Lesson Research Proposal for 2nd Year Higher Level - Functions

For the lesson on 7 February 2018018

At Saint John Bosco Community College, Kildysart, Co. Clare - Anna Quilter's class

Instructor: Anna Quilter

Lesson plan developed by: Dermot Galvin, Michelle Morrissey, Anna Quilter & Angela Rahill

1. Title of the Lesson: The Story of the Gannet

2. Brief description of the lesson

In this lesson students will describe the story of a gannet's dive as represented by a quadratic graph using plain english, the parameters of height and time and mathematical expressions. This will enable students to gain a more complete understanding of quadratic functions and graphs.

3. Research Theme

The main objectives of our Lesson Study will be to ensure that:

- The teacher responds to individual learning needs and differentiates teaching and learning activities as necessary.
- Teachers contribute to building whole-staff capacity by sharing their expertise.
- Teachers value and engage in professional development and professional collaboration.
- Students reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning.
- Students demonstrate the knowledge, skills and understanding required by the post-primary curriculum.
- Students experience opportunities to develop the skills and attitudes necessary for lifelong learning.

4. Background & Rationale

Our Lesson Study is going to be based on the application of quadratic functions to real life situations. This lesson is aimed at a higher level Second Year class of varying ability.

Why we chose this topic:

- Based on our own experience as teachers, students fail to see the relationship between roots, factors and the developing function. This difficulty continues throughout Junior Cycle and on to Senior Cycle and results in a lot of re-teaching due to the fact that the procedural approach to teaching does not equip students to visualise this relationship.

- Furthermore, students fail to see the connection between quadratic functions, graphs and the real life applications and uses of these graphs, for example they are unaware that the 'y - axis' can represent the profits of a company, the height of a ball thrown in the air or the flight of a plane, etc. and that they can use quadratic graphs to determine height/profit/maximum area etc. after a given period of time.

- Factorisation and quadratic functions are the foundation of several topics at both Junior Cycle and on to Senior Cycle.

5. Relationship of the Unit to the Syllabus

Related prior learning outcomes	Learning Outcomes for this Unit - Junior Cycle	Related later learning outcomes - Senior Cycle
Students already have some experience of problem solving and this lesson provides another skill in problem solving. This topic is not only a key part of the Junior Cert syllabus in its own right; it also forms the foundation of the students' understanding of several other topics both at Junior Certificate level and later at Leaving Certificate level.	**4.2 Representing situations with tables, diagrams and graphs** Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. Students look at various patterns and make predictions about what comes next. − use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations (exponential relations limited to doubling and tripling) − − present and interpret solutions, explaining and justifying methods, inferences and reasoning **4.4 Examining algebraic relationships** Features of a relationship and how these features appear in the different representations. Constant rate of change: linear relationships. Non-constant rate of change: quadratic relationships. Proportional relationships. − show that relations have features that can be represented in a variety of ways − distinguish those features that are especially useful to identify and point out how those features appear in different representations: in tables, graphs, physical models, and formulas expressed in words, and algebraically − use the representations to reason	**4.2 Solving equations** – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • f(x) = g(x), with f(x) = ax+b, g(x) = cx+d where a, b, c, d ∈ Q • f(x) = g(x) with f(x) = g(x) = where a, b, c, e, f, p, q, r ∈ Z • f(x) = k with f(x) = ax2 + bx + c (and not necessarily factorisable) where a, b, c ∈ Q and interpret the results – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to • one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is ± 1 in the linear equation) and interpret the results – form quadratic equations given whole number roots – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: f(x) = g(x) with f(x) = ; g(x) = k where a, b, c, d, e, f, q, r ∈ Z **5.1 Functions** – recognise that a function assigns a unique output to a given input – form composite functions – graph functions of the form • ax+b where a,b ∈ Q, x ∈ R • ax^2 +bx + c where a, b, c ∈ Z , x ∈ R • ax^3 +bx^2 + cx+d

	about the situation from which the relationship is derived and communicate their thinking to others − recognise that a distinguishing feature of quadratic relations is the way change varies **4.5 Relations without formulae** Using graphs to represent phenomena quantitatively. − explore graphs of motion − make sense of quantitative graphs and draw conclusions from them − make connections between the shape of a graph and the story of a phenomenon − describe both quantity and change of quantity on a graph **4.6 Expressions** Using letters to represent quantities that are variable. Arithmetic operations on expressions; applications to real life contexts. Transformational activities: collecting like terms, simplifying expressions, substituting, expanding and factoring. − evaluate expressions of the form • ax + by • a (x + y) • x^2 + bx + c • • axy where a, b, c, d, x, y ∈ Z • ax^2+ bx + c • x^3 + bx^2 + cx + d where a, b, c, d, x, y ∈ Q **5.2 Graphing functions** Interpreting and representing linear, quadratic and exponential functions in graphical form. − interpret simple graphs − plot points and lines − draw graphs of the following functions and interpret equations of the form f(x) = g(x) as a comparison of functions **• f(x) = ax^2 + bx + c, where a ∈ N; b, c ∈ Z; x ∈ R • f (x) = ax^2 + bx + c, where a, b, c ∈ Z, x ∈ R •** − use graphical methods to find approximate solutions where f(x) = g(x) and interpret the results **− find maximum and minimum values of quadratic functions from a graph**	where a,b,c,d ∈ Z, x ∈ R • abx where a ∈ N, b, x ∈ R – interpret equations of the form f(x) = g(x) as a comparison of the above functions – use graphical methods to find approximate solutions to • f(x) = 0 • f(x) = k • f(x) = g(x) where f(x) and g(x) are of the above form, or where graphs of f(x) and g(x) are provided – investigate the concept of the limit of a function – recognise surjective, injective and bijective functions – find the inverse of a bijective function – given a graph of a function sketch the graph of its inverse – express quadratic functions in complete square form – use the complete square form of a quadratic function to • find the roots and turning points • sketch the function – graph functions of the form • ax2 +bx + c where a,b,c ∈ Q, x ∈ R • abx where a, b ∈ R • logarithmic • exponential • trigonometric – interpret equations of the form f(x) = g(x) as a comparison of the above functions – informally explore limits and continuity of functions **5.2 Calculus** – find first and second derivatives of linear, quadratic and cubic functions by rule – associate derivatives with slopes and tangent lines – apply differentiation to • rates of change • maxima and minima • curve sketching

6. Goals of the Unit

- Students will recognise quadratic equations.
- Students will be proficient in factorisation of quadratic equations and will be able to form quadratic equations from their roots.
- Students will use algebra, graphs and tables to solve quadratic equations.
- Students will be competent in the formation of a quadratic equation to represent a given problem.
- Students will need to recognise the relationship between roots, factors and quadratic functions.
- Students will be in a position to sketch the shifts in graphs and consequently to graph quadratic functions and will understand how different changes in variables of a formula would cause the graph to shift position.
- Students will understand that quadratic graphs represent real life situations and will be able to visualise the relationship between the X-axis and the Y-axis.
- Students will be confident in the interpretation of these graphs and will understand how a change in the X variable of the function corresponds to a change in the Y variable.
- Students will understand how the maximum and minimum points influence the shape of the graph.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	● Factorising of Quadratic Equations
2	Solving Quadratic Equations ● Students can see the relationship between the factors and the roots
3	Functions ● Students are introduced to the concept of domain and range and how these can be used to draw a linear/quadratic graph to describe the relationship between the two variables in a Function.
4	Graphing functions - to include sketching and shifting ● Students can see how changing the variables in the function will results in a shift in position of the graph. ● Students can see how the maximum and minimum points

	will influence the shape of the graph ● Students can predict the change in the shape and position of the graph as the variables in the formula change
5	Matching functions to their related graphs ● Students will practice recognising graphs from a variety of functions using matching exercises. This is designed to increase the students awareness and confidence of the shape of the graphs
. . 6.	Applying real life problems to function graphs ● Students are given a quadratic function and are given the opportunity to change the X variable and to observe the change in the overall function as a result. ● Students now realise that the X variable is actually seconds of time and the Y variable is the corresponding location of a ball thrown in the air. .
7	Research Lesson

8. Goals of the Research Lesson:

a) Mathematical Goals

- Students will understand and use a 2 dimensional quadratic graph and use their knowledge of factorisation and roots to solve a real world problem.

b) Goal of the Unit

- Students will become confident of their knowledge of quadratic functions, they will become more positive about learning by developing a positive disposition towards investigating, reasoning and problem solving by seeing patterns, trends and relationships.

- Students will understand that the factors and roots of a quadratic expression have a real meaning when solving a real world problem

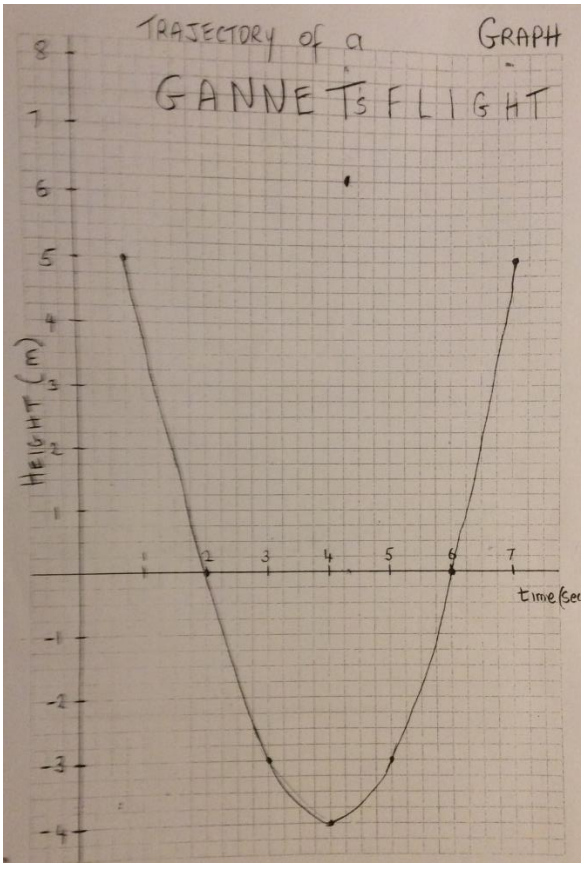
- Students will appreciate that a problem solving using a quadratic function can be applied to real world situations.

c) Key Skills and Statements of Learning

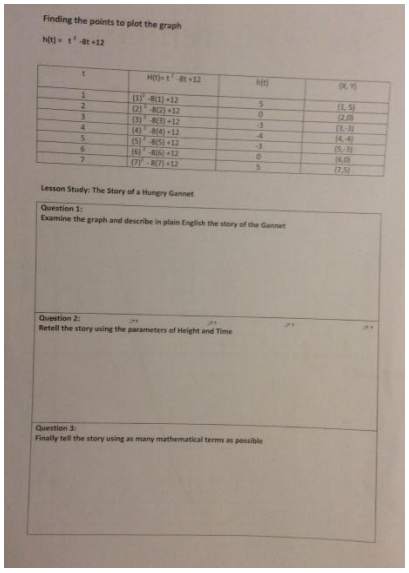
- Students will express ideas clearly and accurately. *SL1 - Communicates effectively using a variety of means in a range of contexts in L1*
- They will become confident about performing and presenting in front of their classmates as everybody's contribution is value.
- Students will be respectful of difference in that other students may have approached solving the problem in a different way.
- Students will improve their literacy by expressing ideas clearly and accurately. *SL16 - Describes, illustrates, interprets, predicts and explains patterns and relationships.*
- Students will manage their own learning by reflecting on what they learn. *SL17 - Devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.*
- Students will engage purposefully in meaningful learning activities. *SL15 - Recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.*

9. Flow of the Research Lesson:

The problem



The answer sheet



Finding the points to plot the graph

$h(t) = t^2 - 8t + 12$

t	$h(t) = t^2 - 8t + 12$	h(t)	(X, Y)
1	$(1)^2 - 8(1) + 12$	5	(1, 5)
2	$(2)^2 - 8(2) + 12$	0	(2,0)
3	$(3)^2 - 8(3) + 12$	-3	(3,-3)
4	$(4)^2 - 8(4) + 12$	-4	(4,-4)
5	$(5)^2 - 8(5) + 12$	-3	(5,-3)
6	$(6)^2 - 8(6) + 12$	0	(6,0)
7	$(7)^2 - 8(7) + 12$	5	(7,5)

Lesson Study: The Story of a Hungry Gannet

Question 1:
Examine the graph and describe in plain English the story of the Gannet

Question 2:
Retell the story using the parameters of Height and Time

Question 3:
Finally tell the story using as many mathematical terms as possible

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction and Posing the Task (5 min) Use an online picture and student input to explain what a gannet is. Explain the layout of classroom materials.	Supply the students with an A4 poster of the graph for the quadratic h(t) = t2 -8t +12 in the domain 1 <=t <=7 and a picture of a diving gannet. *(We ASSUME this quadratic to fit the gannets journey)*	
Problem Solving 1 (5 min) Task 1 Using the graph you have been given, Tell the story of the gannet using five plain English sentences Anticipated responses include ● The gannet is diving. The gannet drops from the sky into the water. ● The gannet is getting food; fish in the water.	*Supply students with 3 pieces of coloured card. Each piece will be a different colour to represent the 3 different questions and will be used for student contributions to the ceardaíocht.* Also, supply students with a completed table of the relationship between time and height for each second from time = 1 seconds	Assess as the teacher goes, around the classroom, choosing individuals for ceardaíocht.

● The gannet goes down and then up. ● The gannet goes downwards into the water, then comes up out of the water. ● The bird dives very fast. ● The gannet dives down deep in the water. ● The gannet turns and rises up, then leaves the water. ● The gannet gets wet.	to time = 7 seconds Ask the students to describe what is happening for up to 7 seconds, i.e. to tell the story in plain english. This will take 5 minutes.	
Ceardaíocht 1 (5 – 10 min) SpecificStudents are asked in a specific order to tell their part of the story. Card is used to record their input. (See boardwork)	Individual students will then be asked to come up to the board and describe and explain their findings After the story has been told, - 1 student will synopsize the whole story. What direction does the gannet take after diving into the water? Do you think he is travelling fast? and why? When does the gannet start to come up to the surface ? What happens after it comes out of the water? Was the gannet hungry? How long was it under water?	Discussion is elicited from the students using open ended questions such as: ● What do you think? ● Why is that? ● Did anyone else solve it the same way? *Possible assessment questions arising from observations may include*: What does submerge/Dive mean? Why is the gannet going into the water?

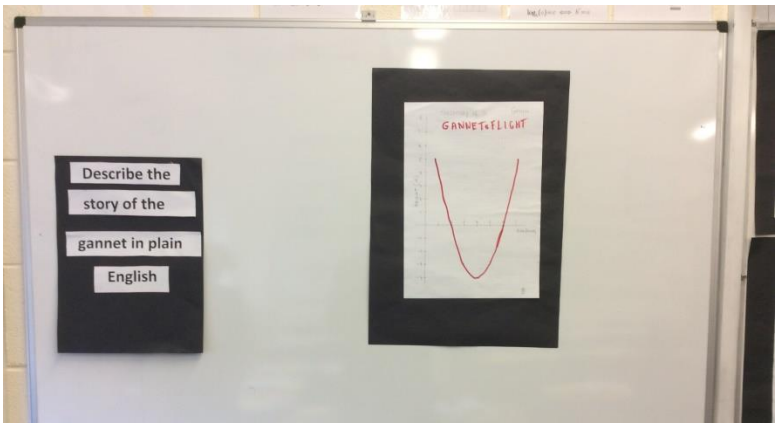
Problem Solving 2 (10 - 15min) **Task 2** Tell the same story with more detail using the parameters height and time Anticipated responses ● After 1 second he is five metres in the air/above the water. ● After 2 seconds he hits the water, so his height is 0. ● After three seconds he is 3 metres underneath the water. ● After 4 seconds he is 4 metres underneath the water. This is the deepest he goes and now he starts to come back up. ● After 5 seconds he is now 3 metres deep, i.e. 3 metres under the water. ● After 6 seconds, he hits the water again and comes up through it/breaks the water. ● After 7 seconds he is five metres above the water again and heading up ● He is heading off to space. Never to return. (infinity). **Task 3** Tell the story using as many mathematical terms as possible Anticipated responses ● Roots of 2 and 6 (2 seconds and 6 seconds) ● Minimum value (4, -4) - Turning point ● Starting to dive at 5 metres ● Axis of Symmetry ● Negative y - Under axis/under water ● Positive y - Above axis/above water	Now, ask the students to tell the story of the gannet within the parameters of height and time. Tell student they have 10 minutes to do this task . Finally, ask the students to analyse the graph using as many mathematical terms as possible in five minutes.	As per problem solving 1.

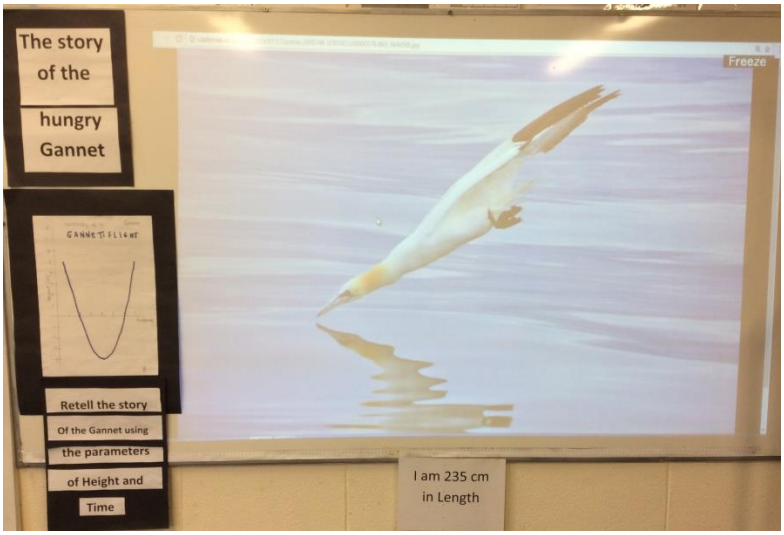
Ceardaíocht 2 (15 - 20 min) Specific students are asked in a specific order to tell their part of the story. Card is used to record their input. (See boardwork) Please use terms that you have learned relating to quadratic expressions	Teacher will ask individual students to take observation they have made at each of the above three steps and write it on the card. Individual student will then be asked to come up to the board after finished both task 2 and 3. Students will be asked to describe and explain their findings Higher order questions may be asked linking junior cert Science in terms of Speed and Velocity and how to calculate each term for task 3 Minimum point and minimum value. Turning point Distance time speed as a relationship between distance and time. SLOPE ?? steepness Increasing decreasing Time Total journey AT the end Q What parameters/variables of this ASSUMED story would affect	**As per Ceardaíocht 1.** *Possible assessment questions arising from observations may include:* From what height did the gannet start the dive? Teacher asks how long did it take the gannet before it entered the water? How long was the bird under the water? After how many seconds did the gannet come back out of the water? What was the deepest dive of the gannet? After how many seconds was the gannett at it deepest? Describe the actions of the gannet in the first 5 seconds? Using SHOW ME BOARDS Is the dive steep? Is the upward journey steep? Teacher should observe if students are defending their own ideas and whether they are responding to each other's ideas. *We expect students to use the terms:* FACTORS ROOTS t axis represents…. h axis represents….

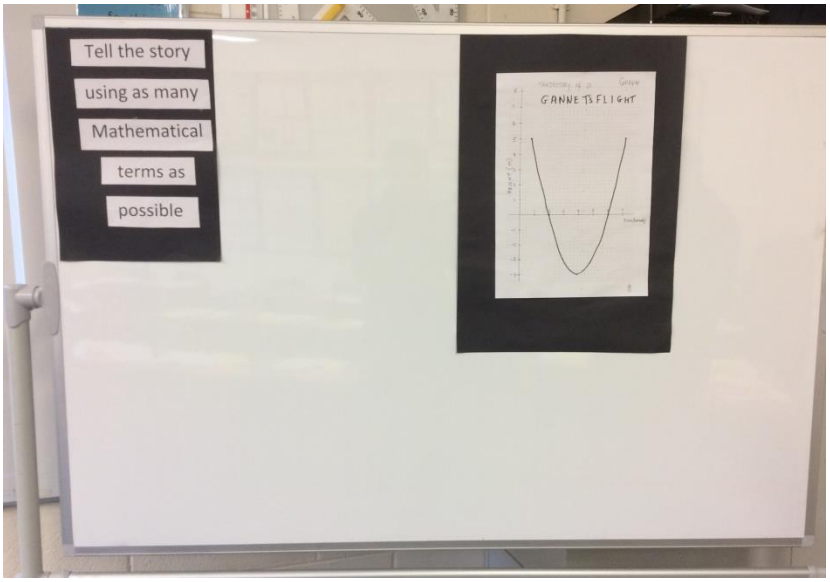
Summing up & Reflection Now for those gannets in action! http://video.nationalgeographic.com/video/weirdest-cape-gannets. Students asked about the whether a gannet follows a quadratic curve. Topic is summarized and students are asked to reflect on their learning by completing a Reflection Questionnaire, including reflection on: ● What they learned ● What is still unclear ● How they feel about the topic ● If they feel they can interpret: ○ when the gannet was going down ○ When the gannet was going up ○ The relationship between the steepness of the graph and the speed of the gannet. **Extension/Homework** Students are given another graph relating to profit made by a business over a number of weeks. Can they tell the story of that business using both mathematical knowledge and reasoning but also explaining in plain english.	Following discussion and arrangement of the coloured card on the board, the teacher or students will use the final layout on the Board to summarise the story of the gannet. Teacher will issue a Reflection Questionnaire on which students will record responses to the questions above.	Reflections used for assessment.

10. Board Plan

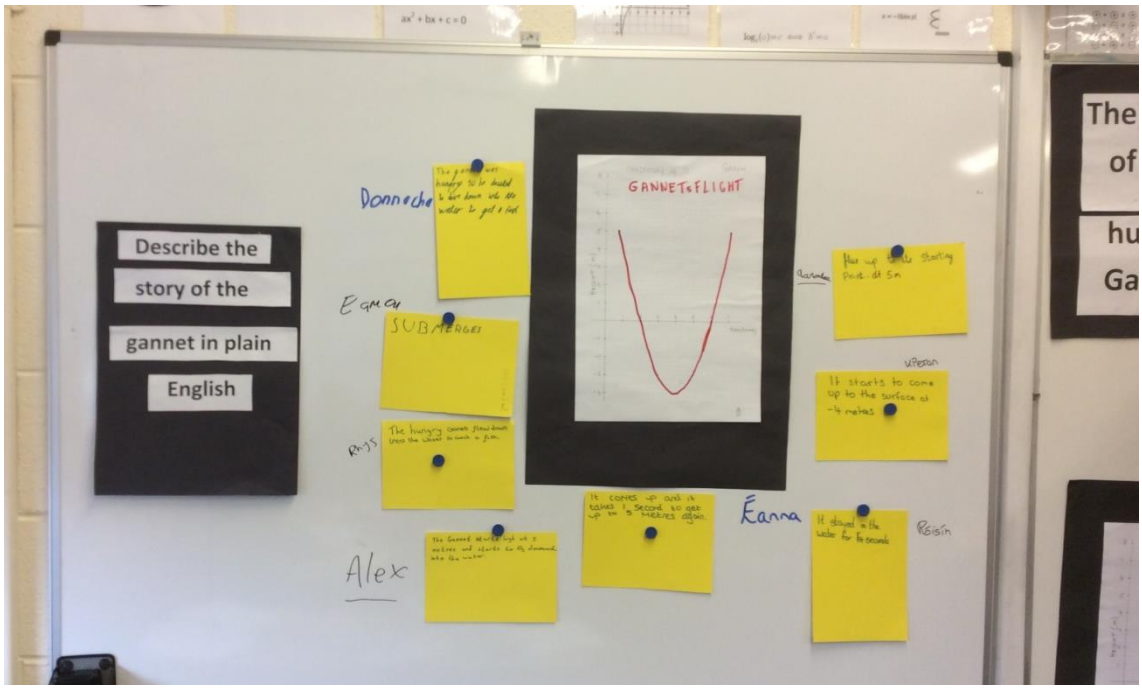
Before

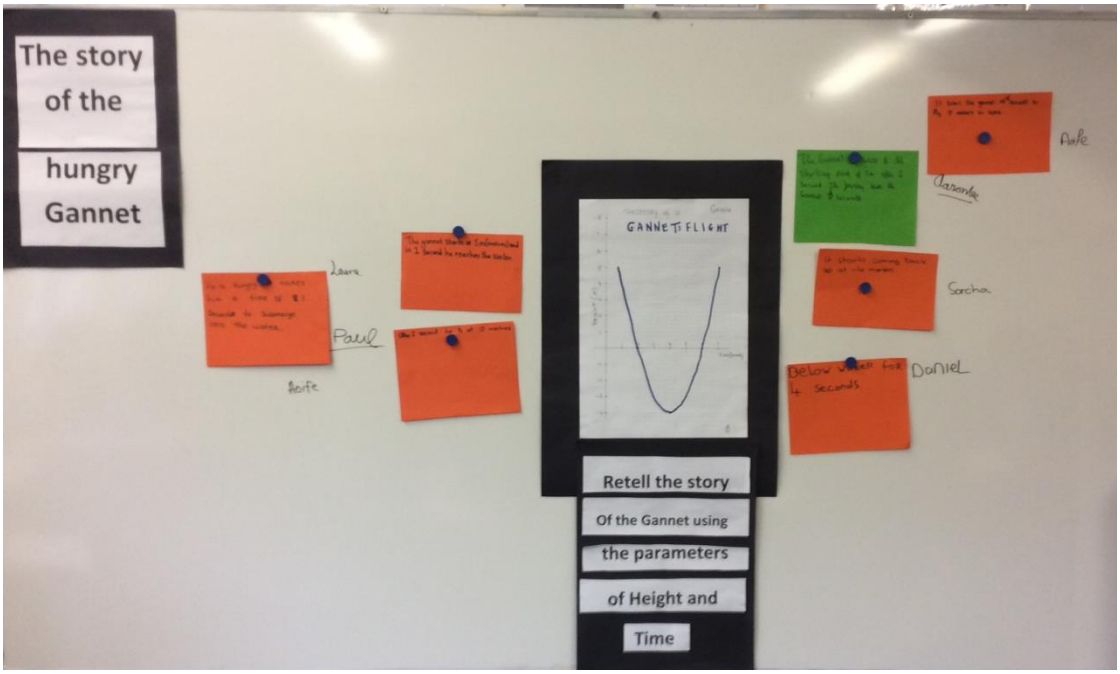


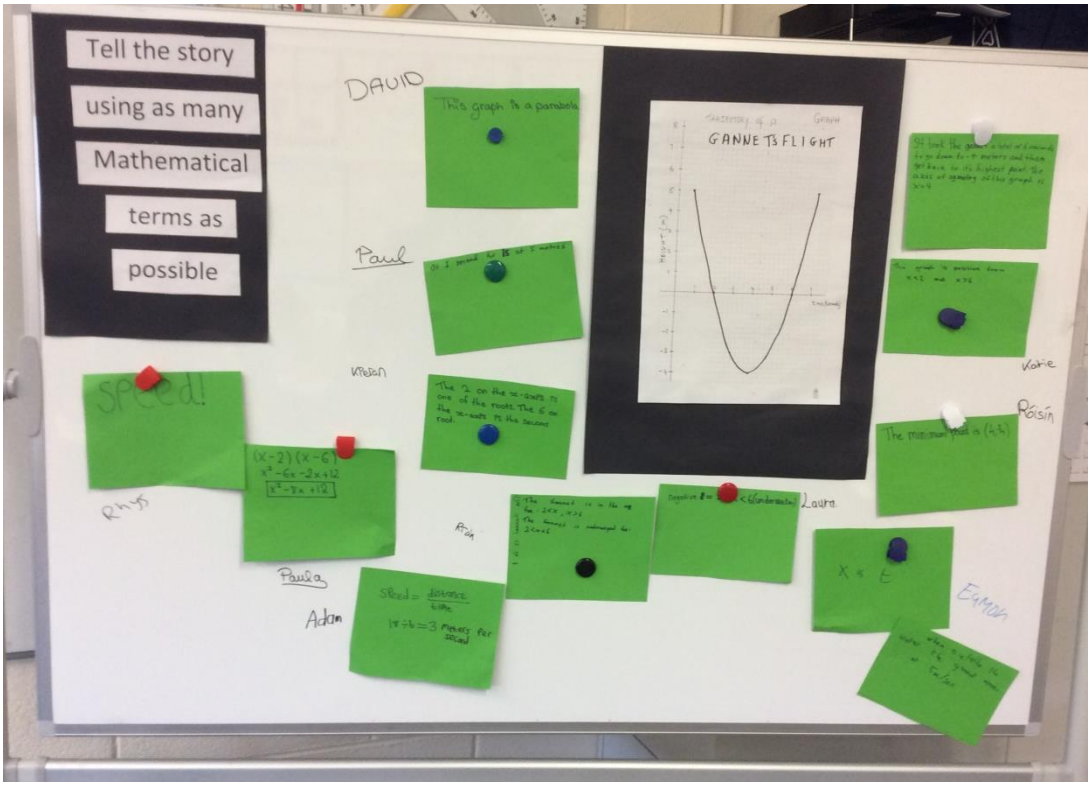

The story
of the
hungry
Gannet
GANNET FLIGHT
Retell the story
Of the Gannet using
the parameters
of Height and
Time
Freeze
I am 235 cm
in Length


Tell the story
using as many
Mathematical
terms as
possible
GANNET FLIGHT

After


Describe the
story of the
gannet in plain
English
GANNET FLIGHT
Donnacha
SUBMERGES
Alex
Eanna
Róisín
The
of
hu
Ga


The story
of the
hungry
Gannet
GANNET FLIGHT
Retell the story
Of the Gannet using
the parameters
of Height and
Time
Laura
Paul
Aoife
Sorcha
Daniel



11. Evaluation

The consensus was that the lesson was successful, and the goals stated by the group at the outset had been met. It was clear to the team that the students were engaged in the task from the beginning. The students responded well to the prior knowledge and once the task was set they engaged immediately in trying to find solutions. All students were successful in writing at least one aspect of the story of the gannet for all three questions.

Once the Ceardaíocht began, and the students saw their classmates work, they were confident in suggesting further solutions and in answering questions posed by the teacher. The class learned from their colleagues presentations of their work to the class. 30 student observations were presented to the class. This gave the students a comprehensive picture of story of the gannet and of the quadratic graph.

The next stage of the Ceardaíocht went very well. A number of the students (at all levels) recognised speed and velocity as aspects that could be developed. Overall, the students work and their reflections demonstrated a strong knowledge and understanding of quadratic functions and their graphical representations. This lesson meets its research targets.

12. Reflection

The team had hoped to observe students understanding of the relationship between the graph and the diving gannet. The team also hoped to observe students learning from one anothers input to develop whole class learning. It was also hoped that the diving speed of the gannet would be recognised.

A very engaged class with significant mathematical discussion was observed. The students understanding of the topic developed throughout the class and is strong. Students worked excellently throughout the class. The majority of the work was individual work and students were immersed in the problem in trying to find new words, relationships and mathematical expressions to describe the gannet's flight. Students written reflections indicated that the topic was clear and that they found the class engaging and rewarding. Students enjoyed coming to the board and discussing their points.

The lesson objectives were achieved. The synopsis of the board work volunteered by the students after all three questions illustrated that they were engaged and had a deep understanding of the quadratic graph. It was observed that the engagement and the understanding developed by students in the first two questions helped students to answer and engage with the 3rd question. It was also observed that a number of students used the parameters of time and height to answer question 1.

Misconceptions observed were that x was being used instead of t for the horizontal axis. Other students observed that the gannets trip took 7 seconds as opposed to 6 seconds. These misconceptions were corrected by the students colleagues.

It was observed that students didn't break down the journey of the gannet on a second by second basis and that this could be further encouraged by rewording question 2 or question 1.

The boardwork, picture, video and coloured card added immensely to the lesson.
The research lesson took an hour to complete.

It was stated by team members that a derivative of this lesson could be used in senior cycle as an introduction to calculus as the changing velocity of the gannet would be easily understood.