Lesson Research Proposal for 2018

For the lesson on 26/1/18
At Gaelcholáiste Choilm class; 5th year higher level Mathematics class
Instructor: Tara Costelloe
Lesson plan developed by: Michelle Sliney, Dan Murphy

1. Title of the Lesson: Unpacking 3D Shapes

2. Brief description of the lesson

Students will examine a square based pyramid, identify all 2D shapes possible using the points given and find the areas of these shapes.

3. Research Theme

We want our students to enjoy learning, to feel motivated to learn and to expect to achieve. We want our students to have a sense of ownership of their work and to enjoy engaging and persisting with increasingly challenging work.

Specifically we want to give our students the skills and therefore the confidence to solve 3D trigonometric problems. Our objective is to explore this approach to enable students to break down a 3D shape into simpler 2D shapes so that students can transfer and apply skills previously learned.

4. Background & Rationale

(a) This area of learning is on the Leaving Certificate Higher Level syllabus.
(b) This area has been a concern amongst Leaving Cert Higher Level teachers in Coláiste/Gaelcholáiste Choilm over past number of years. This area has been identified as one of the biggest challenges of the Leaving Cert Higher level course and tends to be an area where students encounter difficulties in state exams.
(c) We aim to provide students with opportunities to understand how to approach these problems and hence give them the confidence to tackle similar problems in different contexts.
(d) Students need to develop skills to visualise 3D shapes and to identify 2D shapes within the 3D shape. In particular they need to able to identify right angles in the 2D representation of the 3D shape. These right angles are not always apparent so students need practice in identifying perpendicular sides of a shape.
(e) Students need exposure to these types of problems and should experience physically constructing the shapes to aid their visualisation
(f) Equipping students with these skills should also aid students as they progress to third level education, particularly if they choose to pursue further study in mathematics or areas of applied mathematics.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Junior Cert Syllabus **Right-angled triangles**. – apply the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances **Trigonometric ratios.** – use trigonometric ratios to solve problems involving angles (integer values) between 0˚ and 90˚ **Working with trigonometric ratios in surd form for angles of 30˚, 45˚ and 60˚** – solve problems involving surds **Right-angled triangles.** – solve problems involving right angled triangles **Decimal and DMS values of angles.** – manipulate measure of angles in both decimal and DMS forms	**Leaving Certificate Syllabus** Use of the theorem of Pythagoras to solve problems (2D only) – use trigonometry to calculate the area of a triangle – solve problems using the sine and cosine rules (2D) Use trigonometry to solve problems in 3D	To develop problem skills which can be applied to all strands of the Leaving Cert syllabus. Any third level courses involving mathematics and problem solving, e.g. engineering, sciences, mathematics.

6. Goals of the Unit

- Students can solve 2D and 3D trigonometric problems
- Students can apply trigonometric facts and formula to solve problems.
- Students recognise the potential uses of mathematical knowledge, skills and understanding in all areas of learning
- Students devise and evaluate strategies for investigating and solving problems using mathematical knowledge, reasoning and skills

7. Unit Plan

Lesson	Learning goal(s) and tasks
1 The Research Lesson	• Brainstorm on tools available to solve triangles. • Present students with complex 3D Trigonometric problem
2	• 3D prism problems
3	• 3D pyramid problems
4	• Exam papers style questions
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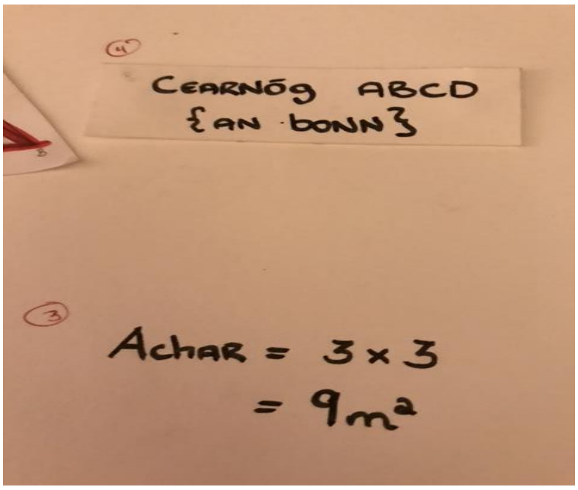
8. Goals of the Research Lesson:

- Students to closely analyze a three dimensional diagram by identifying the triangles, sketching the triangles, filling in the information given and supplementing this information from prior knowledge.
- To help students to see the connection between 2D and 3D diagrams
- From 2D diagrams students visualize 3D reality and can identify the right angles that are not always apparent on paper.
- Students make decisions about which trigonometric formula can be used to calculate other dimensions eventually leading to the solution required.
- Students are confident enough to attempt an approach, take a leap even if they don't arrive at the required solution.
- Students have the resilience to attempt multiple approaches if necessary.
- Students recognize that all the information given is significant and applicable to the problem at hand.
- Students have confidence to work independently

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction **5 min introduction** Revision of prior knowledge needed to tackle trigonometric problems. **<u>Right angled triangles</u>**: • Pythagoras' Theorem • Sin/Cos/Tan Ratios • Area = ½ b x h **<u>Non-right angled triangles</u>** • Sine Rule	Review approach to 2D problems	Students are asked to provide any rules about triangles that they know.

• Cosine Rule • Area = 1/2abSinC		
Posing the Task (5 min) **<u>Problem:</u>** Below is the square based pyramid ABCDEO. ¦AB¦ = 3m, the point O is in the center of the square base ABCD. The point E is directly above the point O making ¦OE¦ the perpendicular height of the pyramid and ¦OE¦ = 2.5m. Using the points ABCDE and O, identify as many 2D shapes as possible. Find the area of each of these shapes.	Given certain sets of data, prompt students to list strategies and formulae used to solve 2D trigonometric problems.	Do students recognize all strategies and formulae available to them?
<u>Anticipated solutions:</u> 1. Square base ABCD; Area = $9m^2$  2. Half the base, this will give the four equal triangles ABC, ADC, BCD, and BAD; Areas = $4.5m^2$	A worksheet will be handed out with the problem and a labeled diagram of the pyramid. **Additional resources:** Laminated diagram,	By posing questions make certain that students understand the problem.

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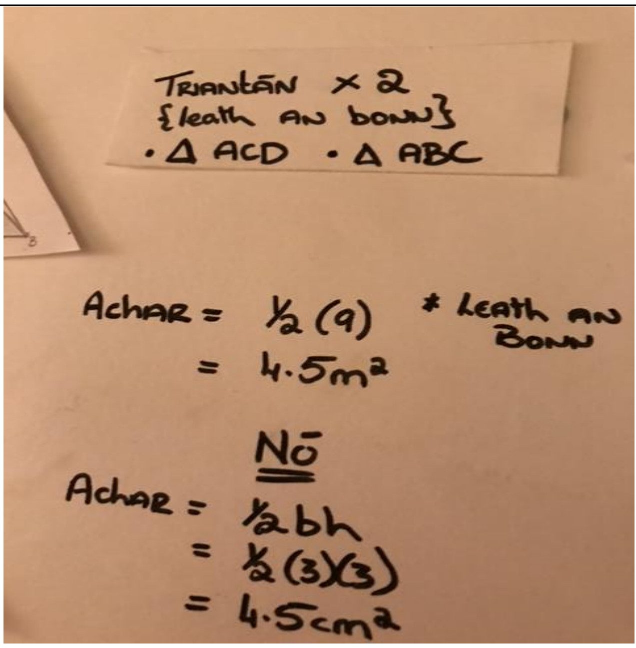
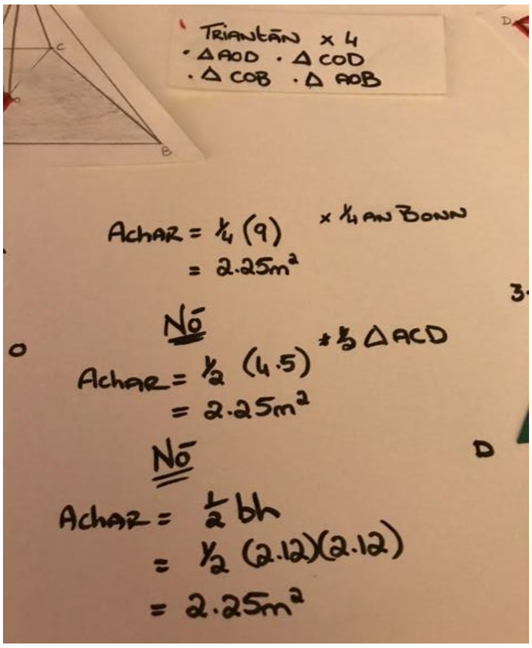


3. Quarter of the base, this will give four equal triangles, AOD, COD, BOC, and AOB; Area = 2.25m^2



4. Four vertical triangles, DOE, COE, AOE, and BOE; Area = 2.65

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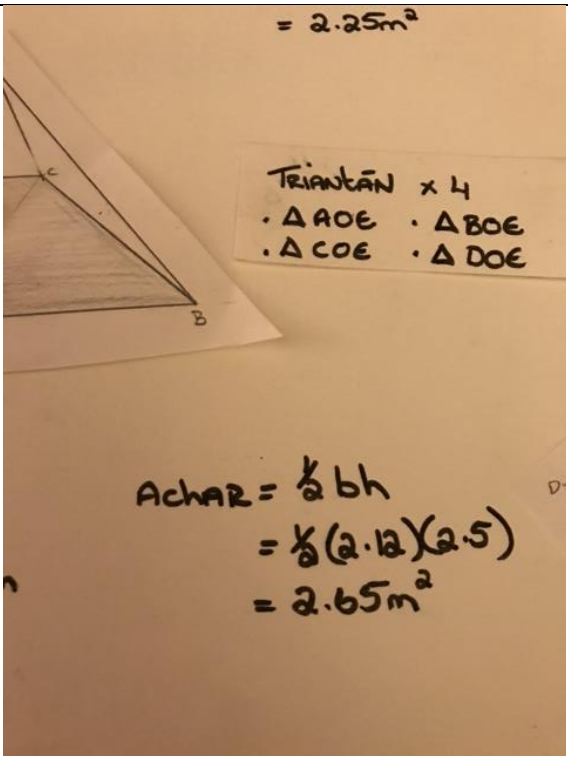
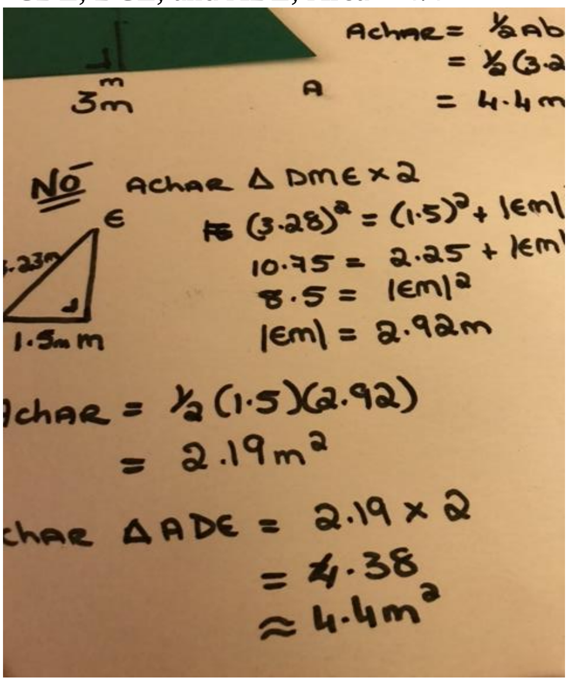


5. The four equal faces of the pyramid, ADE, CDE, BCE, and ABE; Area = 4.4



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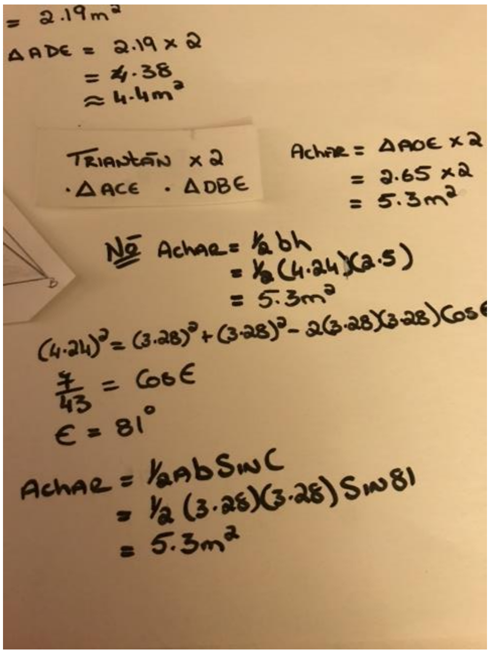
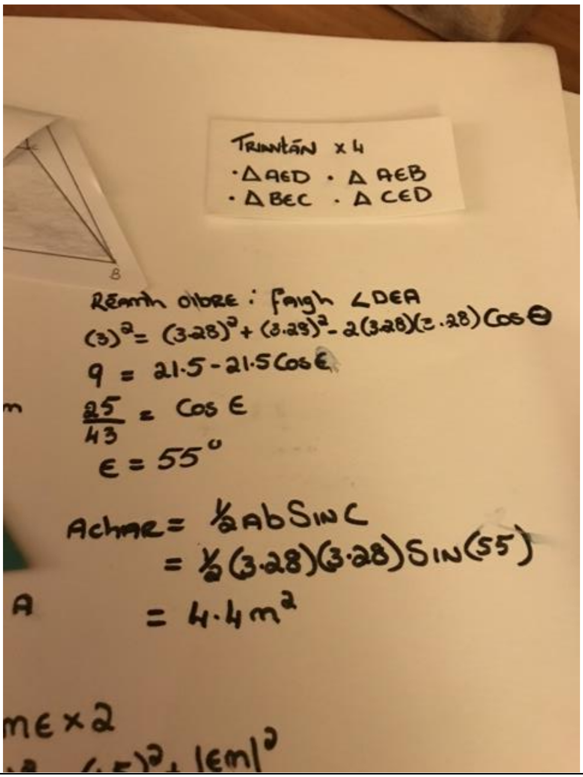
6. Two triangles across the diagonal of the base, DBE And ACE; Area = 5.3



7. Two triangles across the diagonal of the base, DBE And ACE; Area = 4.4

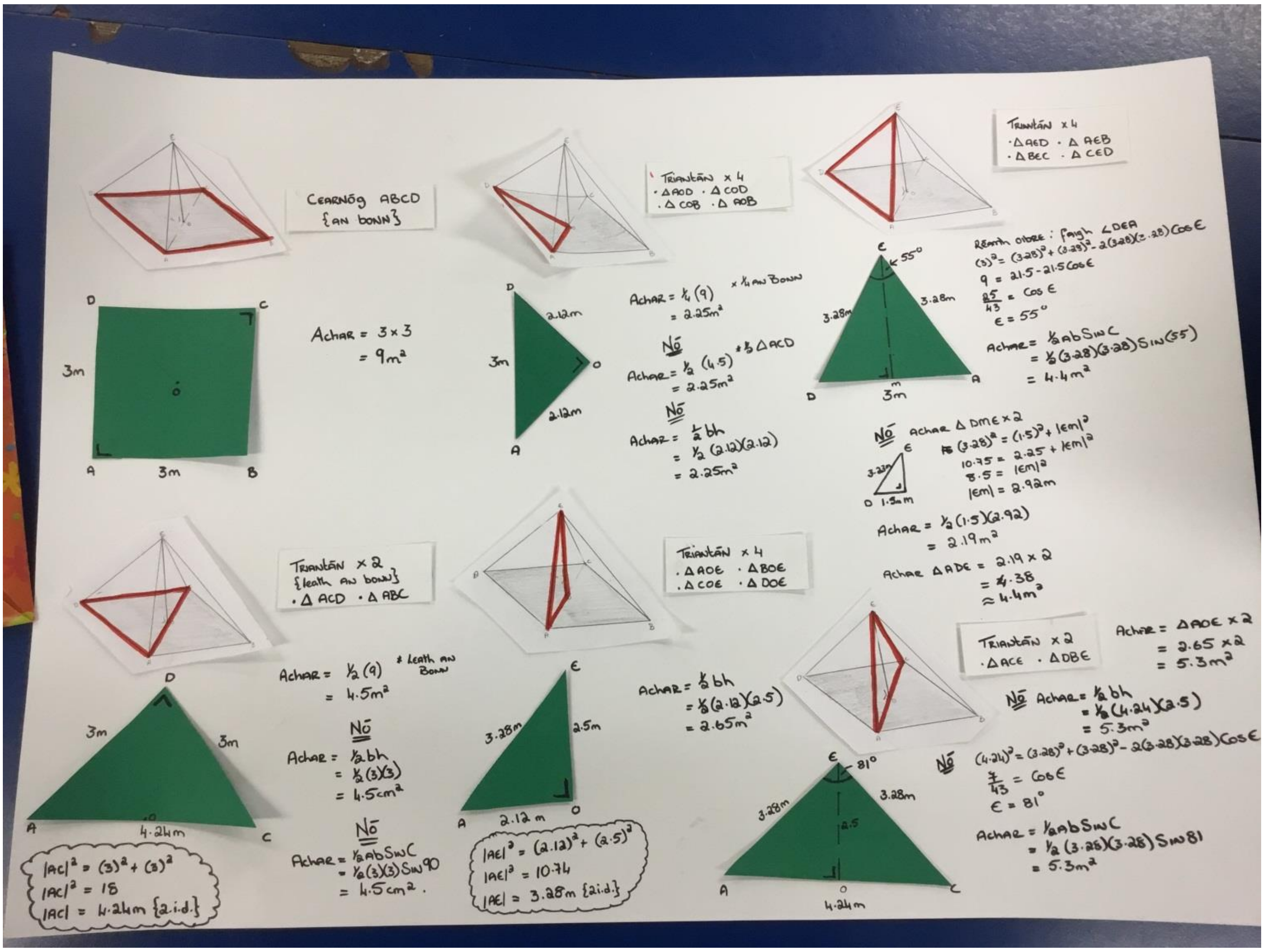


19 shapes in total to be identified.		
Ceardaíocht /Comparing and Discussing (15min) (6 unique shapes are to be found, 19 shapes in total) Ask students how many shapes they think there are to be found. Invite 6 students up to share their solutions. In each case students will be asked to highlight the relevant shape on a 3D diagram of the pyramid, draw a 2D representation of this shape (a cut out will be put up on the board then) , fill in all dimensions showing any work needed and then find the area of the shape. Then they will highlight any repetitions of this shape on the 3D diagram. Shape 1- Square Base Shape 2 – Half Base Triangles Shape 3 – Quarter Base Triangles	Walking around the class keep note of students' progress. Encourage students through praise and urge them to look for more solutions. Ask students that finish early to try to find as many dimensions as possible. Remind students who having difficulties of the resources available that might help Did everyone spot the square and calculate the area correctly? After the first student explains the work, ask the class for a similar triangle, push for all 4 possible triangles. With all possible methods for calculating the area as shown above. Similar to the approach taken for the half base.	How many students are confident to start straight away? Are students using the available resources? Can students make the transition from 3D to 2D diagrams? Do students transfer dimensions when they apply to more than one diagram? Can students apply the correct formula to find the area of each shape? Which formula did you use to find this area? Is there another way to find the area? What type of triangle is this? What other information does this tell us about the triangle? Is there another of these shapes? Is it contained in any other shape on the board?

Shape 4 –Four Vertical Triangles Shape 5 – Pyramid Face Triangles Shape 6 – Vertical cross-sectional triangles	After first student explains their method, call on others to show alternatives for the calculation and to point out similar triangles. Now that we have the diagonal i.e. |AE| we can do the faces. Again have students suggest alternatives after student has completed the work on the board. Similar approach to above, spotting different methods for area and spotting the similar triangle.	Do you need to find any new dimensions? How? Ensure students are aware of the need to calculate the hypotenuse for the next step. Do students understand why the top angle is needed for one of the methods?
Summing up & Reflection (15 min) Recap on all strategies used and any lessons or tricks learned. Students will then be given another 3D trigonometric problem to solve individually (5 min) Students will also be asked to identify how they applied the strategies/ resources learned today in class when solving this new problem. The solution to this problem will then be presented by a student and the class will discuss the strategies/resources used and compare different approaches.	 **<u>Focus on:</u>** Isolating shapes. Drawing 2D representations and filling in all known dimensions. Applying the correct formula to find any other required data. Highlight strategies using resources. Identifying duplications of shapes.	 Have students shown any strategies that help them pick one shape out of the pyramid? Do students see when shapes are duplicated? Are students recognizing which dimensions are to be used for each shape? Did students explain how they used the resources available? Did students explain why they chose the formula they chose?

	Using the board, recap on the strategies used in class. Monitor students' progress with the second problem. Facilitate a class discussion on the different strategies that could be used for this problem and highlight when each of these was used in the earlier problem.	Are students attempting to use any of the strategies learned earlier? Do more students make use of the resources or are students using a different resource? Do any students attempt multiple approaches to solving this problem?

10. Board Plan



11. Evaluation

It took a lot longer than planned for, but the students did experience deep learning and the lesson achieved all of the stated goals and it should set the students up well for further 3D Trig problems. Giving each of the students the problem on a laminated sheet allowed exploration and experimentation with finding the shapes.

Students really appreciated all of the work being left on the board, but found there was a lot of time pressure to find the shapes as well as calculate the areas.

We should look at splitting the problem in two.

- Identify all possible shapes, then do boardwork on this.
- Calculate the area of each of the shapes, more boardwork looking at different methods.

12. Reflection

A huge benefit of the process was the ability to work with other experienced teachers in setting lesson goals and creating lesson to reach these targets.
Definitely made me reconsider my board work and I will aim to use the “No Rubbing Out” technique.
Gave me an insight into amount of work needed to plan a lesson like this, however the hard work is definitely rewarded with the benefits students gain from the lesson.