# Lesson Research Proposal for 2nd Year - Volume 

For the lesson Friday 26th January 2018
At Old Bawn CS, 2nd year
Teacher: Méabh Murray
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## 1. Title of the Lesson: "The Formula Speaks Volumes"

## 2. Brief description of the lesson

During this lesson, the students will explore the volume formula for cuboids. They will then develop a formula for volume of all prisms leading to the discovery of the formula for the volume of a cylinder.

## 3. Research Theme

At Old Bawn Community School we want students to:

- enjoy their learning, be motivated to learn and expect to achieve as learners.
- engage purposefully in meaningful learning activities.

As teachers of mathematics, we will actively support the achievement of these goals by:

- developing lessons that are engaging and allow students to achieve success by ensuring the problems chosen can be solved in a wide variety of methods.
- providing students with an opportunity to explain their reasoning clearly and accurately.


## 4. Background \& Rationale

This lesson is aimed at second-year students. The teaching of area and volume is important subject material as it is related to everyday life and is a basic numerical skill that students will need over their lifetime. We have observed that students find the topic of volume difficult to grasp and are often making mistakes when differentiating between area and volume. The science department have observed a confusion amongst students on the topic of volume versus mass. There is also an issue with students understanding of the units to use (i.e. where they come from) as opposed to memorising which units to use.

We normally teach the volume of cylinders immediately after the volume of a cuboid. Traditionally, the formula is explained as $V=\pi r^{2} \times h$, leading to drill exercises on finding the volume of a cylinder. The focus has been on using the formula to solve problems without much time spent on discovering where the formula comes from.

The chief examiners report, $2015^{1}$, highlighted that students frequently chose the incorrect formula when attempting to find the volume of a cylinder, e.g. using the formula for a cuboid instead of a cylinder. This suggests that understanding in some cases is limited to filling in a formula, but a disconnect occurs between the formula and the shape which becomes evident when students are required to identify which formula to use themselves. We hope our lesson will address this disconnect.

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## 5. Relationship of the Unit to the Syllabus

| Related prior learning Outcomes |
| :--- |
| In primary school, students have: |

Discovered that the area of a rectangle is length by breadth

Used $\mathrm{cm}^{2}$ as units for area.
Calculated area using $\mathrm{cm}^{2}$ and $\mathrm{m}^{2}$.
Calculated the area of regular and irregular 2D shapes.

Compared visually $\mathrm{cm}^{2}$ and $\mathrm{m}^{2}$.
Met capacity and rename measures of capacity.

They should be able to ..
...select and use appropriate instruments of measurement.
...find the volume of a cuboid experimentally.

In second level (i.e. in 1st year) students have learned how to:

Find the length of the perimeter and the area of the following: disc, triangle, rectangle, square and any combinations of these

Draw and interpret scaled diagrams.
Investigate nets of rectangular solids
Find surface area of rectangular solids

Solve problems involving the surface area of a cuboid.

Learning outcomes for
In second year, students are expected to learn how to:

Find the volume of rectangular solids and cylinders.

Solve problems involving volume of cylinders.

Solve problems involving volume of cuboids and cylinders.

Related later learning outcomes
In 3rd year, the students will:

Investigate nets of other prisms (e.g. triangle based prisms), nets of cylinders \& cones.

Solve problems involving the surface area of a cylinder.

Solve problems involving the curved surface area of cylinders.

Solve problems involving the surface area of triangular base prisms and cones.

Solve problems involving cones and spheres.

Perform calculations to solve problems involving the volume of cones, triangular base prisms, spheres and any combination of these.

## 6. Goals of the Unit

- To help students discover that area relates to 2D shapes and volume relates to 3D shapes.
- To help students develop problem-solving skills through examining a variety of 3D shapes.
- To help students understand that the units used for 2D and 3D shapes stem from the number of variables used to calculate area and volume.
- To allow students to transfer their knowledge of volume from the context of a cuboid to the context of all prisms.
- To help students discover the formula to calculate the volume of a cylinder.
- To help students to discover the formula to find the volume of all prisms.


## 7. Unit Plan

| Lesson | Learning goal(s) and tasks |
| :---: | :--- |
| $\mathbf{1}$ | Students should know the difference between a 2D and 3D shape. This will <br> consist of examples of each and the real-life contexts when each is used. <br> Introduce the idea of prisms. Identify the face/base of a prism. <br> Know that a prism has a uniform cross-sectional area. <br> Identify a cuboid as a rectangular based prism. <br> Recall the area of a rectangle. |
| $\mathbf{2}$ | Recall how to find the volume of a cuboid. Practice finding the volume of simple <br> cuboids. |
| $\mathbf{3}$ | Discover how to calculate the volume of a rectangular solid using <br> area of face $x$ height. <br> Establish the formula for the volume of a cylinder. |
| $\mathbf{4}$ | Students convince themselves that this process can be applied to all prisms to <br> find the volume through investigation. |
| $\mathbf{5}$ | Solve problems involving the volume of a cuboid/other prisms |
| $\mathbf{6}$ | Solve problems involving the volume of a cylinder |

## 8. Goals of the Research Lesson:

## Mathematical Goals

Students will:
a) Understand how to calculate the volume of a cuboid using face area $x$ height
b) Understand how to calculate the volume of a cylinder

Key Skills \& Statements of Learning
a) Being Numerate: By engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.
b) Working with Others: Students will learn with and from each other by discussing different approaches to problem solving.
c) Communicating: During Ceardaíocht, students will present and discuss their mathematical thinking.
d) Managing myself: Students will have the opportunity to reflect on their own learning when the teacher asks them to write a reflection at the end of the lesson.
e) Managing information and Thinking: Students will be encouraged to think creatively and critically.
f) Being Creative: Students will explore options as they actively participate in the construction of knowledge.
g) Staying Well: By engaging in tasks that are appropriate to their abilities, students' confidence and positive disposition will be promoted.
h) Being literate: Through Ceardaíocht, students will have the opportunity to express their ideas accurately and clearly.

This lesson is also designed to meet the following JC Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.
2. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
3. The students describes, illustrates, interprets, predicts and explains patterns and relationships.
4. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

## 9. Flow of the Research Lesson:

| Steps, Learning Activities <br> Teacher's Questions and Expected Student <br> Reactions |
| :--- |
| Introduction |
| Before we start today's problem, I want to quickly <br> review some maths we learned over the last two | weeks.

- Place a cuboid on each desk. The cuboid will have the dimensions written on the face and the depth will have already been split into 101 cm blocks using a permanent pen.

$\rightarrow$ denotes student responses
Q. How is a cuboid a prism?
$\rightarrow \quad$ Matching faces
$\rightarrow \quad$ All faces are flat
$\rightarrow \quad$ Four parallel lines joining the faces
Q. What can we measure on a prism?
$\rightarrow \quad$ Area of a face
$\rightarrow$ Total Surface area
$\rightarrow \quad$ Length
$\rightarrow$ Width
$\rightarrow$ Depth/Height
Q. What is each dimension called?
$\rightarrow \quad$ Length
$\rightarrow \quad$ Width
$\rightarrow \quad$ Height
Q. Can anyone remember how to find the area of a rectangle?
Area rectangle $=$ Length x width

Put up a picture of a rectangle. Label length and width.

Remind students what a prism is.

Explain why height and depth mean the same thing.

Ensure each child correctly identifies the label on each dimension.

Put up card on board that describes
Area $=$ length x width


This is to ensure all students can correctly label the face of a cuboid, and correctly find the area.

It is important that all students clearly understand the language used.

## Posing the Task 1

Presenting a cuboid.
The cuboid will have some dimensions written on it
$\mathrm{L}=4 \mathrm{~cm}$
$\mathrm{W}=3 \mathrm{~cm}$
$\mathrm{H}=10 \mathrm{~cm}$ (graded into one cm widths)

## Task 1

Can you come up with at least three ways to calculate the volume of the cuboid on your desk?

Give students five minutes silent thinking time to come up with as many ways as possible to calculate the volume of a cuboid.

Students will be given the cuboid.
Each student will be given a sheet with empty boxes to place suggestions in. See Appendix A.Encourage students to write down the formula they are using every time. These worksheets will be put up on the board during Ceardaíocht. This worksheet will be glued into the students' copy at the end of the lesson.


Teacher encourages students who find 3 ways to come up with as many as possible.

Place a formula on the board
Volume of Cuboid $=$ Length $x$ Width $x$
Height

As the teacher circulates the room, she looks for good examples of the various methods- Finding volume treating the cuboid as one block, two blocks, three blocks.

The order will be noted of the ways the solutions will be displayed on the board. Students names will be noted e.g. who to ask questions etc during Ceardaíocht.

For students who find the volume using $4 \mathrm{~cm} \times 3 \mathrm{~cm} \times 10 \mathrm{~cm}$ they could be asked: Could you try and find the volume a different way?
The teacher can prompt by saying "keep the length and width fixed"

Can students recognise the length, width and height of the shape

Can they substitute correctly into the formula, the length, the width and the height?

Can they work out the volume in different ways but keeping the length and width the same.

Are they using units correctly?


## Ceardaíocht /Comparing and Discussing

The solutions will be presented in the order outlined above. A discussion will take place around units. A student who chose the incorrect units or omitted the units for method one will be first called to the board, followed by a student who did the correct units.

- Do you understand?
- Can you explain it back to me?
- Did anyone else calculate the volume of the block in one go?
- Can you explain it back to me?

When the block was broken into two pieces what dimensions always remained the same?

What are we calculating when we multiply length x width?
$\rightarrow \quad$ Area

What is a prism?
$\rightarrow$ Matching faces, joined by parallel lines; should be on the board

When the block is in three parts what dimensions remained the same?
$\rightarrow \quad$ Area of the face

## Conclusion:

Volume $=$ Area of face $x$ height

For each response/ solution presented on the board:
Another student who didn't use the method (as noted when walking around) will be asked to explain the solution.

How can I calculate the units for volume?


For each student solution on board; the teacher will circle the product (i.e. 3 x 4) that gives area in all the calculations on the board with a red marker.


Teacher can ask:
Why does this product keep showing up in all calculations?
How does this product relate back to the shape we're looking at?
What's staying the same?
What's different?
Prompt the students that the area that is never changes in a prism is the face.
Will this method work for all shapes?
Can you think of an example of a
different shape it will work for?
Why will it work on this shape?
Can you think of an example it won't work for?
Why won't it work for that shape?

Do you think these units are correct?

Why/Why not?
Why are these units correct.

Can students see that the length and width of the shape don't change?

Can they see that the area doesn't change?

Can they see that the only dimension that changes is the height?

Will they recognise that the volume of the prism is the Area x Height.

Will this lead them to discover the volume for other prisms?

Will they be able to come up with formula for the volume of a cylinder?

## Posing Task 2:

Students will be given a worksheet (see Appendix B) with various shapes ( 2 cuboids, 2 triangular prisms, and 1 cylinder). On each of these shapes the area of the face will be given and the height will be given. Students will be asked to find the volume of each shape. The last question on the worksheet will be "can you come up with how to calculate the volume of a cylinder in words? Now can you create a formula using letters to find the volume of a cylinder? Make sure you list what every variable you use stands for"

Students will have 15 minutes to complete this task.
Ceardaíocht /Comparing and Discu
The solutions to the worksheet will be
the board.
A blown up copy of each shape will b

1. Cuboid: Area face x height
$\rightarrow 3 \mathrm{~cm}^{2} \times 6 \mathrm{~cm}$
$\rightarrow 18 \mathrm{~cm}^{3}$

- Did anyone else get this answer?
- Do you understand?

2. Cuboid: (Area of face) $x$ height
$\rightarrow 5 \mathrm{~cm}^{2} \times 3 \mathrm{~cm}$
$\rightarrow 15 \mathrm{~cm}^{3}$
3. Triangular prism: Area of face $x$ height
$\rightarrow \quad 3 \mathrm{~cm}^{2} \times 8 \mathrm{~cm}$
$\rightarrow 24 \mathrm{~cm}^{3}$

- Did anyone else get this answer?
- Do you understand?

4. Triangular prism: $\frac{1}{2}$ (Base $x$ Perpendicular height) x height
$\rightarrow \frac{1}{2}(2 \mathrm{~cm})(2 \mathrm{~cm}) \times 10$
$\rightarrow 2 \mathrm{~cm}^{2} \times 10 \mathrm{~cm}$
$\rightarrow 20 \mathrm{~cm}^{3}$

- Did anyone else get this answer?
- Do you understand?

Prompt: Remind students about the formula they have discovered:

Volume $=$ Area of face $x$ Height


Are all of the shapes prisms?
Which one is not a prism?
Why is it not a prism?
Can we transfer knowledge on finding the volume of a prism to this shape to find its volume?

Teacher reminds by asking

- What never changes?

On board place the formula
Triangular Prism =

## Area of face $x$ depth

(Base x Perpendicular Height x depth)


Students may need to be prompted "can you remember how to find the area of a circle?"

Put the area of the circle on the board
Area of circle $=\pi r^{2}$

Do students know how to find the volume of a cylinder using Face Area x Height

Will students recognise that a cylinder is not a "prism" although it can be treated like a prism.

Can students find the area of these shapes?

Can they all see it's area of face $x$ height?

Will they remember the area of a circle?

Will they come up formula themselves for the volume of a cylinder?

10. Proposed Board Plan


## 11. Evaluation

The classroom will be divided up in 3 sections, approximately 9 students per observer, see Appendix C. The 4th observer will use the LessonNote ${ }^{2}$ app to observe the whole lesson. Observers will take note of student interactions, engagement etc. keeping in mind the goals of the lesson. Student worksheets will be collected and photographed to be reflected on later. The completed board will be photographed to be reflected on later. A post-lesson meeting will take place immediately after the lesson for reflection to take place. When observing the lesson, the following questions will be kept in mind:

- Were students motivated during the lesson?
- Were they engaged at all times?
- Did students enjoy the lesson?
- Could students solve the problems in a variety of methods?
- Were students given an opportunity to explain their reasoning clearly and accurately?
- Do students understand that the volume of a cuboid is the face area $x$ height?
- Do students understand that this formula can be used for other prisms?
- Did students understand that this formula cannot be used for all shapes?
- Were students able to discover the formula for the volume of a cylinder?
- Did students have sufficient reflection time?

[^1]
## 12. Reflection

## What had we hoped before the lesson:

We had hoped ...

- to create a lesson that would engage students and motivate them to learn.
- to see evidence of key skills embedded in the lesson.
- to see if students could transfer knowledge from one lesson into the next and build on this prior knowledge.
- that students would understand the origins of the formula for the volume of a cylinder.
- that the experience of lesson study would help inform our teaching and help us recognise the value of collaboration and reflection.


## What was actually observed during the lesson:

During the lesson, students were fully engaged and were on task at all times. Giving students the opportunity to present at the board was powerful as other students were very engaged when this was happening. They were actively taking down their peers' board work without prompting from the teacher.


When the teacher was walking around, she did not spend too much time with one student. The teacher got to go around to everyone and was discussing higher order content with some students. We felt the structured format of the lesson allowed for this to happen and doesn't allow the focus of the lesson to become about what students do not know. The teacher was checking for what the students had discovered as opposed to what they had not understood. As a result, more able students had the time to explore higher order questions with the teacher because the teachers' time was not absorbed by one or two students who were struggling. For example, one student wanted to discuss whether the formula "Area of any face x height" would work for any prism, or only certain ones. Such a discussion may not have occurred in a class where the teacher spent her time helping students fill in the LxWxH formula.

Moreover, we did not feel the learning of the students who struggled was compromised. Students who had problems during task time were noticed actively engaging while their peers wrote a correct solution on the board. It was felt there was also a value in struggling at times.

Students really enjoyed having the physical object in front of them. It was noted how much they were turning it around. One student noted in the reflection section of her workbook that the task was much easier to understand because she had the block in front of her and could examine it carefully.

The lesson started at a slow pace and then moved quicker at the end. Yet we don't feel the learning was compromised in any way.

The deputy principal attended the lesson and noted "A lot of learning in one hour, how they got from something so simple that stepped up to the next level".


It helped having the physical object

"A lot of learning in one hour, how they got from something so simple that stepped up to the next level".

## Major points raised during the post-lesson discussion, and the team's own opinions:

It helped having the physical blocks for each student. Many thanks to the co-operative woodwork department for making these for us!

Differentiated learning was taking place. The well able students still carried on with the tasks as they tried to find other ways to solve the problems. The weaker students learned from when their peers were at the board. The teacher could reinforce learning through asking students to repeat back what others had presented.

When we observed the students' worksheets after the lesson, we came across two students who used an interesting way to work out volume of a triangular prism. They attempted to turn the triangular prism into a cuboid (by adding to the diagram, see photo below), and then trying to find the volume of the prism. While, in theory, this method is correct, the student used the wrong length and width for the cuboid. It was thought the origins of such a strategy lay in the fact that students had previously discovered the area of a triangle by halving the area of a parallelogram. This would definitely be worth exploring in another lesson.


Finding volume of cuboid and halving to find volume of triangular prism. An example of where the method was correct however the wrong dimensions used.

One student only got a third of the way through the worksheet yet was able to achieve success. She correctly answered two parts. Despite this, she was clearly engaged in all the work her peers were doing on the board as she still achieved the aim of the lesson and correctly discovered the volume of the cylinder formula. We agreed that such differentiated learning and the opportunity for all students to achieve success during the lesson was a successful outcome.

The teacher brought the students from their comfort zone at the start of the lesson (going over prior knowledge) to a point where they discovered something new. The lesson flowed smoothly. Students were taking ownership of their learning as they presented their solutions at the board. At the end of the lesson it was clear that the mathematical goals of the lesson had been met. This was evident when students were asked to write down on their whiteboards what they thought would be a "general" formula for working out the volume of a cylinder. Using the whiteboards in this way was a very effective way of assessing the "new" learning of the lesson.


Students displaying their whiteboards at end of lesson.


Evidence that goal of lesson was met: Students understood how to find the volume of a prism using (Area * Height) and apply this to a cylinder.

## Ideas for future study

It would be worth exploring the following in a future or follow-on lesson:

- Exploring the alternative way to find the volume of a triangular prism through halving the volume of a cuboid (as a student described above).
- Exploring to see if students could discover the volume of a cone through building on what they've learned about the volume of a cylinder.

We see the value in having quiet time for students, e.g. giving students the time to do individual work. We are all used to having our students working with each other on solving problems and hearing discussion in our classes. While we still value this, we see that there is added benefits in having quiet time.

## Reflection: <br> Write 3 sentences about what you learned today.


"Finding answers on my own was a lot more rewarding and helped me feel more confident"
An idea for future lessons would be to mix between group and individual work. Task 1 could have been done as a paired exercise and maybe students would have come up with more of the "expected responses" through discussion.

We also see the value in having one-hour lessons. We feel that this lesson could not have been done in the usual 40-minute period.

## What did we as teachers learn from process?

Students really enjoy coming up to the board. We will aim to adopt this practice more in our classes. We will have periods of quiet time in our classes where students can work individually and use this time to look for "correct" solutions that can be presented at the board to class. We will ask students who we think are struggling to reinforce or repeat the message that's been presented at the board. We will try in the future to find a balance between students working quietly individually and working with others.

The most valuable feedback of the lesson resulted from the conversations that we had as a group after the lesson. We will aim, as teachers, to bring this powerful message back to our schools. We can see the benefits of collaborating together on designing a lesson and how the in-depth and deep discussion after the lesson informed us.

## Summary

There were two mathematical goals of the lesson. We felt that both goals were achieved. Students clearly understood how to calculate the volume of a cuboid using face area $x$ height. They also discovered the formula for calculating the volume of a cylinder. This was evident through the work they completed on the worksheets and the mini-whiteboards.

Students enjoyed the lesson and were motivated to learn. This was evident from the reflections and from the conversations we could hear them having among themselves after the lesson. Key skills of numeracy, managing information and thinking, communicating and working with others were evident in the lesson.

Finally, as a group, we felt the process was very worthwhile. The structured methodology, from researching prior knowledge (back to 4th class in primary) to having the board-work pre-planned provided a unique opportunity for us to reflect on what students know, what we want them to know and the value in planning a precise route to allow them to discover the learning intention themselves.

The process of collaborating with our colleagues both within the school and teachers from other schools provided insight into what goes on in other classrooms. This was very valuable and has provided the basis for future collaboration for similar lessons. We will aim to adopt "Lesson Study" approaches and methodologies in our lessons.


Yvonne Marley (Old Bawn CS), Luiza Stoicescu (Tallaght CS), Méabh Murray (Old Bawn CS), Irene Stone (Lesson Study Associate), Cailín Foy (Old Bawn CS)

## Appendix A: Task 1

$|$| Find the volume of the cuboid on your |
| :--- |
| desk. The cuboid as dimensions as |
| follows: |
| Width $(w)=3 \mathrm{~cm}$ |
| Length $(I)=4 \mathrm{~cm}$ |
| Height (h) 10 cm |
| Come up with as many ways as you can |
| (at least 3) of finding the volume of |
| the cuboid. |



Appendix B: Task 2

| Cuboid | Find the volume of the shape using <br> the information provided in the <br> diagram and what you learned today. |
| :--- | :--- |
| Area of face $=3 \mathrm{~cm}^{2}$ |  |

## Triangular Prism <br> Area of face $=3 \mathrm{~cm}^{2}$ <br> 

## Triangular Prism

Perpendicular Height $=2 \mathrm{~cm}$



Write down a formula for the volume of the cylinder. Clearly state what each variable you use stands for:

## Reflection:

Write 3 sentences about what you learned today.

1. $\qquad$
$\qquad$
2. $\qquad$
$\qquad$
3. $\qquad$
$\qquad$
$\qquad$

Appendix C: Classroom layout

Appendix D: Sample of notes taken on Lesson Note during observation

The Formula Speaks Volumes
$\square$
00:45 (+2s) - Note (4s)

00:45 (+7s) - T01 (23s)

shoos me?


00:46 (+1:25) - Note (15s)


00:48 (+2:47) - Note (24s)
Face
Paralled ling U \& bidx. $\square \times \square^{-}$Anee.

00:48 (+3:26) - T01 (9s) hapl os hous
hors

00:49 (+3:42) - T01 » S25 (7s)
Dimercion as?
00:49 (+3:51) - T01 (21s)
Dose it malter
Whit enepery
Aus
Assosiativ!
comruituher
00:49 (+4:22) - T01 (3s)
Task 1

00:49 (+4:27) - T01 (26s)
Tcsic 1
Cubold





[^0]:    ${ }^{1}$ Chief Examiner's Report JC Maths 2015: https://www.examinations.ie/misc-doc/EN-EN-25073660.pdf

[^1]:    ${ }^{2}$ Designed for the collaborative improvement for teaching and learning http://lessonnote.com/
    Maths Development Team: Lesson Study 2017-2018

