Lesson Research Proposal for 4th years - The trapezoidal rule

For the lesson on 10-01-18
At St. Louis Community School, Louise Kelly's class
Instructor: Louise Kelly
Lesson plan developed by: Gearoid O'Suilleabhain, Louise Kelly and Mairéad Quinn

The area of irregular shapes.

1. Brief description of the lesson

In this lesson students will engage with a problem to find the area of an irregular shape and by doing so will find a formal method for dealing with the area of irregular shapes.

2. Research Theme

At St. Louis Community School, we want students who:

a) engage purposefully in meaningful learning activities.
b) enjoy their learning, are motivated to learn, and expect to achieve as learners.

As mathematics teachers, we will actively support the achievement of these goals by paying attention to the following entry points in my every day classes:

a) The students understand and can explain the purpose of the learning tasks their engaged in, and can extend and develop the activity meaningfully.
b) The students are motivated to learn through having a clear sense of attainable and challenging learning outcomes.

3. Background & Rationale

1. Why we chose the topic
   The teaching of length, area and volume is an important subject material from the point of view that it brings together previously learned material and extends this to real life situations.
   When tackling area problems students have great difficulty in moving from the regular shapes to irregular or compound shapes. When it comes to teaching area it is important to simplify area so that the concept can be perceived in a simple way.

2. Our research findings
   In teaching mathematics in Transition Year, schools and teachers should focus on increasing student engagement with mathematics, and building confidence in students' mathematical abilities by re-visiting basic practical work and basic procedures.
   Transition Year is an opportunity to consolidate the mathematical knowledge and skills learned during the junior cycle, and to develop these in a manner that would act as a useful bridge to the new Leaving Certificate course.[1]

   Transition year should be used as an opportunity to highlight the importance and relevance of mathematics in the real world.[2] The topic of Area and Volume presents us with an opportunity to achieve the above.

   The focus of Transition year in St. Louis' is to allow students that opportunity to engage in challenging activities in the key areas: Number, Statistics, Geometry and Area and Volume. The

[1] www.pdst.ie
[2] www.projectmaths.ie

department believes that a deepening understanding of these areas will provide students with a stronger base for LC and develop problem solving skills.

4. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
From JC students should have the following covered: • Understand the term area as a region, and the concept of area in a mathematical sense. • Students should have a practical competence in the calculation of areas of the following shapes: circle, square, triangle, rectangle and the parallelogram. • The approximation of the area of a circle. • Area of Compound shapes composed of triangles can be calculated as a sum of those triangles. • Area of compound shapes composed of standard shapes have an area of the sum of those shapes. • Ability to access shape area formulae. • Appropriate use of units. • Relate 2D, surface area, area and nets. • Distinguish area from/and volume. • Students should be in the position to employ suitable strategies in calculation of area. • Students should be able to draw scaled diagrams of shapes.	In Transition Year, students should: • Recognize the difference between 2D and 3D. • Investigate the nets of cubes, cuboids and prisms. • Draw the nets of cubes, cuboids given their dimensions. • Construct 3D models from nets. • Calculate the surface area and volume of cubes and cuboids. • Use the trapezoidal rule to approximate the area of irregular shapes.	In the LC the students will study the following: • Axes and symmetry in simple shapes. • Calculate the area of a triangle 2D shapes and 3D solids, including nets of solids. • Problems involving perimeter, surface area and volume. • Modelling real-world situations and solving a variety of problems (including multi-step problems) involving surface areas, and volumes of cylinders and rectangular solids. • Select and use suitable strategies to estimate the area of a combination of regular and irregular shapes. • Select and use suitable strategies to find the volume and surface area of rectangular solids, cylinders shape. • Draw and interpret scaled diagrams. • Curve sketching. • Use integration to find the average value of a function over an interval. • Determine areas of plane regions bounded by polynomial and exponential curves. • Area is not taken to be a primitive term or a given property of regions.

5. Goals of the Unit

a) Students will be comfortable with area and the formulae that we use.
b) Students understand the difference between different units and their order e.g. cm compared to cm^2 compared to cm^3.
c) Students will understand the difference between two dimensional and three dimensional.
d) Students will understand the connection between real life situations and classroom procedure.
e) Students will understand and recognize the value of efficiency for calculations.

6. Unit Plan

Lesson	Learning goal(s) and tasks
1	Through investigation students look at deriving the trapezium rule.
2	Students solve problems using the trapezium rule.
3	**Research lesson students solve a problem on an irregular shape and hence derive the trapezoidal formula.**
4	Students solve problems using the trapezoidal rule
5	Students use the trapezoidal rule to calculate area of countries

7. Goals of the Research Lesson:

a) Mathematical Goals

Students will:

- Apply a number of different strategies for calculating area with irregular shapes.
- Students should understand that regular shapes maybe used to estimate area of irregular shapes.
- Compare the efficiency and accuracy of their methods.
- Develop their clarity on estimation and approximation.

b) Key Skills and Statements of Learning

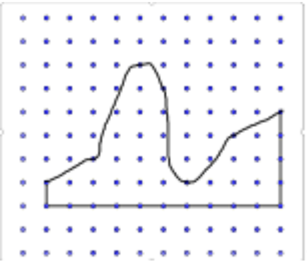
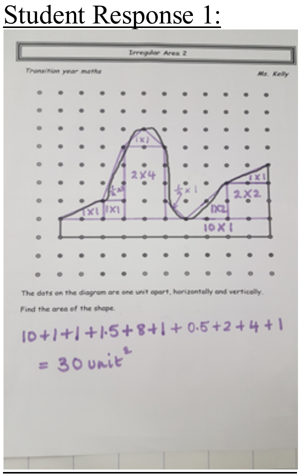
In the planning and design of this lesson the Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will implement and promote JC Key Skills in the following ways:

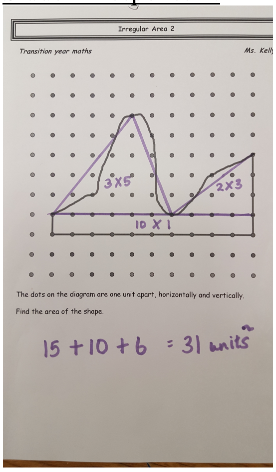
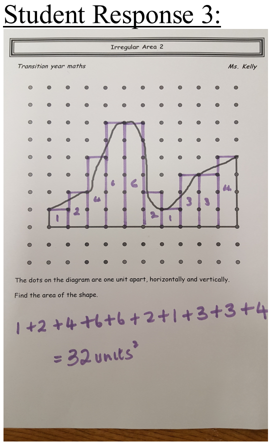
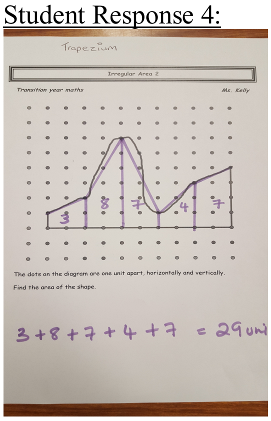
1. Being Literate: Students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: It will develop a positive disposition towards problem solving.
3. Managing Myself: Student's will have the opportunity to reflect on their own learning.
4. Staying Well: Students' confidence and positive disposition to learning will be promoted.
5. Communicating: Students will present and discuss their mathematical thinking.
6. Being Creative: Students' will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with Others: Students will learn with and from each other.
8. Managing information and thinking: Students will be encouraged to think creatively and critically.

This lesson is also designed to meet the following JC Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The students describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

8. Flow of the Research Lesson:

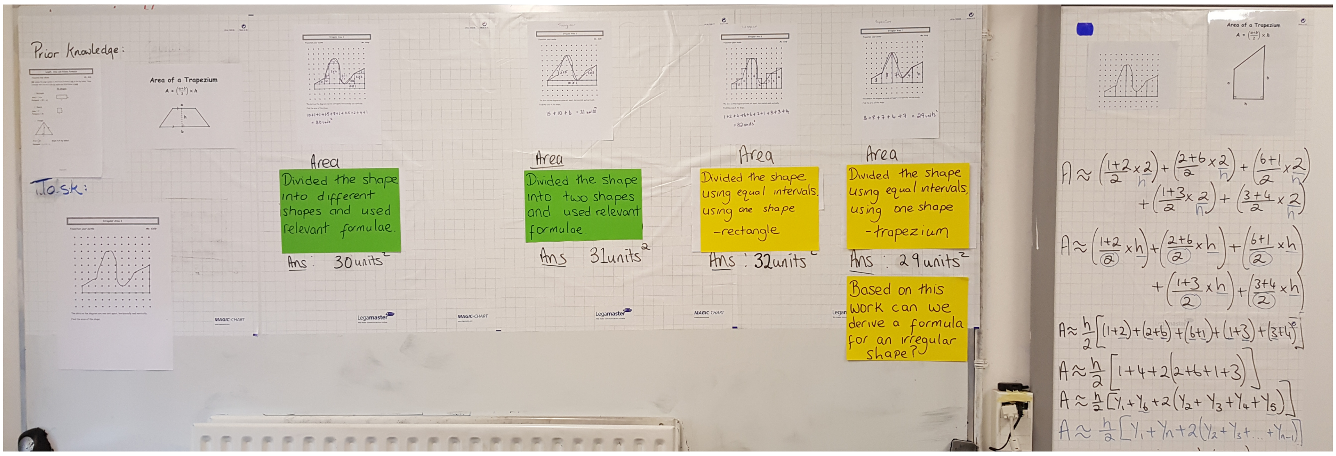
Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Today we are going to continue our work with area. Yesterday we looked at problems involving area of an irregular shape (the trapezium). We will carry out investigative work today around area and we might then link this to another subject (link to geography). Previously we looked at efficiency of methods as we look for the most accurate.	Teacher will display a poster of the trapezium formula derived previously in class and remind students of their formula sheet (containing junior cert formulae).	Are student's motivated?
Posing the Task  The dots on the diagram are one unit apart, horizontally and vertically. Find the area of the shape. Clarifying the problem: Let's go over the problem. • The dots are **<u>both</u>** horizontally and vertically one unit apart. • A box using 4 dots and stating that it is 1 unit squared • Parts of the shape are linear in sections and non-linear in other sections. • Explain your answer, this means write in words the method you used.	Hand out problem to students. Problem will be displayed on the board.	Do students understand the task? Are students eager to solve the problem? Do students understand that they may find different ways of solving this problem?
Student Individual Work <u>Student Response 1:</u> 	As teachers walk around the class, look for good examples of the various methods – using various 2D shape formulae. You might ask students with more than one shape drawn "Is this the most efficient method?" You might ask "How many different formulae are you using to find the area here?"	Are students able to tackle the problem? Do students understand that using more than one formula within the same problem is inefficient? Do students realize there are various answers depending on the method used?

Student Response 2:  	If students from the same group have responses like 1 and 2, you might ask "Which method is more efficient?" You might ask "Is it a good idea to have multiple shapes?"	Do students tend to use the same shapes? Are students aware of including space external to the shape? Are students aware of excluding space internal to the shape? Are student's adapting other student's methods?
Student Response 3:  	You might ask students "Is there a way to improve accuracy by using another 2D shape?" You might ask "Are there a lot of rectangles there?" You might ask "Can you become more efficient?"	Are students gaining better understanding working together or by themselves?
Student Response 4:  	You might ask "How many shapes are you using?" You might ask "What is the name of this shape?" You might ask students "Is this method accurate?"	Are students using the word trapezium? Are students realizing their answers have increased in accuracy?
	If students do not see uniformity or see just one shape. You might ask "In your pairs can you fill this irregular shape with a number of the same type of shape. Calculate its area, can you reduce the number of shape?"	Are students working in pairs?

Ceardaíocht /Comparing and Discussing (see board work)		
Response 1: Using multiple shapes	Response 1: Please raise your hand if you used this method, or if you would like to use this method. Why?	Are students defending their ideas? Are they responding to each other's ideas?
Response 2: Using two shapes	Response 2: Please raise your hand if you used this method, or if you would like to use this method. Why? What is the difference between the first method and the second method? Which is more efficient?	Are students recognizing that some methods are more efficient than others? (i.e. various shapes to one shape).
Response 3: Using histograms	Response 3: Please raise your hand if you used this method, or if you would like to use this method. Why? Why is this method more efficient than the previous methods?	Are students recognizing that some methods are more accurate than others?
Response 4: Using trapeziums	Response 4: Please raise your hand if you used this method, or if you would like to use this method. Why? Which is the best method? Why?	Are students responsive to the method? Do students realize this method is the most accurate and efficient?
Today we solved a problem involving the area of an irregular shape. We decided that using the trapezium method was the most efficient and accurate. Now let's use the formula that we have decided is the most efficient method (i.e. the trapezium formula) to derive a general formula for our shape.	Teacher displays the trapezium formula again (this time rotated) beside the original problem. What pattern do the students notice? What do each of the twos represent in relation to our trapezium formula? (teacher points to numerical values). Teacher labels the multiple 2, h. Since we have terms common to each area interval, what do we call this? Teacher takes out the $\frac{h}{2}$ as common. What other pattern do the students notice? Teacher rewrites the area taking out the first and last value and writing the other values in a bracket with 2 being multiplied. Teacher asks students if we can generalize this as a formula?	Do students observe that there are two twos common to each interval area? Do students notice that the multiple 2 is h and the division 2 represents the 2 in the trapezium formula? Do students recognize the HCF? Do students notice that other than the first and last value every other value appears twice? Do students understand to write a general formula we need to

	Teacher points to the y-axis on the problem sheet. Teacher assigns each if the original values $y_1 \ldots y_6$. Finally, the teacher poses the idea of an extra interval i.e. y_7 The teacher asks the students to come up with a solution to represent the various intervals we may have. The teacher writes the completed formula on the board.	introduce variables in place of the numerical values? Do students understand the formula in abstract form? Do the students understand that for the general formula the last term needs to be represented by y_n?
Summing up & Reflection Today we learned: • regular shapes may be used to estimate area of irregular shapes. • that particular methods are more efficient and accurate than others. • In this case, the trapezium is the most appropriate shape but this is not always the case. • how to find approximate answers when we have non-linear sections in 2D shapes. • how to derive the trapezoidal formula. Tomorrow we will use this formula to solve problems.		Do students agree with the teacher's view of what has been achieved in the lesson?

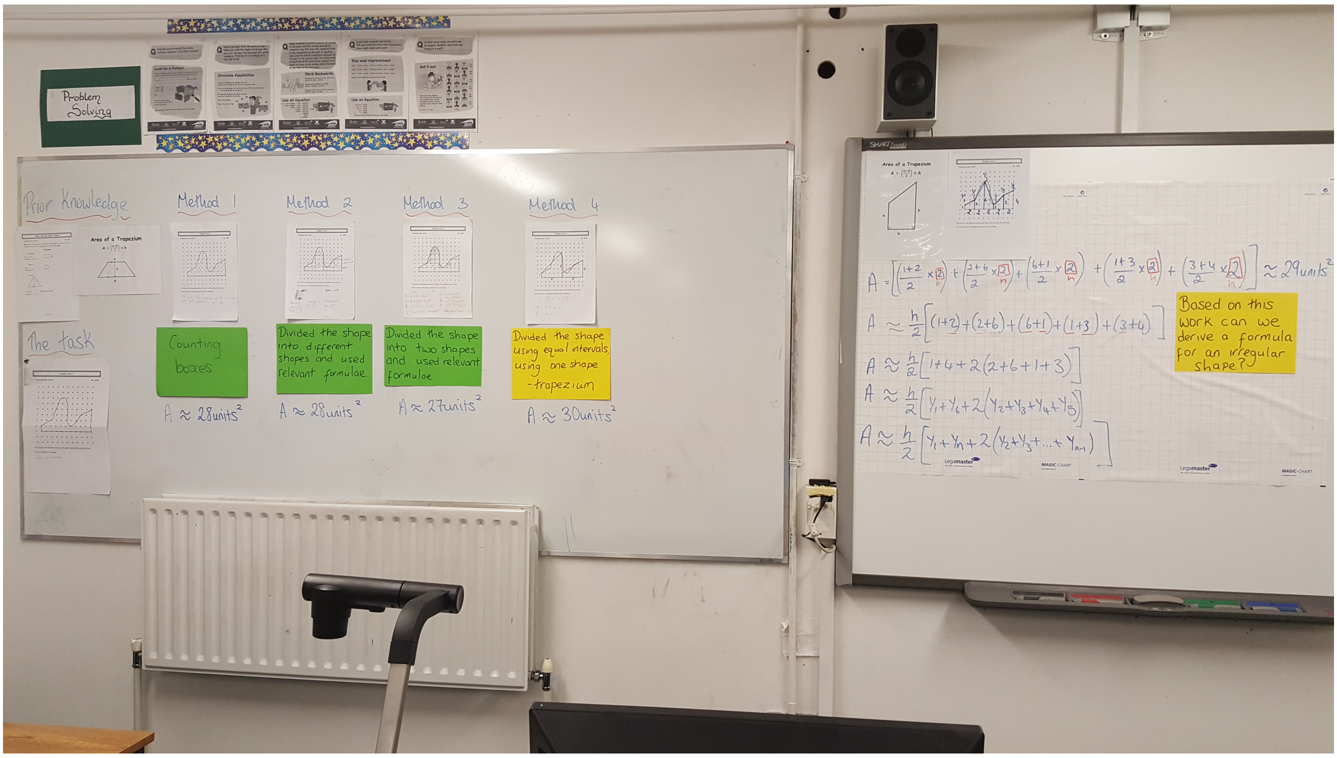
9. Board Plan



10. Evaluation

In order to evaluate the lesson, the group's main focus will be:
How did the students engage with the task? Did the students achieve the goals of the lesson?

Boardwork:



To assess this, the observers will record students work and conversations that occur as a result of the group task set. The group will use an observation sheet which will be used to gather evidence on how students engaged with the research theme, what key skills the students used when completing the task and also record whether the students achieved the goals of the lesson (see Appendix for Observation sheet).
Alongside this, teachers will record any misconceptions students had, identify students who did not understand the problem, students who self-corrected or students who peer corrected.

Reflection (Post Lesson Discussion):

The teacher that taught the lesson gave her feedback initially, she felt that the lesson had gone well, there was a slight deviation to the plan but this had allowed students to achieve the goals of the lesson. The teacher felt that students would have benefited from more time to discuss the task. The students were clearly engaged in the task and some of them believed it was a "fun lesson". The students brought a lot of terminology into the conversation about this problem without teacher prompts e.g. estimation and approximation.

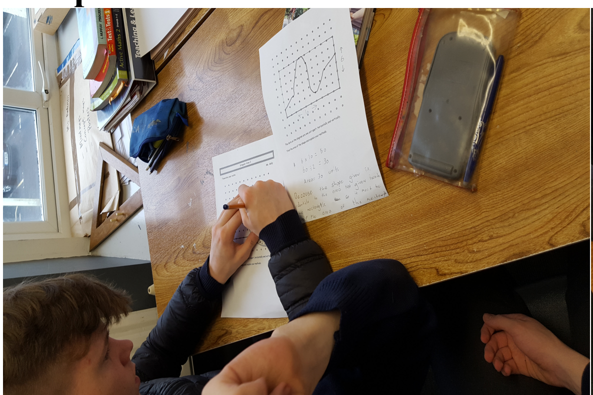
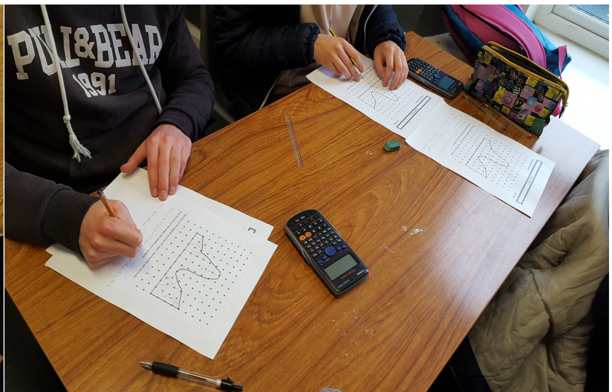
Each of the observers were very positive and enthused by the whole experience. The observers commended the teacher's delivery for its excellent organisation, the questioning during the Ceardaiocht

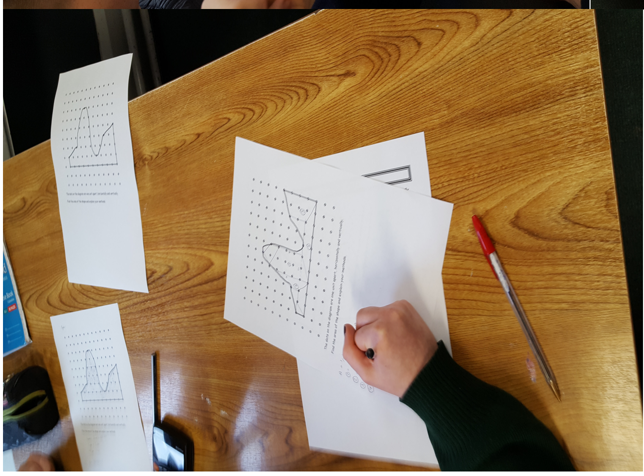
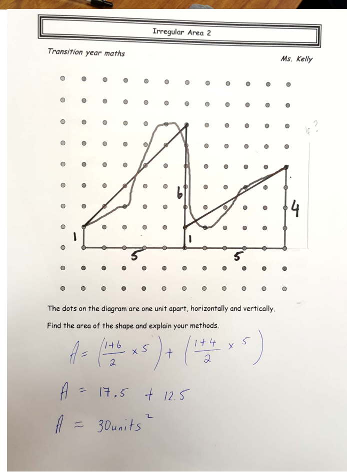
and the thorough explanation around the variables found in the derived formula.
The evidence gathered showed students engaged in a very rich learning experience. Students were heard discussing what area actually was? Explaining and correcting their peers if they misunderstood an aspect of this e.g. a weaker student believed the area of a box that had 3x2 dots was $6u^2$, they were corrected by their group and told area was about the space it took up.

The evidence gathered supported this problem as a group task, as there was a lot of discussion about how they would go about dividing the shape up. It was observed however that after 10 min, many of the groups had come to a progressive standstill, one suggestion was that some of the stronger students may have had their progress hindered through too much discussion. As a result of this, the teacher adjusted the plan and introduced that idea of uniformity to the lesson, and questioned whether they could be more efficient by using the same size of shape. This scaffolding enabled some of the stronger students to solve this task through the use of the trapezium, thus creating a set of solutions which would allow the teacher to achieve the goal of deriving the formula.

The observers commented that it was a privilege to be part of this lesson and see the rich conversation about area and also to see the many different approaches that students starting from. There were some similarities in the groups many of the students were drawn to divide the shapes horizontally; they did not approach it from the vertical perspective. There was another observation that some groups preferred to work with one page and would rub out work rather than use an extra sheet.
Overall, the students achieved the goals of the lesson, they used a range of different shapes to calculate the area, through the Ceardaíocht students considered estimation and approximation. They finished on the deriving the trapezoidal rule, and its value as an efficient tool that may be used to calculate areas of irregular shapes.

Samples of Students Solutions:

Appendix 1: Observation Record

Observation: Lesson Research Proposal for 4th years – The trapezoidal rule

Research Theme:

c) engage purposefully in meaningful learning activities.
d) enjoy their learning, are motivated to learn, and expect to achieve as learners.

Goals of the Lesson: Students will be able to:

- apply a number of different strategies for calculating area with irregular shapes.
- compare the efficiency and accuracy of their methods.
- develop their clarity on estimation and approximation.

Key Skills to be developed during lesson:

1. Being Literate: Students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: It will develop a positive disposition towards problem solving.
3. Managing Myself: Student's will have the opportunity to reflect on their own learning.
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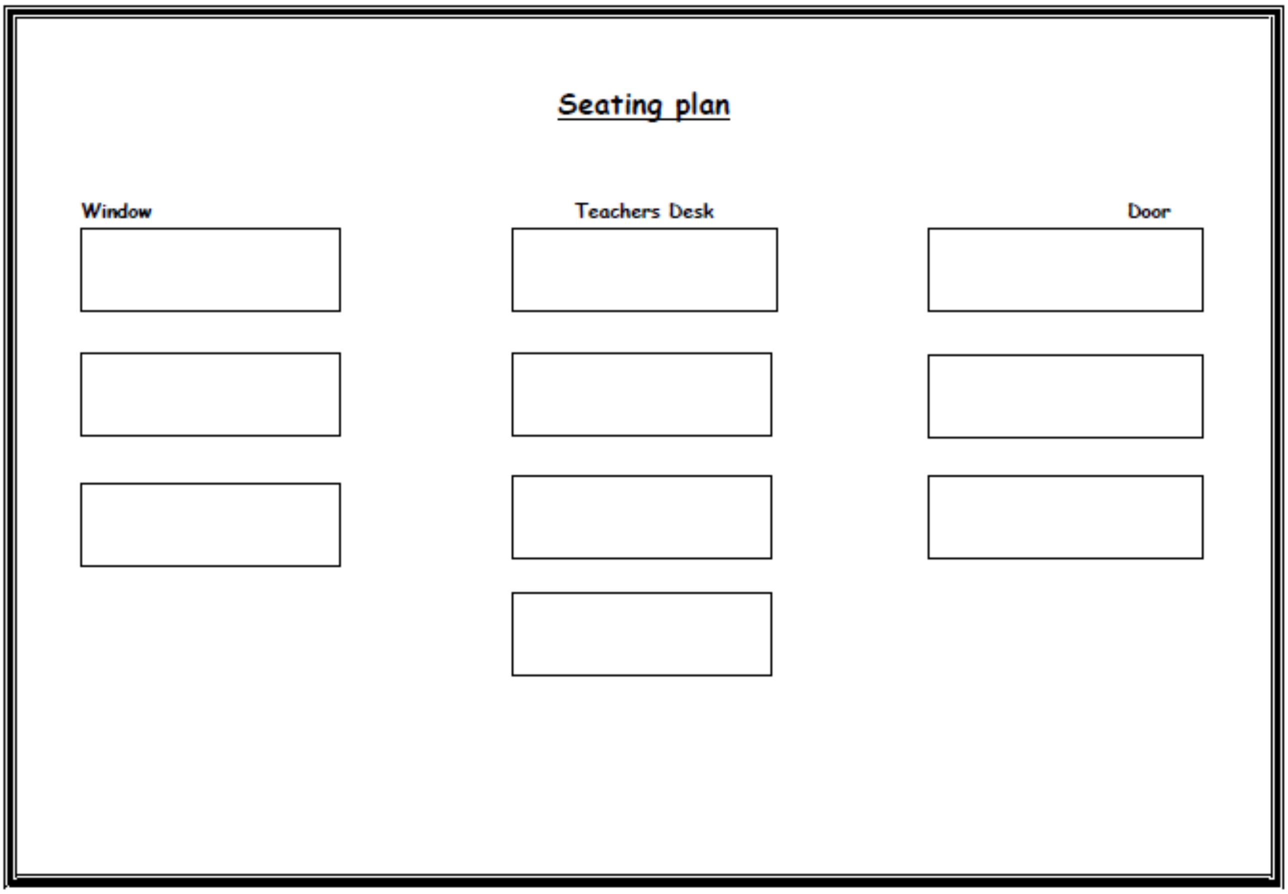
Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8

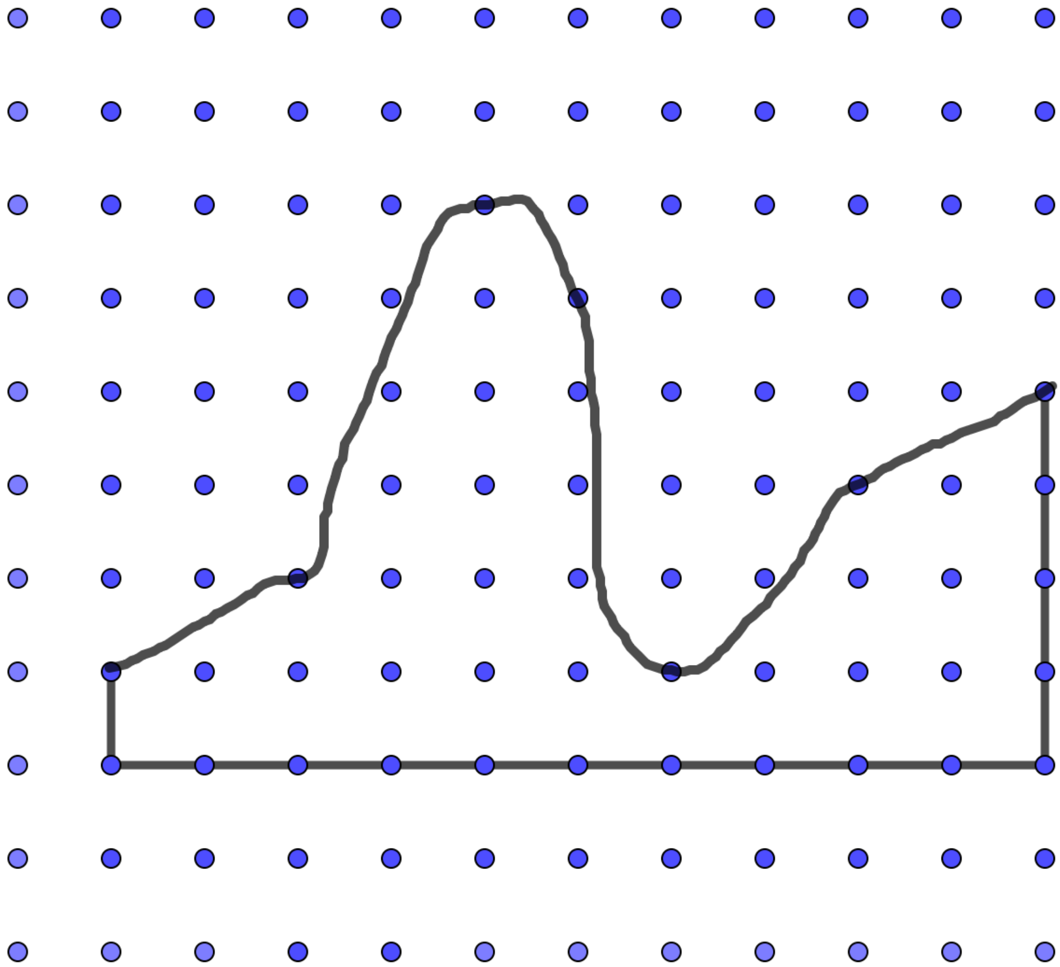
Appendix 2



Appendix 3:

Irregular Area 2

Transition year maths *Ms. Kelly*



The dots on the diagram are one unit apart, horizontally and vertically.
Find the area of the shape and explain your methods.