Lesson Research Proposal for 2nd Year Simultaneous equations

For the lesson on Thursday 18th January 2017
At Marian College, Alison Bird class
Instructor: Arlene Murphy
Lesson plan developed by: Alison Bird, Geraldine Conlon, Sean Mc Ginty and Carl Brien

1. Title of the Lesson: Simultaneously solving for two variables

2. Brief description of the lesson

Students will be given two linear equations with two unknown variables and will be required to solve for the two unknown variables using as many methods as possible in order to recognise the link between co-ordinate geometry and algebra and so that they will understand that there are many different ways to solve problems mathematically.

3. Research Theme

At Marian College, we want:

1. Students' enjoyment in learning is evident and arises from a sense of making progress and of achievement. Their engagement with learning contributes to their sense of well-being.
2. They have a sense of ownership of their work, take pride in it, and take responsibility for improving it.

As Mathematics teachers, we will actively support the achievement of these goals by paying attention to the following entry points in my every day classes:

1. Teachers create an inclusive, orderly, student-centred learning environment based on mutual respect, affirmation and trust, in which students regulate and monitor their own behaviour. Lesson design is flexible to allow for emerging learning opportunities.
2. Teachers meaningfully differentiate content and activities in order to ensure that all students are challenged by the learning activities and experience success as learners.
3. Teachers recognise and affirm continuing professional development (CPD) and collaboration as intrinsic to their work.

4. Background & Rationale

The lesson is aimed at 2nd Year mixed ability students. We chose this topic because we as teachers feel there is a negative attitude from our students towards the topic of algebra. By second year it has become apparent that there are clear differences in the student's' abilities when faced with a mathematical problem involving symbols that represent variable quantities.

Why we chose this topic:

We chose the topic of simultaneous equations because we feel that the students will gain valuable problem solving skills from this area of mathematics, and that it will build on the skills they have already established from the introduction to algebra in 1st year (solving linear equations).

Our research findings:

During our most recent Maths Department meeting we had a thorough discussion regarding students' lack of understanding in basic algebra concepts. It was decided that an alternative approach is essential to help students' development in this area. The traditional methods are not engaging a large number of our students for a number of reasons. It was felt that a more student-centred learning approach might help students to engage and help them understand the methods of solving simultaneous equations. It is hoped that the lesson proposal will empower the students to comprehend the relationship between the two equations and to use their prior knowledge to understand there are a multiple ways to solve simultaneous equations.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes

In 6th class students learn to: · understand the concept of a variable, and substitute values for variables in simple formulae, expressions, and equations · translate verbal problems into algebraic expressions · acquire an understanding of properties and rules concerning algebraic expressions · solve simple linear equations · use acquired concepts, skills and processes in problem-solving In 6th Class students, · explore the concept of a variable in the context of simple patterns, tables and simple formulae and substitute values for variables · The child should be enabled to, translate word problems with a variable into number sentences In 1st Year: · Students will learn to understand the concepts of variables, constants, coefficients and algebraic expressions. · Students will also understand how to evaluate expressions, add, subtract and multiply terms. · Students should be able to multiply terms with a bracket and understand how to multiply two expressions. · Students should have a thorough knowledge of how to solve linear equations.	In 2nd year students: · explore the properties of points, lines and line segments including the equation of a line · Find the slopes of parallel and perpendicular lines · Find the point of intersection of two lines · Engage with the idea of mathematical proof · Consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value · Find the underlying formula written in words from which the data are derived (linear relations) − find the underlying formula algebraically from which the data are derived (linear, quadratic relations) · Show that relations have features that can be represented in a variety of ways − distinguish those features that are especially useful to identify and point out how those features appear in different representations: in tables, graphs, physical models, and formulas expressed in words, and algebraically − use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others · decide if two linear relations have a common value − investigate relations of the form y=mx and y=mx +c	In 5th and 6th year Higher level, students are expected to: · select and use suitable strategies (graphic, algebraic, numeric, mental) for finding solutions to: o Simultaneous linear equations with two unknowns and interpret the results o One linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of *x* or the coefficient of *y* is +/-1 in the linear equation) and interpret the results · select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to: o Simultaneous equations with three unknowns o One linear equation and one equation of order with two unknowns and interpret the results

6. Goals of the Unit

a) Students will understand that some types of problems do not have a single solution, rather they may have a multiple set of solutions.
b) Students will apply algebra to real life situations using problem solving methods.
c) Students will learn to enjoy problem solving and to engage with individual and group work.
d) Students will understand the characteristics of the line.
e) Students will recognise that situations involving linear equations may be expressed and/or solved mentally, numerically, graphically and algebraically.
f) Students will gain confidence with algebra and feel a sense of empowerment from autonomous learning.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Students will investigate previous knowledge of the concepts of algebraic terms. Students will engage in problem solving activities based on prior learning.
2 Research Lesson	Introduce two linear equations with two unknown variables and ask the students to solve for the two unknowns in as many ways as possible.
3	Graphing linear equations on Cartesian planes using the equation of the line and substitution. Crossing the y-axis and crossing the x-axis. Students using linear equation to draw a line on a plane and rearranging the linear equation in the form $y = mx + c$ and drawing the line and showing graphically that the lines are the same. Solving linear equations of the form $y = mx + c$. Encouraging students to draw a table to graph linear equations. Students will be able recognise linear equations from the given graphs.
4	Solving simultaneous equations algebraically using the substitution method
5	Solving simultaneous equations algebraically using the elimination method
6.	Using word based problems to solve algebraic linear equations

8. Goals of the Research Lesson:

a) Mathematical Goals
Students will:

- Understand the concept of simultaneous equations and their solutions
- Understand the characteristics of linear equations and use them to solve simultaneous equations
- Understand that there are different methods of solving simultaneous equations mentally, numerically, graphically and algebraically

b) Key Skills and Statements of Learning
In the planning and design of this lesson the Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will implement and promote the JC Key Skills in the following ways:

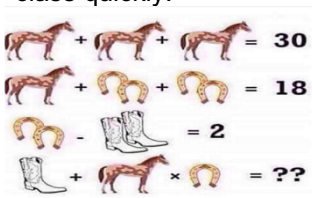
1. Being Numerate: Expressing ideas mathematically and developing a positive disposition toward investigating, reasoning and problem-solving.
2. Working with others: Co-operating and learning with others
3. Being Creative: students should be able to explore options and alternatives and to learn creatively.
4. Staying well: being positive about my own learning
5. Managing myself: Being able to reflect on my own learning
6. Being literate: Expressing ideas clearly and accurately
7. Managing information and thinking: thinking creatively and critically
8. Communicating: Using number and discussing and debating and using number

This lesson is also designed to meet the following Junior Cycle Statements of Learning

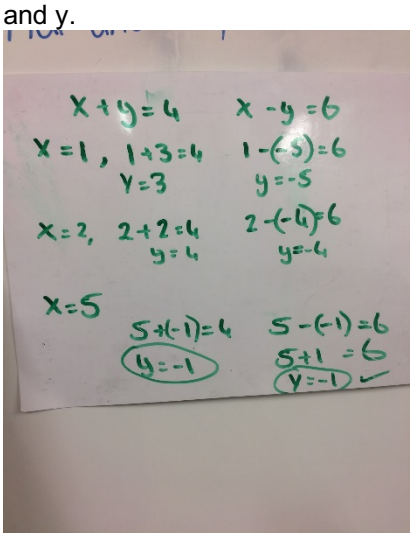
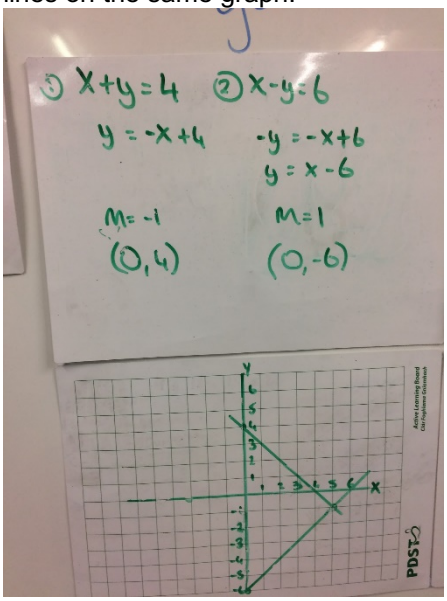
1. Communicates effectively using a variety of means in a range of contexts in English.
15. Recognises the potential uses of Mathematical knowledge, skills and understanding in all areas of learning.
17. Devises and evaluates strategies for investigating and solving problems using mathematical reasoning and skills
21. Applies practical skills as she/he develop models and products using a variety of materials and technologies.

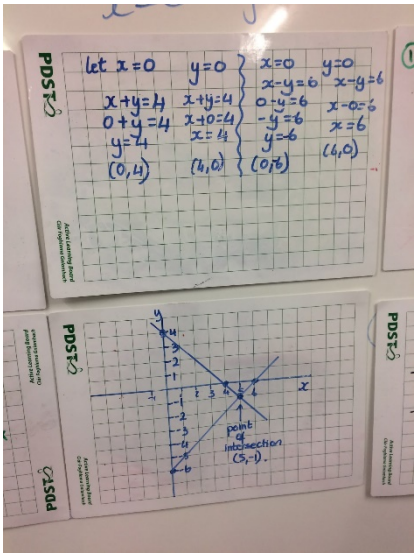
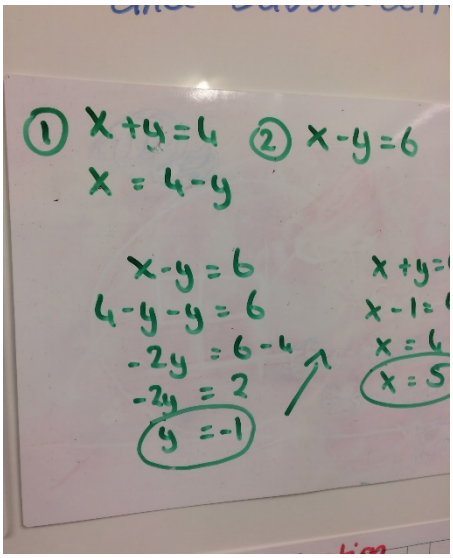
9. Flow of the Research Lesson:

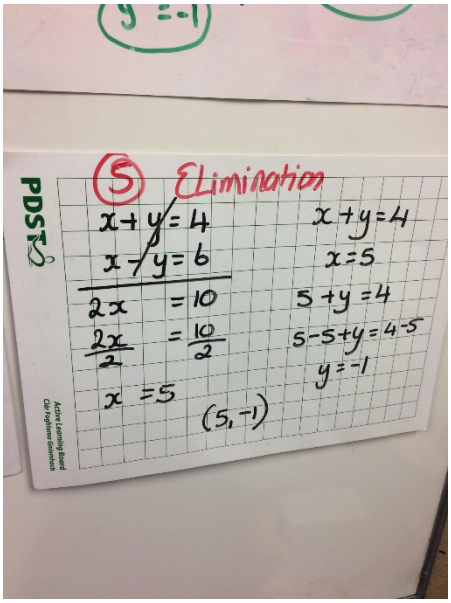
Steps, Learning Activities Teacher's Questions and Expected Student Reactions	**Teacher Support**	**Assessment**

Introduction Students enter the classroom and take their seats. Each student will have an A3 show me board, whiteboard marker and an eraser. Task provided on PowerPoint at beginning of class for students to mentally work out the values of pictures.	Ask the students to find out the answer to the final question. Place emphasis on values of each of the 3 unknowns (horse, horseshoe, and boot). Replace names with variable titles, x,y etc and introduce the 1st equation.	This question will be presented on the board as the students enter the class. This will help the students to settle into the class quickly.  This introduces the fact that there can be more than one variable in an equation. Students should be able to solve this in their heads to conclude that 1 horse = 10 1 horseshoe = 2 1 boot = 1 Final answer = 21

Posing the Task The teacher switches the PowerPoint slide **Task:** How many ways can you find the values of *x* and *y* if: $x + y = 4$ *and* $x - y = 6$ The teacher will ask the students to recognise that these are linear equations. The teacher then clarifies the question and challenges students to find the values of x and y in as many ways as possible. In order to do their workings out students will be provided with A3 whiteboards, markers and erasers.	Question will be on the next PowerPoint slide	Students will orally agree that they know what the task is asking them to do. Do the students understand the task? Do the students understand that they need to find a value for x and a value for y that satisfies both equations? Do the students understand that they will be using multiple ways of finding a value for x and y?

Student Individual Work Anticipated student responses: Response 1: Trial and Improvement It is anticipated that because of the introduction task, most students will begin by using this method to find values for x and y.  Response 2: Graphical method: using the linear equation y = mx + c to draw the two lines on the same graph.  Response 3: Graphical method: where the line crosses the x and y axis. Students should let x = 0 to find where the graphs cross the y axis and let y = 0 to find where the graphs cross the x axis.	Teacher will walk around the class observing the individual work, and will encourage and prompt if necessary. Students who get stuck, or students who finish early will be encouraged to try other methods of solving the equations. If the students are solving for the first equation, do these values solve the second equation too? Encourage the students to use both sides of the show me boards, as they may start to graph the equations. Is there another way to write the equation of a line? If an x in by itself, what number is actually here? (1) Can they solve for the slope of the line using rise over run as well?	Are the students able to tackle the problem? Can all students find values for x and y that will satisfy both equations using trial and improvement? Will the students get the same types of graph despite using two different methods?

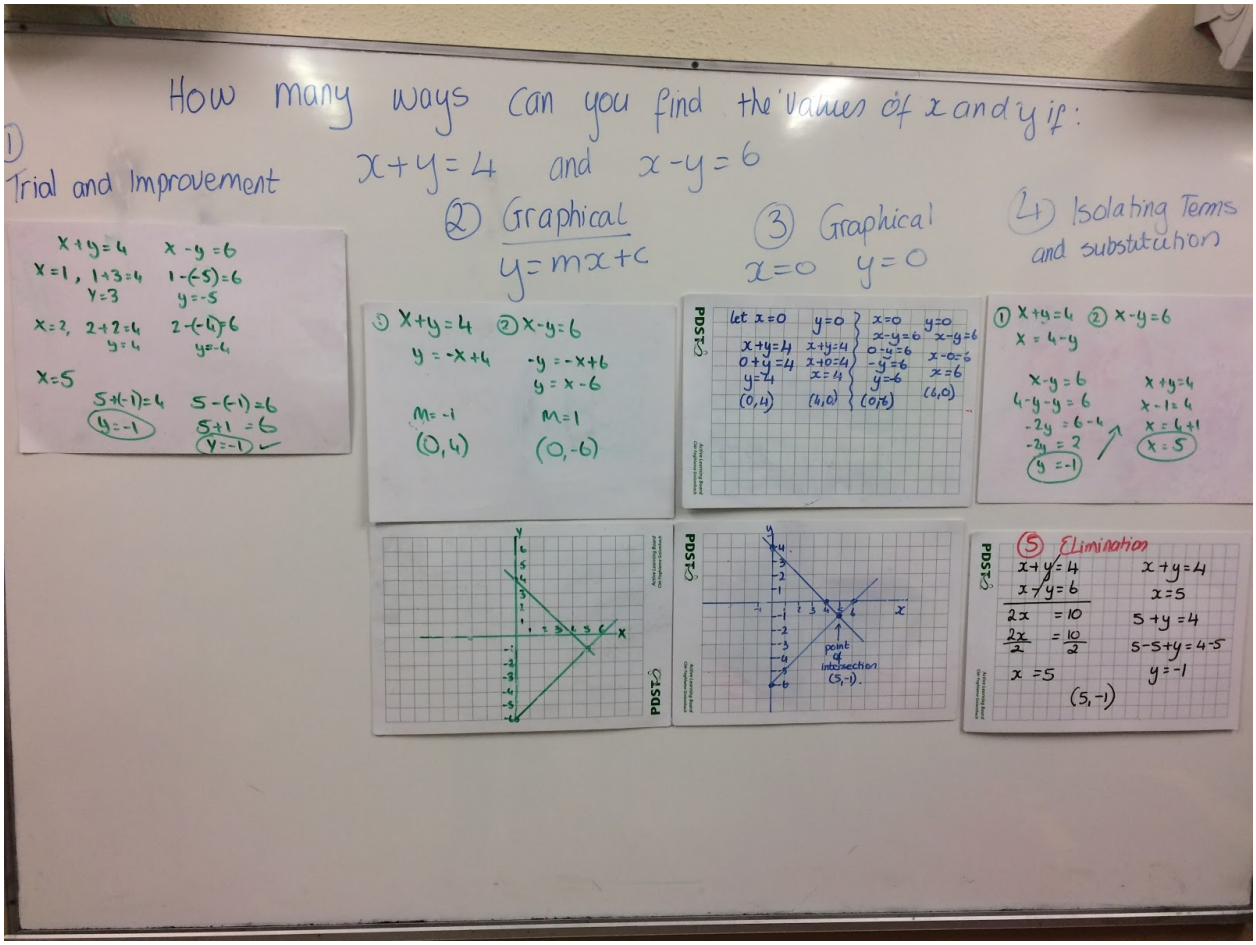
 Response 4: Isolating the x or y value and substituting the value to solve for one variable.  Response 5: Eliminating one variable to find a value for the other.	Ask the students about the x-y plane, what happens to an x value as you go along the y axis? And vice versa? Teacher will encourage students to move the position of the equations to see if that makes a difference.	The students should be encouraged to manipulate their formulae and substitute. Students should move the equations into a format where one equation is above the other. They will notice that there is a +y and a -y and they will eliminate these variables to solve for x. Students will be asked to explain this method to each other.

		Students will be asked if they understand that all of these methods can be used to get the same answer.
Ceardaíocht /Comparing and Discussing Response 1: Trial and Improvement Response 2: Graphical y = mx + c Response 3: Graphical x = 0, y = 0	Response 1: Please raise your hand if you used this method. Response 2: Please raise your hand if you used this method. Were you surprised that you got the same answer even though you used a different method? Response 3: Please raise your hand if you used this method.	Are students defending their ideas? Are they in agreement with using this method? Can the students explain their use of this method? Can the students explain their use of this method? Do they understand that the graph is the same as the graph drawn in response 2?

	Would you like to use this method? Which graphical method do you prefer?	Do the students understand why this method works?
Response 4: Isolating terms and substitution	Response 4: Please raise your hand if you used this method. Does this method seem easier than the other methods using graphs?	Can the students explain their use of this method? Are they responding to each other's ideas?
Response 5: Elimination method	Response 5: Is there a more suitable mathematical method that can solve this problem and show the same values for x and y?	
Extending the students learning Pose a new question to students using variables with coefficients. $3x + 2y = 11$ $2x - 2y = 14$	Question started in class and should be finished for homework. Ask the students to use as many of the methods that they have learned to solve the two equations.	
Summing up & Reflection We learned that: • We can use trial and improvement to solve two variables in two linear equations • We can use two different methods of graphing linear equations to find their point of intersection • The point of intersection represents values of x and y where the two lines cross • We can use algebra to isolate a term and substitute it into the second equation to find a missing value, then use this value to find the other variable • We can use the elimination to solve for two variable in two linear equation		

Ask the students to write a reflection		Do the students' reflections represent the teacher's view of the lesson?

10. Board Plan



11. Evaluation

From the beginning of the lesson the students focussed on the horse question on the board. They were given a few minutes to solve the puzzle as the teacher took the attendance and made sure everyone was settled into the lesson. This successfully promoted student-to-student discussion as the students were engaged and interested in discussing their findings to the puzzle. The observers felt that it was an excellent start to the lesson. The lesson continued in a positive atmosphere which was cohesive to learning; the tone was set and the theme introduced in a fun way.

During individual work, most students solved the problem using one method, but not many students were able to solve the question in two or more ways. This type of lesson may be more suitable to team-teaching as with this large class the teacher did not get to see every student's piece of work and so was unable to support all students as much as she would have liked. During the introduction of the task the teacher reminded the students of graphs they had previously studied which caused many students to draw a graph without understanding the reason. Students were slow to link the two equations.

The student discussion was very successful. The students who were invited up to the board were able to explain clearly what they had done to solve the two equations. They owned their work and other students reacted well to being taught by their peers. The other students could then understand the links between methods. One observation was that the work done by the students was too small and could not be seen clearly on the board, perhaps teachers should give larger working space to the students. Another point by an observer was that the students may have been more successful in the task if they had been put into groups.

This was a very enjoyable lesson and the goals of the lesson were met. Students were satisfied that there was more than one way to solve this type of problem and were sent home with a similar question to try for homework. In the next lesson the teacher said that the students were a lot more confident in the theme and were more engaged in the topic. The students' enjoyment of the lesson was made evident at the end of the class when a number of students took pictures of the board work.

12. Reflection

After the research lesson it was evident that the students were engaged in the lesson – they took ownership of their work; showed enthusiasm; full participation; learned not to give up and that there was more than one method of solving problems.

The most thing that was observed was the enjoyment and participation of the students. Although some gave up before the 10 minutes were up, most did not. There were lots of questions during the discussion and photographs taken of the board work afterwards.

The team felt that this was a positive learning experience for the teachers and the class involved. They noted that you don't have to use 'teacher talk' to get a lesson taught, that it's good to let students make mistakes and to think about their teaching methods. This lesson may be a good lesson for team-teaching. Teachers would love to create more lessons like this but the most obvious issue is lack of time.

The students' reflections on the class were very positive, they felt it was an enjoyable learning experience and liked the fact that they were left to their own devices and encouraged to think outside the box. This is a lesson that the observers are looking forward to using in their own classrooms and would recommend it to teach simultaneous equations.