Lesson Research Proposal for 5th Year Ordinary Level.
Coordinate Geometry

For the lesson on 19/01/2018
At Hartstown Community School, Claire Moore's class
Instructor: Claire Moore
Lesson plan developed by: Claire Moore, Rowan Webb, Darragh Connolly and Rebecca McNulty

Title of the Lesson: The story of two drones.

Brief description of the lesson

Students will be presented with a real life scenario of two drones on a collision course. Using coordinate geometry, functions and/ or algebra to find the point of intersection of the two flight paths in as many ways as possible.

Research Theme

A Lesson Study usually has two research objectives:
(1) At Hartstown Community School, we want students to:

a) grow as learners through respectful interactions and experiences that are challenging and supportive
b) Enjoy their learning, are motivated to learn, and expect to achieve as learners.

As mathematics teachers, we will actively support the achievement of these goals by paying attention to the following entry points in my every day classes:

a) Creative attention towards the problem
   Attempt to create problems that arouse the students' interest and motivation, and stimulates their mathematical thinking process as well. Problems that allow students to find a solving method according to his / her abilities.
b) Support for encouraging a problem-solving mindset
   Ask questions and give supportive comments and advice to stimulate the students' thinking process. Use teaching materials and aids that support the thinking process. Be mindful to allot sufficient "thinking-time".

Background & Rationale

1. Why we chose this topic

The teaching of simultaneous equations is important subject material from the point of view that it brings together previously-learned materials and extends students to understand them at an even deeper level[1].
It is commonly recognised that when tackling problems algebraically, students experience difficulty in changing "real" situations into mathematical statements. In second year, differences in the students' abilities become more pronounced and there are quite a few students who do not fully understand what a mathematical statement is and how to set one up.
For these reasons when it comes to teaching simultaneous equations one cannot simply teach it as a procedure, rather students need to have a good grounding in the concept of forming equations ,graphing them and solving them either graphically or algebraically ,multiple methods of solving one problem with two variables.

2. Our research findings

Through discussions of members of the maths department we realise that our teaching of simultaneous equation was imbalanced towards procedures and we introduced a common practice to ensure all students used the same method no matter what type of equations presented themselves in a question . We now use the manipulation method and substitution method as it helps with manipulation of Formulae ,finding slope in the form y=mx+c ,when one equation is quadratic i.e Patterns and sequences or proving a line is a tangent to a circle or just finding the points of intersection between a line and a circle.

Because of these deficits we have decided to commence teaching of simultaneous equations using a problem-solving situation which naturally gives rise to this concept.

In designing the research lesson we believe it is important to engage students enthusiastically with the subject matter. The lesson proposal tries to devise creative ways to make it easier to comprehend this concept by illustrating the problem and using suitable teaching aids. The approach depends on allotting students plenty of time to think about the problem and figure it out on their own.

3. *References*

Junior Certificate Mathematics Syllabus 2016
Found at ; https://www.curriculumonline.ie/getmedia/4f6cba68-ac41-485c-85a0-32ae6c3559a7/JCSEC18_Maths_Examination-in-2016.pdf

Leaving Certificate Mathematics Syllabus 2015
Found at;
https://www.curriculumonline.ie/getmedia/f6f2e822-2b0c-461e-bcd4-dfcde6decc0c/SCSEC25_Maths_syllabus_examination-2015_English.pdf

Hartstown Community School Leaving Cert Maths Competency Exam
Available upon request.

4. *Relationship of the Unit to the Syllabus*

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Co-ordinating the plane. Properties of lines and line segments including midpoint, slope, distance and the equation of a line in the form. y - y1 = m(x - x1) y = mx + c. ax + by + c = 0 where a, b, c, are integers and m is the slope of the line. Intersection of lines. Parallel and perpendicular lines and the relationships between the slopes. − explore the properties of points, lines and line segments including the equation of a line − find the point of intersection of two lines − find the slopes of parallel and	explore the properties of points, lines and line segments including the equation of a line − find the point of intersection of two lines − find the slopes of parallel and perpendicular lines	– select and use suitable strategies (graphical, numerical, algebraic, mental) for finding solutions to • simultaneous linear equations with two unknowns and interpret the results • one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is ± 1 in the

perpendicular lines Patterns and the rules that govern them; students construct an understanding of a relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output. − use tables to represent a repeating-pattern situation − generalise and explain patterns and relationships in words and numbers − write arithmetic expressions for particular terms in a sequence Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. Students look at various patterns and make predictions about what comes next. − use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations (exponential relations limited to doubling and tripling) − develop and use their own generalising strategies and ideas and consider those of others − present and interpret solutions, explaining and justifying methods, inferences and reasoning		linear equation) and interpret the results. – discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions
Finding formulae Ways to express a general relationship arising from a pattern or context.	− find the underlying formula written in words from which the data are derived (linear relations) − find the underlying formula algebraically from which the data are derived (linear, quadratic relations) 4.4 Examining algebraic relationships Features of a relationship and how these features appear in the different representations. Constant rate of change: linear relationships. Non-constant rate of change: quadratic relationships. Proportional relationships. − show that relations have	

Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
From Junior Cert ● Coordinate geometry ● Algebra ● Functions	− explore the properties of points, lines and line segments including the equation of the line. − find the point of intersection of two lines -decide if two linear relations have a common value − investigate relations of the form y=mx and y=mx +c -decide if two linear relations have a common value − interpret simple graphs − discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula −plot points and lines -draw graphs of the following functions and interpret equations of the form f(x) = g(x) as a comparison of functions −use graphical methods to find approximate solutions where f(x) = g(x) and interpret the results	– select and use suitable strategies (graphical, numerical, algebraic, mental) for finding solutions to • simultaneous linear equations with two unknowns and interpret the results • one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is ± 1 in the linear equation) and interpret the results – interpret equations of the form f(x) = g(x) as a comparison of the above functions

6. Goals of the Unit

- Coordinate the plane
- Plot a point
- Construct a line/ lines
- Find the midpoint of a line
- Find the distance between two points
- Find the slope of a line (using a graph and the formula) + link it to parallel/ perpendicular lines
- Find the equation of a line using graph + formula
- Interpret a graph
- Recognise the link between a graph and real life
- Find the point of intersection from a graph visually/ algebraically
- Find the area of a triangle (where one of the vertices is the origin + where none of the vertices is the origin)

7. Unit Plan

Lesson	Learning goals and tasks
1	Coordinate the plane + Plot points + Join points (to form a line)
2	Find the midpoint of a line + Find the distance of a line
3	Find the slope of a line from a graph/ formula (rise / run)
4	Find the equation of a line from a graph ($y = mx + c$)
5	Find the equation of a line ($y – y1 = m [x – x1]$)
6	Research Lesson H/W: Use knowledge/ skills to find the points of intersection of a circle and a line. 19th January 2018 - 09:00 - 09:40
7	Identifying parallel lines by showing you can't find a point of intersection (graphically or algebraically)

8	Identifying perpendicular lines
9	Find the area of a triangle (one vertex is the origin)
10	Find the area of a triangle (no vertices are the origin)

8. Goals of the Research Lesson:

- Interpret a graph
- Recognise the link between a graph and real life
- Find the point of intersection from a graph visually/ algebraically
- Students will have a conceptual understanding of the meaning of solutions to simultaneous equations

b) **Key Skills;**

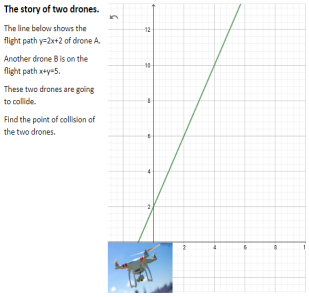
- Communicating
- Being Creative
- Managing Information & Thinking
- Working with Others
- Staying Well
- Being Numerate
- Being Literate
- Managing Myself

Statements of Learning;

15. Recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. Describes, illustrates, interprets, predicts and explains patterns and relationships.
17. Devises and evaluate strategies for investigating and problem solving using mathematical knowledge reasoning and skills.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
	The teacher will explain point of collision and a flight path in the context of coordinate geometry.(5 Mins)	The teacher will look for any of the following solutions; graphical, algebraic, trial and error or using functions. The observer will be looking for students to have; 1. graphed the line, identifying the point of intersection and/ or 2. Solved the problem simultaneous equati therefore connecting two.
Introduction PowerPoint presentation. Show first slide of two drones on a collision course. Explain that we are going to use coordinate geometry to find the point of collision of the two drones. Explain the terms flight path and collision. Show the next slide of the graph of the flight path of drone A. Pose the problem.(5 mins)		We are giving the students a relatable real life scenario when posing the question while simultaneously meeting project and curriculum goals.

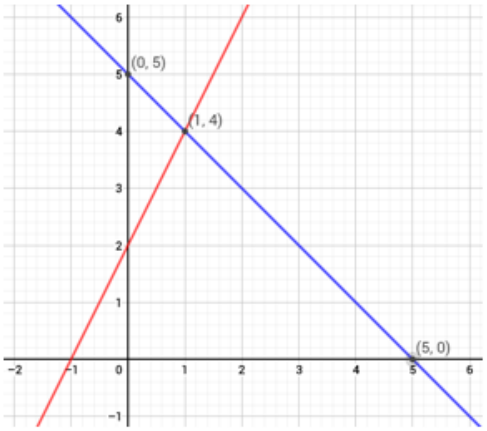
Posing the Task **The story of two drones.** The line below shows the flight path y=2x+2 of drone A. Another drone B is on the flight path x+y=5. These two drones are going to collide. Find the point of collision of the two drones. Student problem solving segment. Students asked to solve the problem by coming up with as many solutions as possible. Teacher will patrol the room observing students work and looking for expected solutions. The PowerPoint will prompt any students who do not understand the concept of collisions and flight paths without providing examples that will restrict the students' way of thinking about the problem.	The problem will be presented using a PowerPoint as well as a hard copy given to each student.  The story of two drones. The line below shows the flight path y=2x+2 of drone A. Another drone B is on the flight path x+y=5. These two drones are going to collide. Find the point of collision of the two drones. **[copy of students' question sheet]**	

Possible Solutions

Graphically

1. Using x and y intercepts

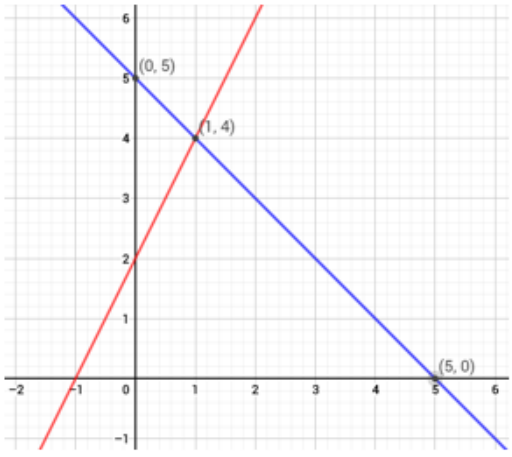
$$x + y = 5$$
$$x = 0$$
$$y = 5$$
$$(0, 5)$$
$$y = 0$$
$$x = 5$$
$$(5, 0)$$



Point of intersection $(1, 4)$

2. Using $y = mx + c$

$$x + y = 5$$
$$y = -x + 5$$
$$m = -1$$
$$c = 5$$



Point of intersection $(1, 4)$

The students will be prompted by the teacher if they are on the wrong course get stuck at any point during the lesson.

The teacher will look out for;

- Use of the wrong formula.
- Mixing up x and y.
- Only finding x or y.
- Drawing the graph wrong.

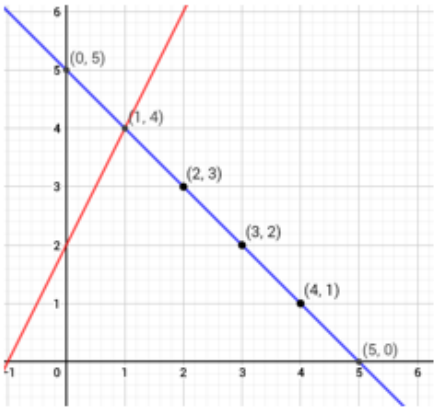
The teacher will leave some misconceptions to be addressed at the ceardaíocht discussion section of the lesson.

3. Plotting Points

$$y = -x + 5$$

x	$-x+5$	y
0	0+5	5
1	-1+5	4
2	-2+5	3
3	-3+5	2
4	-4+5	1
5	-5+5	0

(0,5) , (1,4) , (2,3) , (3,2) , (4,1) , (5,0)



Point of intersection $(1, 4)$

Algebraically

4. Substitution for y

$$y = 2x + 2$$

$$x + y = 5$$
$$y = -x + 5$$

$$2x + 2 = -x + 5$$
$$3x = 3$$
$$x = 1$$

$$y = -1 + 5$$
$$y = 4$$

Point of intersection $(1, 4)$

5. Substitution for x

$$y = 2x + 2$$
$$x = \frac{y-2}{2}$$
$$x + y = 5$$
$$\frac{y-2}{2} + y = 5$$
$$y - 2 + 2y = 10$$
$$3y = 12$$
$$y = 4$$
$$x = \frac{4-2}{2}$$
$$x = 1$$

Point of intersection $(1, 4)$

6. Elimination

$$y = 2x + 2 ①$$
$$x + y = 5 ②$$

$$y = 5 - x ②$$
$$② \times -1 \quad -y = -5 + x$$

$$y = 2x + 2$$
$$\underline{-y = x - 5}$$
$$0 = 3x - 3$$
$$3x = 3$$
$$x = 1$$

$$y = 2(1) + 2$$
$$y = 4$$

Point of intersection $(1, 4)$

7. Elimination

$$y = 2x + 2 ①$$
$$x + y = 5 ②$$

$$2x - y = -2 \quad ①$$
$$x + y = 5 \ ②$$
$$② \times -2 = \ -2x - 2y = -10$$

$$2x - y = -2$$
$$\underline{-2x - 2y = -10}$$
$$-3y = -12$$
$$y = 4$$

$$x + y = 5$$
$$x + 4 = 5$$
$$x = 1$$

Point of intersection $(1, 4)$

8. Trial & Error

$$x + y = 5$$
$$1 + 4 = 5$$

$$y = 2x + 2$$
$$2(1) + 2 = 4$$

Point of intersection $(1, 4)$

9. Functions

$$y = -x + 5$$

x	$-x+5$	y
0	0+5	5
1	-1+5	4
2	-2+5	3
3	-3+5	2
4	-4+5	1
5	-5+5	0

(0,5) , (1,4) , (2,3) , (3,2) , (4,1) , (5,0)

$y = 2x + 2$

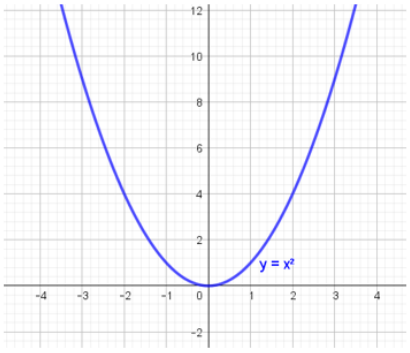
x	$2x + 2$	y
0	2(0)+2	2
1	2(1)+2	4
2	2(2)+2	6
3	2(3)+2	8
4	2(4)+2	10
5	2(5)+2	12

(0,2) , (1,4) , (2,6) , (3,8) , (4,10) , (5,12)

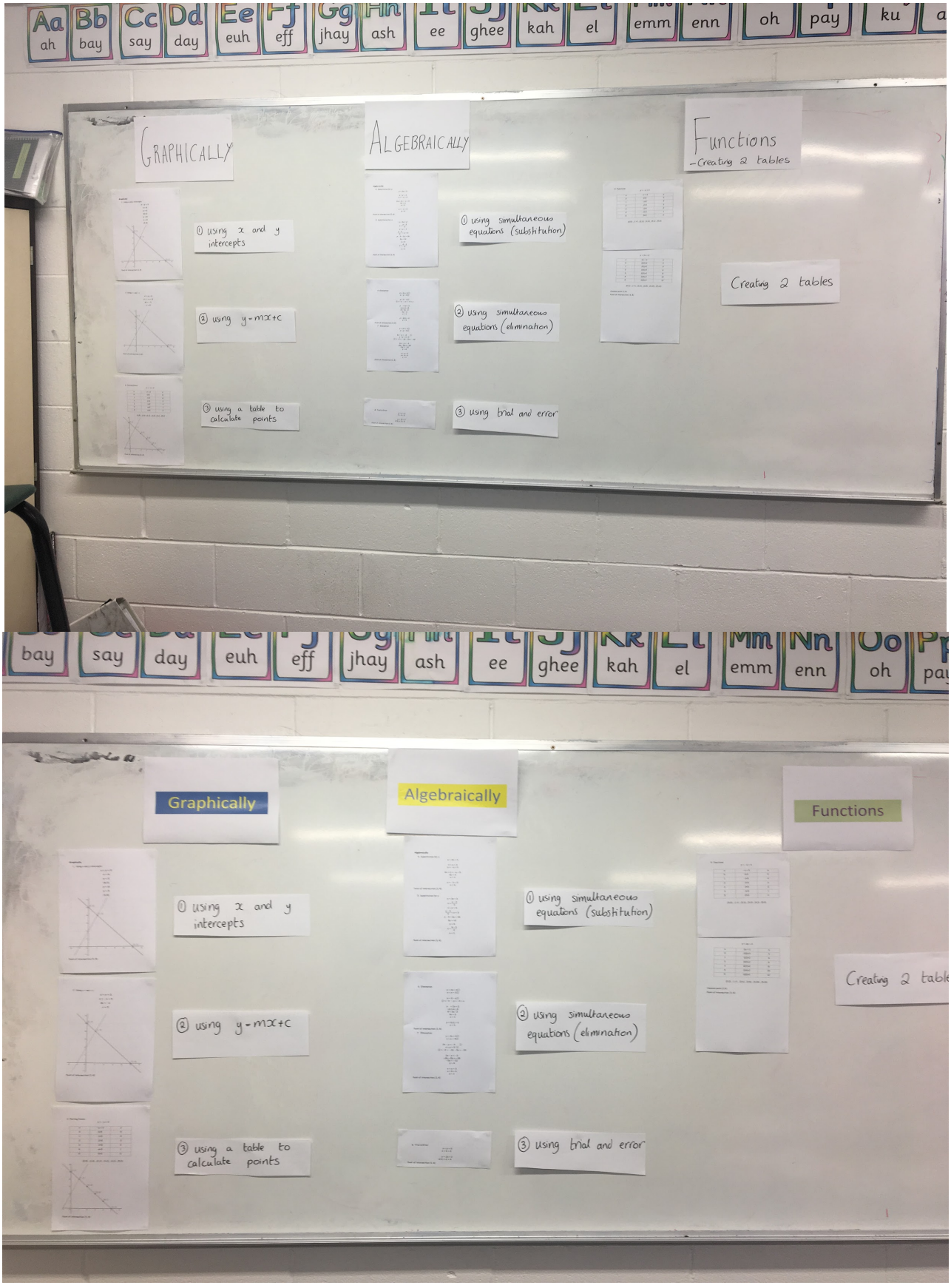
Common point (1,4)

Point of intersection $(\mathbf{1}, \mathbf{4})$

Ceardaíocht /Comparing and Discussing 1. Graphically; - using x/ y intercepts - using y = mx + c - a different way 2. Trial and Error / Substitution - verifying solution from graph 3. Simultaneous Equations 4. Functions - Tables (possibly using calculator)	The problem can be answered using a variety of ways from topics already covered top date. - **Graphically** prompt questions about drawing the 2nd drone's flight path. - **Algebraically** "do you know any other way of finding the point of intersection/ collision?" - **Trial and error** "do you know any other way of checking if your answer is correct?" - **Functions** **"**could you track the points each drone has flown through?"	Ask the students; "in your opinion which way is the best way to solve this problem?" "will this always be the best way?" "is it important to be able to solve simultaneous equations graphically and algebraically?" "is there a quicker method between the two in this case?" "will this method always be the quicker of the two?"

Summing up & Reflection The teacher will summarise by helping the students realise how important it is for the students to be able to find the point of intersection between two simultaneous equations both graphically and algebraically. The teacher will give students a post it note each and they will have to write one thing they found easy and one thing they found difficult during the lesson on the post it. The teacher will then give students an extended homework problem to be solved from the learning done in class. **Extended Homework Task** **Question** **Given the line $y = 2x + 3$ and the curve $y = x^2$ shown below, find the point(s) of intersection**  	The teacher will relate back to the work placed on the board by students.	

Board Plan


GRAPHICALLY
ALGEBRAICALLY
Functions
-Creating 2 tables
① using x and y intercepts
② using y=mx+c
③ using a table to calculate points
① using simultaneous equations (substitution)
② using simultaneous equations (elimination)
③ using trial and error
Creating 2 tables
Graphically
Algebraically
Functions

11. Evaluation

. To evaluate the lesson, the 4 observers were each given an area of the room, each observer had seven or eight students to observe. All observers were given a seating plan and took notes of interactions between students and interactions between teacher and students. They recorded the number of students solving the problem and in what way they solved them.

.to grow as learners through respectful interactions and experiences that are challenging and supportive and to enjoy their learning, are motivated to learn, and expect to achieve as learners, were the research themes of the lesson study so evaluation of this by means of framing the reflection appropriately was important.

Students' were asked to reflect on their feelings about the class using post it notes which they handed up before the end of the lesson. A portion of post Lesson discussion centred on this theme.

c. Students completed the task and therefore were able to reflect on the mathematical goals of the lesson.

12. Reflection

a) The team were hoping to see the predicted answers to the lesson and that students attempted one or more methods. The team also hoped that the students weren't overly focused on the actual answer to the problem but more so focused on the journey to the answer.
b) The team observed students working together in groups/ pairs to come up with a variety of methods to solve the problem. Some students were observed finding the problem difficult and that students worked predominantly independent.
c) The team observed 6 students/ groups solved the problem by trial by error and the most taught concept was used initially. Solving simultaneous equations using elimination was not observed. The team discussed having numerous copies of the question and/ or answer sheets on the desk for time purposes (avoid students having to constantly draw the axes).
d) To rearrange the order of solutions.
e) -Give the students the number of possible methods to solving the answer to push them to find all.
f) -Having numerous copies of the question and/ or answer sheets on the desk for time purposes (avoid students having to constantly draw the axes).
g) -Ensure students are sitting in groups to work collaboratively when needed.
h) The research lesson was at 9:00 and some students arrived late making it difficult for them to come up with multiple solutions.