Lesson Research Proposal for 3rd Year Higher

For the lesson on 2/02/18
At Chanel college, Loran Deely’s class
Instructor: Lorcan Deely

Lesson plan developed by: Lorcan Deely, Sam Sheridan, Catherine Walsh, Bernard Foley.

- **Title of the Lesson:** Snowflakes

- **Brief description of the lesson**
Expressing linear patterns

- **Research Theme**
  - Students engage purposefully in meaningful learning activities
  - They will be able to report on, present and explain the process and outcome of learning activities to a competent level
  - Students grow as learners through respectful interactions and experiences that are challenging and supportive
  - They ask questions and they can suggest possible solutions competently and are willing to risk incorrect responses and accept that mistakes are part of the learning process

  Supporting our goals:
  - The teacher selects and uses planning, preparation and assessment practices that progress students’ learning
  - The teacher responds to individual learning needs and differentiates teaching and learning activities as necessary

- **Background & Rationale**
  Justify: This lesson is aimed at third year students. The teaching of patterns is very important as if it is successfully introduced, it can positively influence the understanding of algebra and functions to a higher level.
  Misconceptions: The most common misconceptions are not including the constant or mixing up the coefficient of the variable and the constant. Students also quite commonly confuse the significance of term number and term value.
  Group Discussion: It’s very important the students can find the general rule for an arithmetic sequence by analysis rather than by using a formula as it requires an understanding of the topic and leads to more meaningful learning. It’s key that they can link the common difference and the coefficient of the variable.

  Our Research Findings: From our analysis of the Primary Curriculum, it was discovered that students should be able to identify relationships and record symbolic rules for number patterns. The NCCA Junior Cycle states that students should be able to write arithmetic expressions for particular terms in a sequence and also write a generalized expression for linear (and quadratic) patterns in words and algebraic expressions and fluently translate between each representation.
  We felt it was very important that over the course of the topic, students would be able to articulate the connections between the equation of a line in coordinate geometry, graphing a linear function and a linear pattern.
• **Relationship of the Unit to the Syllabus**

<table>
<thead>
<tr>
<th>Related prior learning Outcomes</th>
<th>Learning outcomes for this unit</th>
<th>Related later learning outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>In third class students learn to:</td>
<td></td>
<td></td>
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<tr>
<td>• explore, recognize and record patterns in number, 0–999</td>
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• **Goals of the Unit**

(A) Students will need to categories linear, quadratic and exponential patterns using their defining characteristics as they appear in the different representations.

(B) Students shall need to know how to write a generalized expression for linear and quadratic patterns in words and algebraic expressions and fluently translate between each representation.

(C) Students need to know how to represent these patterns and relationships in tables and graphs.

(D) Students will need to understand the link between linear patterns and linear functions.

(E) Students need to know how to identify the variable change within the pattern and topic and to generate and interpret expressions in which letters stand for numbers.

(F) Students will make the connection between algebra, patterns and functions.

• **Unit Plan**

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Learning goal(s) and tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 The Research Lesson</td>
<td>• Take a linear pattern and ask the students how the pattern was formed. Is there any way of expressing this algebraically instead of words which will work for all terms of the pattern.</td>
</tr>
<tr>
<td>2</td>
<td>A follow up to further linear problems</td>
</tr>
</tbody>
</table>
| 3 | • Extend the learning on Linear patterns and move to a quadratic pattern  
• Introduce 1st and 2nd difference in a quadratic pattern. |
| 4 | • Write a generalized expression for quadratic patterns in words and algebraic expressions.  
• Fluently translate between each representation |
| 5 | • Represent these patterns and relationships in tables and graphs.  
• Identify the connection between patterns and functions. |
| 6 | A follow up problem to be decided. |
• **Goals of the Research Lesson:**
  
a) **Mathematical Goals**
  
  - Students will: Write a generalized expression for linear patterns in words and algebraic expressions.

b) **Key Skills and Statements of Learning**
  
  1. Communicating: Students will share their ideas and express their mathematical thinking.
  2. Being numerate: Order of operations, number systems and algebra will be practiced.
  3. Being literate: expressing ideas clearly and accurately and developing mathematical language.
  4. Being creative: Imagine and explore options and investigate alternative solutions.
  5. Managing myself: students will make considerate decisions and will have the opportunity to reflect on their own learning.
  6. Working with others: Students will collaborate and learn from their peers.
  7. Managing information and thinking: Students will be encouraged to think creatively and critically.
  8. Staying well: Students confidence and positive attitude to learning will be promoted.

The lesson is also designed to meet the following JC statements of learning

- 1. Communicates effectively using a variety of means in a range of contexts in L1
- 15. Recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning
- 16. Describes, illustrates, interprets, predicts and explains patterns and relationships
- 17. Devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills
- 23. Brings an idea from conception to realization

• **Flow of the Research Lesson:**

<table>
<thead>
<tr>
<th>Steps, Learning Activities</th>
<th>Teacher’s Questions and Expected Student Reactions</th>
<th>Teacher Support</th>
<th>Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>Today we’re going to use our mathematical knowledge to solve a problem of a changing number of dots. We’re going to try solving it individually. Then finally as a class and use what we discover to express it algebraically.</td>
<td></td>
<td>We want students to get a sense of what is expected of them and a clear idea of the structure</td>
</tr>
<tr>
<td>Posing the Task</td>
<td>Show students a sequenced diagram of increasing numbers of dots of which the one below is the 4th and tell them we want a way to find any pattern in the sequence, even ones further away.</td>
<td>The pattern of dots will be on the board and also on a handout for the students</td>
<td></td>
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</tbody>
</table>
We ask the students to describe the changing pattern and decide how the number of dots can be broken up into sections that help visualize the changing pattern.

<table>
<thead>
<tr>
<th>We ask the students to describe the changing pattern and decide how the number of dots can be broken up into sections that help visualize the changing pattern.</th>
<th>Do the students understand what is expected of them and what they need to find? Are they engaged in the task?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Six arms of four for the fourth pattern with one central dot</td>
<td>Do the students realize that the dots can be divided into sections in different ways?</td>
</tr>
<tr>
<td>Four concentric circles of six dots with one central dot</td>
<td>The students need to explain in their own words how the pattern is made up and also how it evolves</td>
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Maths Development Team: Lesson Study 2017-2018
Three overlapping lines of seven dots with the middle one counted three times

The students identify how many dots are in the 10th

<table>
<thead>
<tr>
<th>Cearaíocht /Comparing and Discussing</th>
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<tbody>
<tr>
<td><strong>Response 1:</strong> What indicates or why is there no pattern before our first one.</td>
<td></td>
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<tr>
<td><strong>Response 2:</strong> Why isn’t the core increasing as well as the outside dots.</td>
<td></td>
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<tr>
<td><strong>Response 3:</strong> Is it six increasing legs with one in the centre</td>
<td></td>
</tr>
<tr>
<td><strong>Response 4:</strong> Is it increasing rings of dots around the central dot amongst others.</td>
<td></td>
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<tr>
<td><strong>Possible misconceptions.</strong> Some students will think that the formula created for this pattern will apply to all.</td>
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<tr>
<td>In creating the formula, they ignore the central dot and fail to include it in the formula.</td>
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<tr>
<td>In response to students work number 3, the students will double count the central dot in the first pattern leading to 9 dots being counted instead of 7, thus carrying this error onto future patterns.</td>
<td></td>
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<tr>
<td><strong>Response 1:</strong> Did anybody have a pattern before the initial one?</td>
<td></td>
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<tr>
<td><strong>Response 2, 3, 4:</strong> Hands up if you have this as one of your methods? Why was this method used?</td>
<td></td>
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</table>

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<tr>
<th><strong>Summing up &amp; Reflection</strong></th>
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<tr>
<td>Throughout this lesson students have discovered how to express patterns and their algebraic equations.</td>
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</table>

Do they understand that a pattern doesn’t have to begin at one dot?

Demonstrate the connection between response 3 and response 4.

Can students relate the formula created to the patterns?
**Board Plan**

**Task 1**

**Response 1**
- How many dots are there? 5
  - “I see 5 like on dice”
  - “I counted 5 dots”

**Response 2**
- How many dots are there? $4 + 1 = 5$
  - “A square of four on the outside and the one on the inside is 5”

**Response 3**
- How many dots are there? $2(2) + 1 = 5$
  - “Two sets of 2 plus the one in the middle is 5”

**Response 4**
- How many dots are there? $2(3) - 1 = 5$
  - “I see a cross. There are 3 dots in each line but I take away 1 because it is double counting otherwise”

**Task 3**

**Response 1**
- How many dots are there? 25
  - “I counted 25 dots”

**Response 2**
- How many dots are there? $6(4) + 1 = 25$
  - “6 sets of 4 and 1 in the middle”

**Response 3**
- How many dots are there? $4(6) + 1 = 25$
  - “4 sets of 6 and 1 in the middle”

**Response 4**
- How many dots are there? $3(9) - 2 = 25$
  - “3 groups of 4 take away 2 because otherwise I’m triple counting”
**Task 4**

If each "arm" had 10 dots how many dots would there be?

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**Response 1**

"There are 6 groups of 10 plus the one in the middle"

\[ 6(10) + 1 = 61 \]

**Response 2**

"There would be 10 circles with 6 dots and the one at the centre"

\[ 10(6) + 1 = 61 \]

**Response 3**

"There would be 3 lines of 21 dots but we take 2 away so that the one in the middle is only counted once"

\[ 3(21) - 2 = 61 \]
Evaluation

- Have a written record of responses, approaches and solutions
- Identify the approach used by each individual student
- Collect student work at end of lesson for assessment
- Identify students who do not understand the problem
- Did students come up with a variety of ways to express a pattern?
• **Reflection**
  The research team had hoped to observe the link between Algebra and Patterns. We were also looking for students to come up with a variety of ways of viewing the patterns.

• Structuring of lesson was seen as a positive. The sequential ordering of questions moving from simple examples to more complex examples worked. The fact that they had handouts and were active throughout the lesson was also seen as a positive.

• Initial reaction from students was to jump to a solution without looking at alternatives until prompted but once prompted they were able to come up with different and unique answers.

• Extension of lesson to quadratic patterns and also graphing patterns was seen as potential future area of study.

Students work during the lesson.
Describe the changing pattern and show that the dots can be broken up into sections:

If each arm had 10 dots, how many dots would there be?

\[ b_1 = 6 \times 10 + 1 \]
\[ 10 \times 6 + 1 \]

Use numbers to describe the change in patterns:

If each arm had 10 dots, how many dots would there be?

\[ b_n + 1 \]
\[ n \times 6 + 1 \]
\[ 3n(2) + 1 \]
\[ 2(3n+3) \]
Individual student work

How many dots are there?

\[ \begin{align*}
\odot \odot \odot \odot \odot \odot \\
\times 6 : 2 \times 6 + 1 \\
= 25 \text{ dots}
\end{align*} \]

How many dots are there?

25 Dots
6 Intersections
If each "arm" has $n$ dots how many dots would there be? 

$n \cdot 6n + 1$ 

$3n + \frac{1}{2}$ 

$2(3n + \frac{1}{2})$ 

$\frac{3n + 1}{2}$
How many dots are in each "arm" if the pattern contains 259 dots?

\[ 6n + 1 = 259 \]
\[ -1 \]
\[ 6n = 258 \]
\[ \frac{6n}{6} = \frac{258}{6} \]
\[ n = 43 \]

Total dots is 6(43) + 1 = 259

= 259