# Lesson Research Proposal for Second Year, Common Level

For the lesson on the 30th of January
At Mount Temple Comprehensive, Second Year Common Level
Instructor: Louise Lawlor
Lesson plan developed by: Veronica Kerin, Louise Lawlor, Cara Herbert, Heather Davison and Derek McNelis

**1. Title of the Lesson:** The Similar Triangle Wrangle

**2. Brief description of the lesson**

Students will be using dotted paper to produce similar triangles and understand that a line parallel to a side produces similar triangles

**3. Research Theme**

As the Mount Temple Comprehensive mathematics department, we strive for students that enjoy their learning, are motivated to learn, and expect to achieve as learners.
The mathematics teachers intend to work together to devise learning opportunities for students across and beyond the curriculum.
The maths department support this through developing teacher knowledge and competencies in the following areas:

a) Subject Knowledge: Teachers will work in collaboration with the department to identify any gaps their subject knowledge and to deepen their understanding of mathematical concepts to provide the best possible mathematical instruction. This will be achieved through professional discussion and collaboration and individual teacher reflection on practice with appropriate use of student input.
b) Pedagogical Knowledge: Teachers will work together to identify teaching strategies which will support student learning, using research based methods and accessing CPD where necessary.
c) Classroom Management: The experience of the students in the classroom will be central to the planning for and implementation of teaching and learning in mathematics. The pedagogical approach used should encourage students to grow as learners, motivated and achieving their fullest potential. The use of active learning strategies will support this goal and teachers will share their successful pedagogical approaches. Teachers are encouraged to experiment and report back to the department on effective strategies, while understanding not all strategies will work equally well with all teachers and student groups.
d) Knowledge of Students: Teachers will actively engage with students to identify their aptitudes and interests with a view to utilizing

**4. Background & Rationale**

a) As a whole school community, the development of our students as independent learners was identified as a collective aspiration for our students. Although students display a positive attitude toward their learning, teachers agreed there was a need to reinforce and encourage independent learning throughout their time at Mount Temple. Our latest whole school inspection noted the expertise within the school and encouraged teachers to share their knowledge. With the teachers in the mathematics department holding a variety of skills from both within and beyond the teaching profession, there is the potential for teachers to work creatively together to build learning experiences which both fulfill and at times, exceed the curricular expectations. In professional discussions among the math's teachers the ongoing problems experienced by students in the use construction instruments,

formulating proof and fully understanding and applying geometric axioms and theorems was noted. Students could manage geometric questions with shapes and lines in familiar orientations. They were competent in class but struggled with corollaries. The teacher felt there was a need for more student-centered approach.

b) In planning the group were very aware of previous lesson study classes which challenged students to look at a problem in many different ways and decided to try to encourage the students to create their own similar triangles and then use these to come to a greater understanding of similar triangles and later to trigonometric ratios.

## 5. Relationship of the Unit to the Syllabus

| Related prior learning Outcomes | Learning outcomes for this unit | Related later learning outcomes |
|---|---|---|
| Throughout primary school, students are exposed to the ideas of shape and space. As students' progress from senior infants to sixth class, the ideas develop from simple special awareness, to exploring such terms as side, angle, triangle and parallel lines.<br><br>In sixth class students learn to<br>- make informal deductions about 2-D shapes and their properties<br>- use angle and line properties to classify and describe triangles and quadrilaterals<br>- construct triangles from given sides or angles -<br>complete the construction of triangles, given two sides and the angle between them or given two angles and the line between them -<br>estimate, measure and construct angles in degrees -<br>explore the sum of the angles in a triangle | In first year, students develop their knowledge of synthetic geometry through investigation and discovery. They should come to appreciate that certain features of shapes or diagrams appear to be independent of the particular examples chosen.<br><br>Students learn about-<br>[Protractor Axiom] The properties of the degree measure of an angle -<br>Vertically opposite angles are equal -<br>If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse). -The angles in any triangle add to 180˚. -<br>Two lines are parallel if and only if, for any transversal, the corresponding angles are equal<br>- If two triangles are similar, then their sides are proportional, in order (and converse).<br>-Construction of Triangle, given ASA data. | In second and third year, these ideas are developed into formal proofs. Extensions of theorems involving transversals are explored, and further constructions undertaken.<br><br>- Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio (and converse). -<br>If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.<br><br>Constructions:<br>- Division of a line segment into 2 or 3 equal segments, without measuring it. -<br>Division of a line segment into any number of equal segments, without measuring it.<br><br>Continued study of trigonometry:<br>Trigonometric ratios using the same similar triangles as an introduction. |

## 6. Goals of the Unit

i) Students will need to recall the properties of angles in relation to a triangle.
ii) Students will effectively use this knowledge of angles to identify qualities in the given triangles.
iii) Students will effectively use mathematical equipment to measure angles.
iv) Students will need to recognize that triangles which are different in size can have equal angles.
v) Students need to be able to identify corresponding angles and their properties.

## 7. Unit Plan

| Lesson | Learning goal(s) and tasks |
|---|---|
| 1 | Transversals that cut parallel lines. Alternate, corresponding, vertically opposite and interior angles. |
| 2 | Theorem 4: Angles in a triangle add to $180^{O}$ |
| 3<br>The Research Lesson | Explore similar triangles. Realise similar triangles do not have to be identical. Prove using angle relationships from previous lessons that the triangles are equiangular. Encourage students to using their prior knowledge of angles created by parallel lines and a transversal to work backwards and actually prove the converse i.e. that the sides are parallel. |
| 4 | investigating the ratios of similar triangles<br>Construction of evenly lines divided into equal lengths and it's relationship to similar triangles |
| 5 | Theorem 13: If two triangles are similar their sides are proportional.<br>Converse of theorem 13: If the sides of 2 triangles are in proportion then two triangles are similar |
| . . . | Trigonometric Ratios and applications |

## 8. Goals of the Research Lesson:

The mathematical goal of this research lesson is that students would utilize previous knowledge of angles in a triangle and angles formed when a transversal crosses two parallel lines to investigate similar/equiangular triangles. Students will be encouraged to examine different sized triangles. They will make connections with the angles of two similar triangles and the sides of the triangles. The structure of the lesson will provide opportunities for constructivist learning and for students to express mathematical ideas. The students will have experience of using and developing mathematics and mathematical ideas. They will work with other and communicate their ideas (KS4, 6 and 7). The students will describe, illustrate, interpret and explain patterns and relationships(SL16).

**Short -term goals (specific to the lesson):**

**For students to review and apply,**

- Use of a protractor to measure angles
- Draw similar triangles with different orientations.
- Recognise when they can use the fact that the three angles of a triangle sum to 180 degrees.
- Identify parallel lines.

- Apply their knowledge of corresponding angles
- Their knowledge of slope.

**Long-term goals:**

**For students to,**

- Communicate their mathematical thinking using mathematical terminology.
- Appreciate the need for creative thinking.
- Make connections between different strands of maths.
- Devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.
- Appreciates the validity of approaching a problem in different ways.
- Observes a process and draws valid deductions and conclusions.

## 9. Flow of the Research Lesson:

| Steps, Learning Activities<br>Teacher's Questions and Expected Student Reactions | Teacher Support | Assessment |
|---|---|---|
| | | |
| **Introduction**<br><br>Before we start today's problem we are going to quickly review some maths we learned over the last two weeks.<br><br>✓ *Denotes student response(s)*<br><br>Tell me what you know about the angles in the triangle.<br>✓ *3 angles in a triangle add to $180^0$*<br><br>Tell me what you know about the angles formed when a transversal crosses over two parallel lines.<br>✓ *8 angles created*<br>✓ *Supplementary angles add to $180^o$*<br>✓ *Angles at a point add to $360^0$*<br>✓ *Alternate angles are equal*<br>✓ *Vertically opposite angles are equal*<br>✓ *Corresponding angles are equal*<br>✓ *Interior angles add to $180^o$*<br><br>Now let's look at today's problem. | Place an image of a triangle on the board. Check students recall that three angles in a triangle add to $180^0$<br><br>Place parallel lines on a board and a transversal through the lines. Check students can identify the different type of angles created i.e. alternate, corresponding, vertically opposite, interior etc. and understand the relationship between the angles, in particular corresponding | Can students describe all the relationships they have studied regarding angles in a triangle and angles at the transversal to parallel lines |
| **Posing the Task**<br><br>You have been given square paper with a triangle drawn on it. I want you to see how many similar/equiangular triangles you can | The problem will be handed out on square paper and students will be given | Do students understand the task? |

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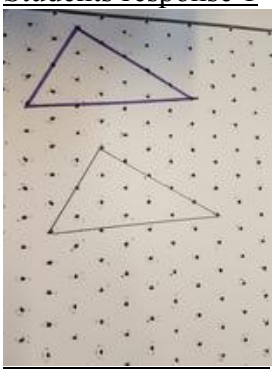
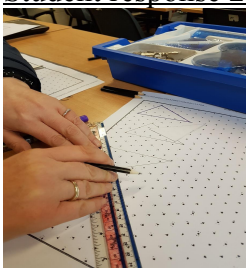
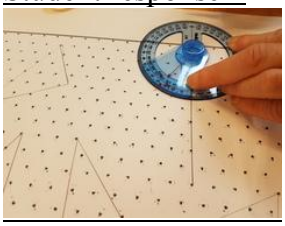
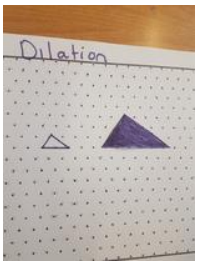
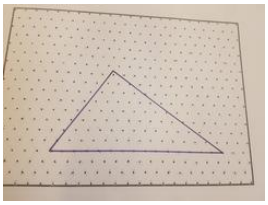
| form. Note: Triangles that are similar/equiangular, are triangles that have the same angles.<br><br>**Clarifying the problem:**<br><br>I want to just show you what I mean by equal angles.<br><br>I want you to work away individually for ten minutes. We will come together as a class and use your work to learn some new mathematics | pencils, rulers, protractors, scissors and extra paper.<br><br>Draw two identical triangles and show the equal angles. | Are students eager to work on the problem? |
| **Student Individual Work**<br><br>Students response 1<br>  <br>Identical triangle<br><br>Student response 2<br>  <br>An identical triangle flipped by 180º<br><br>Student response 3 | Using your seating plan record the approach used by each student and the order in which you will call each student up during Ceardaiocht.<br><br>If students are stuck, help them by appropriate questions or prompts. For example,<br>'Would it help to cut out a copy of the original triangle'?<br>'Would measuring the angles in the triangle help'?<br>What happens when we fit a smaller similar triangle inside a bigger one?<br>What relationship has the two base lines?<br>Can you show this relationship using angle rules? | Are students able to identify that triangles other than the mirror image are similar i.e. rotated triangles, enlarged, dilated etc. |

Student measures the angles in the triangle. May work out the third angle by 3 angles in a triangle property.

Student response 4

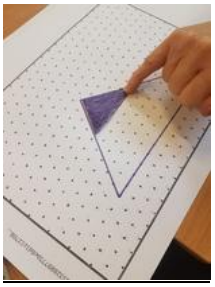




Student starts to draw enlarged or dilated versions of the triangle by ensuring the corresponding angles are equal to the original.

Student response 5



Student places the original image inside the enlarged version.

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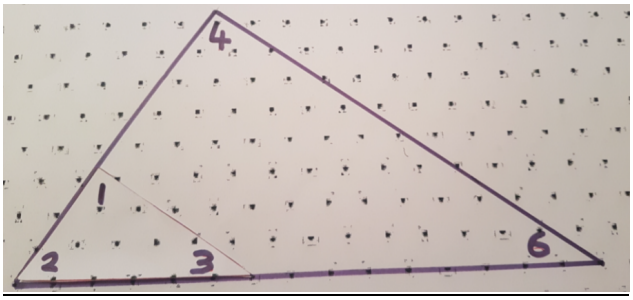
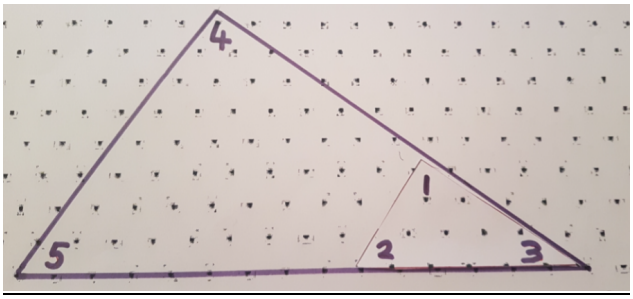
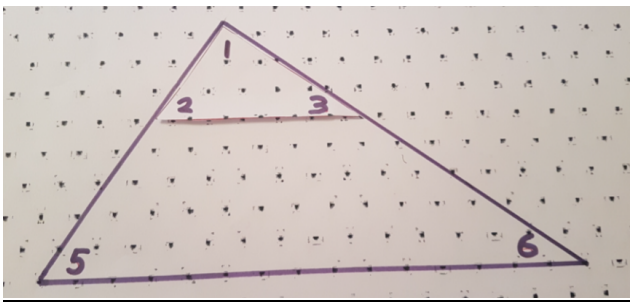
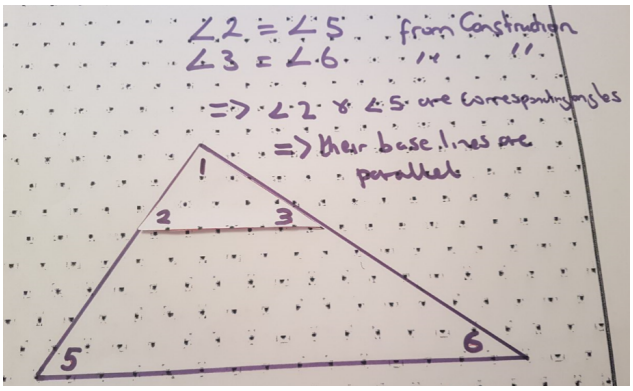
|  <br>  <br>  <br>Student moves the original around inside the enlarged triangle aligning the corresponding angles | | |
| **Ceardaíocht /Comparing and Discussing**<br><br>**How many different ways can we show that angle 2 and angle 5 are equal?**<br>✓ *By construction*<br>✓ *Using parallel line and transversal relationship*<br>**This applies to angle 3/6 and angle 1/4**<br><br>  | When the student presents work at the board make sure to attach their name to it.<br><br>Ask students to raise their hands if they followed the same procedure in getting a similar triangle to the original.<br><br>Encourage students to identify that similar triangles do not have to have sides of the same size.<br><br>Encourage students to using their prior knowledge of angles created by parallel lines and a transversal to work backwards and actually prove the converse i.e. that the sides are parallel.<br><br>Prompt students to use their | Are students' responding to each others ideas? |

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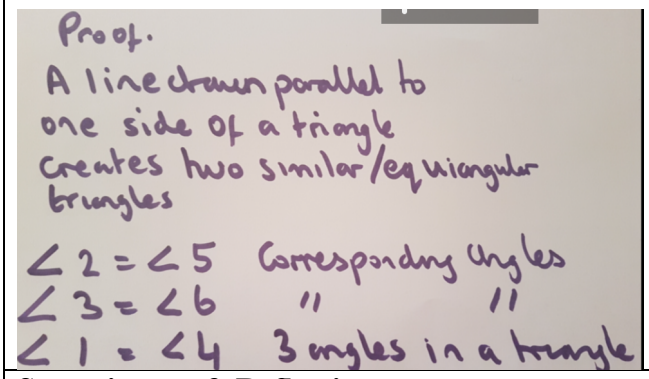
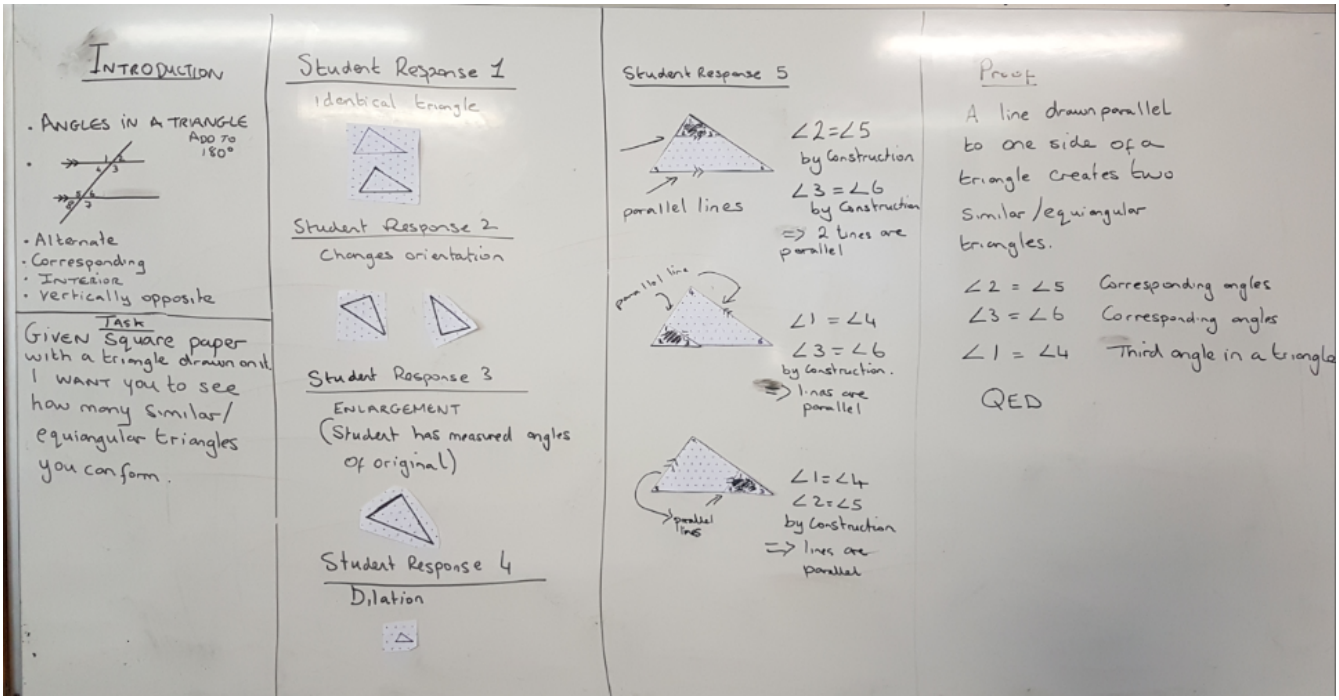
| Prove algebraically the triangles are similar/ equiangular by using angle rules for triangles and parallel lines with a transversal.<br>  <br> | angle relationship knowledge to prove a line drawn parallel to one side of a triangle creates two similar/equiangular triangles. | |
| **Summing up & Reflection**<br><br>We learned that:<br>Similar/equiangular triangles do not have to have sides of the same length and do not have to be mirror images.<br>Similar/equiangular triangles can be of a different orientation or sides can have different lengths once the corresponding angles are the same size.<br>When a smaller similar triangle is placed inside a enlarged version the base lines are parallel.<br>We can not only see they are parallel but we can prove it using angle relationships.<br>I can prove the triangles are similar/ equiangular by using angle rules for triangles and parallel lines with a transversal.<br><br>Ask students to write a reelection using 1-3 points of view<br>One to three points of view<br>Today I understood ….<br>Today I noticed ….<br>Today the questions I have …<br>Today I learned from my friend's ideas….. | | Do the students' reflections represent the teacher's view of the lesson? |

## 10. Board Plan




## 11. Evaluation

During the evaluation process the delivering teacher was congratulated for the class. The teachers noted that the lesson goals were met as evidence by the following. Students enjoyed using the whiteboards to show their existing knowledge of angles. The majority were able to draw identical equiangular triangles, while some progressed to changing the orientation, and the lengths of the sides. A few students cut out a triangle, and this led to them using their knowledge of corresponding angles to deduce that lines in the triangles were parallel. The use of coloured paper on the classroom whiteboard was very effective and students enjoyed coming to the board and explaining their findings to their peers. The main concern of the teachers was the time allowed for each task could have been managed better. The students came to the expected conclusion finding that the similar triangles placed together produced the parallel lines. An extension to the lesson would be to encourage the students to find any other relationships between the triangles moving the students eventually to trigonometric ratios. The theme of the lesson was to encourage students to become more independent in their learning. The lesson was successfully engaged the students to work individually.

## 12. Reflection

The overall process of lesson study was considered very positive by the teachers. The teachers collaborated on teaching and learning. The opportunity for discussion and sharing of ideas was valuable. The collaboration resulted in increased levels of resource sharing, and awareness of the need for more student input in the learning experience.Teachers experienced a sense of achievement when students were empowered to develop ideas, and apply previous knowledge to the task.