

# Lesson Research Proposal for 2<sup>nd</sup> Years – Algebraic Word Problems

For the lesson on 1/2/18  
At Ballincollig Community School  
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## 1. Title of the Lesson: Takeaway Translation

## 2. Brief description of the lesson

In this lesson students will try to solve a word problem using a variety of methods and will discover that these types of problems can be solved using Algebra.

## 3. Research Theme

At BCS we want students to:

- a) Form an algebraic equation from a word problem and solve the equation.
- b) Demonstrate very high levels of interest and participation in learning.
- c) Understand and can explain the purpose of the learning tasks they are engaged in, and can extend and develop the activity meaningfully.

## 4. Background & Rationale

a) Why we chose this topic:

Algebra is a key subject area in mathematics and while there are stand-alone questions on the topic, it crops up in all aspects of the curriculum. Therefore we have decided to make equations the focus of our research topic. The skill of solving equations is a fundamental skill needed by students.

b) Our Research findings

We have found that students often struggle with this especially when equations are taken out of their usual context. Students also have difficulty converting word problems to mathematical statements and thus we are going to work on this by forming equations from word problems. According to the Chief Inspectors report students 'mastered routine procedures and skills, struggled to apply these in unfamiliar contexts and situations' (Chief Inspectors Report 2010-2012). By aiming this lesson at second year students we hope to rectify these problems early on and prevent them from continuing in later years. From our professional discussions we feel students shy away from math's with any mention of words problems and following the New Junior Cycle Key Skills we want students to develop a positive disposition towards investigating, reasoning and problem solving in all areas of the curriculum. Also tying in with the skills document we want student to be confident and positive about their learning and be creative in their work exploring all options and alternatives.

## 5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<p>In Primary School:</p> <ul style="list-style-type: none"> <li>• explore and discuss simple properties and rules about brackets and priority of operation</li> <li>• identify relationships and record verbal and simple symbolic rules for number patterns</li> <li>• explore the concept of a variable in the context of simple patterns, tables and simple formulae and substitute values for variables</li> <li>• translate word problems with a variable into number sentences</li> <li>• solve one-step number sentences and equations</li> </ul> <p>In 1<sup>st</sup> and 2<sup>nd</sup> Year:</p> <ul style="list-style-type: none"> <li>• appreciate the order of operations, including the use of brackets</li> <li>• problems set in context</li> <li>• investigate models to help think about the operations of addition,</li> </ul>	<ul style="list-style-type: none"> <li>• find the underlying formula written in words from which the data are derived (linear relations)</li> <li>• Using letters to represent quantities that are variable.</li> <li>• Arithmetic operations on expressions</li> <li>• applications to real life contexts.</li> <li>• Transformational activities: collecting like terms, simplifying expressions, substituting, expanding and factoring.</li> <li>• explore patterns and formulate conjectures</li> <li>• explain findings</li> <li>• justify conclusions</li> <li>• communicate mathematics verbally and in written form</li> <li>• apply their knowledge and skills to solve problems in familiar and unfamiliar contexts</li> <li>• analyse information presented verbally and translate it into mathematical form</li> </ul>	<ul style="list-style-type: none"> <li>• perform the arithmetic operations of addition, subtraction, multiplication and division on polynomials and rational algebraic expressions paying attention to the use of brackets and surds</li> <li>• select and use suitable strategies (graphic, numeric, mental) for finding solutions to equations of the form: <math>f(x) = g(x)</math>, with <math>f(x) = ax+b</math>, <math>g(x) = cx+d</math>, where <math>a, b, c, d \in \mathbb{Q}</math> and interpret the results</li> <li>• Solving linear equations set in context.</li> <li>• select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: <math>f(x) = g(x)</math>, with <math>f(x) = ax+b</math>, <math>g(x) = cx+d</math> where <math>a, b, c, d \in \mathbb{Q}</math></li> </ul> <p><math>f(x) = g(x)</math> with  <math>f(x) = \pm</math>  <math>g(x) =</math> where <math>a, b, c, e, f, p, q, r \in \mathbb{Z}</math></p> <p><math>f(x) = k</math> with  <math>f(x) = ax^2 + bx + c</math> (and not necessarily factorisable) where <math>a, b, c \in \mathbb{Q}</math> and interpret the results</p>

<p>subtraction, multiplication and division of rational numbers</p> <ul style="list-style-type: none"> <li>• solve equations of the form <math>ax+b=c</math></li> <li>• solve quadratic equations of the form <math>x^2 + bx + c = 0</math> where <math>b, c \in \mathbb{Z}</math> and <math>x^2 + bx + c</math> is factorisable <math>ax^2 + bx + c = 0</math> where <math>a, b, c \in \mathbb{Q}</math> <math>x \in \mathbb{R}</math></li> <li>• solve first degree equations in one or two variables, with coefficients elements of <math>\mathbb{Z}</math> and solutions also elements of <math>\mathbb{Z}</math></li> <li>• solve first degree equations in one or two variables with coefficients elements of <math>\mathbb{Q}</math> and solutions also in <math>\mathbb{Q}</math></li> </ul>	<ul style="list-style-type: none"> <li>• devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.</li> </ul>	<ul style="list-style-type: none"> <li>• select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to simultaneous linear equations with two unknowns and interpret the results one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of <math>x</math> or the coefficient of <math>y</math> is <math>\pm 1</math> in the linear equation) and interpret the results</li> <li>• form quadratic equations given whole number roots</li> <li>• select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form:  <math>f(x) = g(x)</math> with  <math>f(x) = \pm</math>  <math>g(x) = k</math> where <math>a, b, c, d, e, f, q, r \in \mathbb{Z}</math></li> <li>• use the Factor Theorem for polynomials</li> <li>• select and use suitable strategies (graphic, numeric, algebraic and mental) for finding solutions to cubic equations with at least one integer root</li> </ul> <p>simultaneous linear equations with three</p>
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		<p>unknowns</p> <p>one linear equation and one equation of order 2 with two unknowns</p> <p>and interpret the results</p>
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## 6. Goals of the Unit

- Students will understand that maths is a language and word problems can be translated into that language.
- Students may apply prior knowledge of translating word problems with a variable into number sentences.
- Students will understand that not all problems result in the same types of equation.
- Students will understand that some types of problems do not have a single solution, rather they can have single, multiple or infinite solutions.
- This will allow students to consolidate their prior knowledge of manipulating and solving equations.
- Students understand algebraic equations can be used to solve real life situations

## 7. Unit Plan

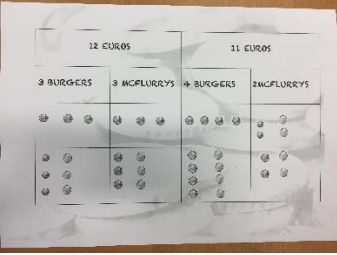
Lesson	Learning goal(s) and tasks
1	Linear and simultaneous equations
2	Quadratic equations
3	Mathematical language
4 The Research Lesson	Word Problem: Burgers and McFlurry
5	Brief overview of other lesson in unit
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⋮	⋮

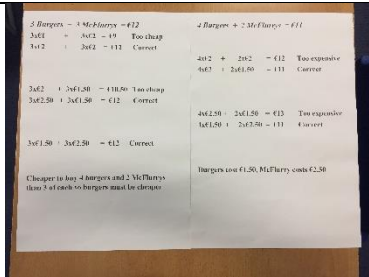
## 8. Goals of the Research Lesson:

- Be aware that there are always several ways to approach any problem
- Appreciate that maths is very important in our everyday life
- Develop their ability to think creatively about various methods of problem solving
- Using key mathematical language to improve their ability to problem solve and explain their method clearly.

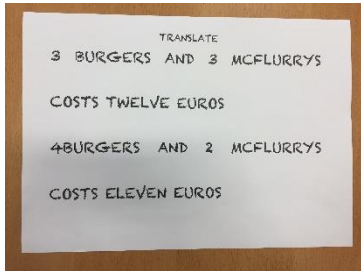
- Make a connection between co-ordinate geometry, line graphs, use of tables, trial and improvement and solving simultaneous equations algebraically
- Be encouraged and enthusiastic to teach their peers and others their method
- Feel a sense of satisfaction from discovery learning and have a will to become better independent learners

## 9. Flow of the Research Lesson:

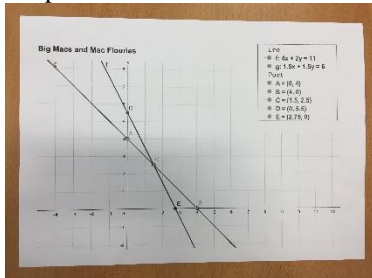
Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
<p><b>Introduction</b> Today we are going to use our mathematical knowledge to solve a word problem. We will try and solve the problem in a variety of ways firstly on our own then we will come together and examine all the different solutions you have come up with.</p>		<p>Are the students engaged? Have they understood that there is individual and group work?</p>
<p><b>Posing the Task</b> One Friday you and some of your friends go to McDonalds and order 3 burgers and 3 McFlurrys and it costs €12. The next Friday the group of you go again but this time you order 4 burgers and 2 McFlurrys and it cost €11. How much do the burgers cost and how much do the McFlurrys cost?</p> <p><b>Clarifying the problem</b> Read the problem aloud to the class.</p> <p>Bring to their attention that there are a selection of resources they may wish to use at the top of the class.</p> <p>Work individually for 10 minutes coming with as many different methods as possible to solve this problem. There are multiple equally correct solutions from different area of maths.</p>	<p>Pose the problem on the board both verbally and visually</p>	<p>Do they understand the task?</p> <p>Are students eager to solve the problem?</p>
<p><b>Student Individual Work</b> (insert photos of each solution)</p> <p>Coins</p>  <p>Trial and Improvement</p>	<p>Note the method/s used each student.</p> <p>Decide the order in which to call students to the board during Ceardaiocht.</p> <p>If students are stuck help them by asking appropriate questions.</p> <p>If students are finished encourage them to come up with another method of getting the same solution. Remind them of resources if necessary.</p>	<p>Are students able to begin the problem solving?</p> <p>Have students used the resources to aid in the problem solving?</p> <p>Can students convert the word problem into a mathematical statement?</p>



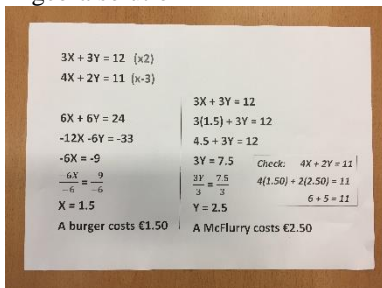
Translate into a formula



Graph



Algebra solution



Ceardaíocht /Comparing and Discussing

Coins

Trial & Improve

After correct solution given, ask another student are there coins that work for one and not the other, what are they?

Why did you start with that value?  
 Why did you make it smaller/bigger?  
 Ask another student to give the

<p>Translate</p> <p>Algebraic Equations</p> <p>Graphing</p>	<p>explanation of why to get bigger or smaller.</p> <p>What letter are you going to use for burger? What letter are you going to use for McFlurry? What other word could we have used instead of “and” &amp; “costs”</p> <p>What do we use to solve it? What do we need to multiple each row by? What do you do next? How do you find the second variable?</p> <p>How many points do you need to draw a line? Is there another way of drawing the line? How do we get our first point? If it’s a negative slope what do we do? If it’s a positive slope what do we do? If you find different points to _____ will your line be different to theirs? Which line represents the first visit to McDonalds?</p>	
<p><b>Summing up &amp; Reflection</b></p>	<p>The teacher recaps briefly the different approaches that can be taken, and reasserts that they are all valid.</p>	<p>Ask students do any of the methods have benefits over the others</p>

## 10. Board Plan

The whiteboard displays the following content:

- Problem Statement:**

One Friday you and some of your friends go to McDonalds and order 3 burgers and 3 McFlurries. It costs €12. The next Friday you go again but this time you order 4 burgers and 2 McFlurries and its costs €11.

How many different ways can you use to find the cost of a burger and the cost of a McFlurry?
- Coins:** A grid showing combinations of burgers and McFlurries for 12 euros and 11 euros.
- TRIAL AND IMPROVEMENT:**

3 Burgers + 3 McFlurries = €12  
 $3x + 3y = 12$  Too cheap  
 $3x + 2y = 11$  Correct

4 Burgers + 2 McFlurries = €11  
 $4x + 2y = 11$  Too expensive  
 $4x + 1y = 10$  Correct

3x + 2y = 11  
 $3x + 2y = 11$  Too cheap  
 $3x + 2y = 11$  Correct

4x + 2y = 11  
 $4x + 2y = 11$  Too expensive  
 $4x + 1y = 10$  Correct

Burgers cost €1.50, McFlurry costs €2.50

Cheaper to buy 4 burgers and 2 McFlurries than 3 of each as burgers must be cheaper.
- Simultaneous Equations:**

3x + 3y = 12 (x2)  
 $4x + 2y = 11$  (x-3)

$6x + 6y = 24$   
 $-12x - 6y = -33$   
 $-48 = -9$   
 $-48 \div -9 = 5.33$   
 $x = 1.5$   
 A burger costs €1.50

$3x + 3y = 12$   
 $3(1.5) + 3y = 12$   
 $4.5 + 3y = 12$   
 $3y = 7.5$  Check:  $4x + 2y = 11$   
 $4(1.5) + 2(2.5) = 11$   
 $6 + 5 = 11$   
 $y = 2.5$   
 A McFlurry costs €2.50
- Big Maths and Mrs. Finlayson:** A graph showing two intersecting lines on a coordinate plane.

## 11. Evaluation

The lesson worked well in that the students were engaged and actively trying multiple ways to solve the problem. Not all anticipated solutions were fully used, but all solutions were attempted. The timing of the lesson worked well, and while the students could have been given more time to try solve it fully the time given to the boardwork was needed.

## 12. Reflection

It was brilliant to see that so many of the students stated in the feedback that they learned that problems can be solved in multiple different ways. We loved that the students got a chance to show off their work and improve their presentation skills. We noticed that a number of students were chatting and discussing the problem and different ways of solving it even after the bell had gone for the end of class. One student commented "I enjoyed the option of materials, and being left to my own devices to figure it out".