experiences to class discussion with confidence. They are respectful of others and experiences of their classmates and to* *demonstrate a high level of motivation, and enjoy engaging and persisting with increasingly challenging work.*

12. Reflection

On reflection, both the teachers involved and the students seemed to really enjoy the Shootin' Hoops lesson. The team hoped that students would be motivated to participate in a research lesson that would engage them into several different areas of the common Transition Year guidelines and the Leaving Certificate curricula. It was evident that this lesson provided enough scope to challenge students to engage with a typical problem-solving lesson. It also provided students with the opportunity to be both personally effective in their methodology and also challenged students to work cooperatively to summarise and articulate their answers. On reflection, this aim was met. The students did work both independently and in cooperation to merge different aspects of their mathematical knowledge to solve a real-life contextualized mathematical problem. Furthermore, in relationship to the background and the reasons for this problem, the research team was thrilled to acknowledge that all of these were met. Students were tasked with tasking a more active role in their learning, that there is more than one way and one solution to a problem and finally, that students would be less fearful of problem-solving.

The research team observed the students' striving for excellence in their approaches to solving this problem. They saw the students practicing in multiple problem-solving strategies including: trial and improvement, drawing a diagram, drawing a table, eliminating probabilities and using an equation. This multitude of approaches highlights that students are confident in their approaches to initiating mathematical problems. The research team positively highlighted the variety of approaches and the logical nature of each of these approaches. It was noted that some students had difficulty in converting from the algebraic approach of roots to that of developing a functions based graph.

Timing was a minor issue in their lesson proposal, this is just to the unforeseen nature of lesson study and the lesson proposal, it was envisaged that students would work much quicker throughout these initial three parts of the research lesson, however in reality it took longer to reach solutions to these initial questions before giving students the fourth part to the problem. On reflection, the research team would probably alter the structure of the lesson, depending on the class and ability, to spend more time establishing and discussing the solutions to questions one, two and three rather than progressing onto question four. It was also noted that there is a great deal of longevity to this lesson plan, as it was commented upon by our knowledgeable other also, and it could service the needs of second and third years and indeed act as an introductory or revision lesson for more senior year groupings.

The student reflection into this lesson and this style of teaching was also very positive. One student stated that she 'liked working together in groups to solve problems and how we went through the solutions on the board clearly showing why each answer was completely correct or not fully correct. Another student stated that 'I learned how to sketch graphs and solve the formula without as much information as we usually have'. Overall, all students displayed very positive comments on this lesson and its format.

In conclusion, this was a brilliant lesson that encompassed the heart of structured problem-solving and cross-strand materials.

Appendix 1

Quality Framework for Post-Primary Schools – Teaching & Learning

Learner outcomes	Students enjoy their learning, are motivated to learn, and expect to achieve as learners	☐
	Students have the necessary knowledge and skills to understand themselves and their relationships	☐
	Students demonstrate the knowledge, skills and understanding required by the post-primary curriculum	☐
	Students attain the stated learning outcome for each subject, course and programme	☐
Learner experiences	Students engage purposefully in meaningful learning activities	☐
	Students grow as learners through respectful interactions and experiences that are challenging and supportive	☐
	Students reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning	☐
	Students experience opportunities to develop the skills and attitudes necessary for lifelong learning	☐
Teachers' individual practice	The teacher has the requisite subject knowledge, pedagogical knowledge and classroom management skills	☐
	The teacher selects and uses planning, preparation and assessment practices that progress students' learning	☐
	The teacher selects and uses teaching approaches appropriate to the learning intention and the students' learning needs	☐
	The teacher responds to individual learning needs and differentiates teaching and learning activities as necessary	☐
Teachers' collective / collaborative practice	Teachers value and engage in professional development and professional collaboration	☐
	Teachers work together to devise learning opportunities for students across and beyond the curriculum	☐
	Teachers collectively develop and implement consistent and dependable formative and summative assessment practices	☐
	Teachers contribute to building whole-staff capacity by sharing their expertise	☐

Appendix 2

The Question:

Sh🏀🏀tin' H🏀🏀ps…

Niamh is playing basketball. She picks the ball off the ground and in one continuous motion she shoots to the basket. The ball hits the floor 4 metres from where it was thrown passing through the basket on the way.

Q1. If the basketball follows a continuous quadratic path, as accurately as you can sketch a graph of the flight of the ball?

Q2. Can you find an expression for the function represented in this graph?

Q3. What can you work out about the greatest height reached by the ball?

Explain your answers and show your workings

Sh🏀🏀tin' H🏀🏀ps…

Q4. The height of the basket was 3 metres. The ball was also 3 metres from the base of the basket when Niamh started to throw it.

Does this effect your answers to the other questions?

Explain your answers and show your workings.