Lesson Research Proposal for 1st Year - Geometry (Exterior Angle Theorem)

For the lesson on **25th January 2017**
At **St. Mark's Community School**, 1st Year
Teacher: Seán Gunnigan
Lesson plan developed by: Seán Gunnigan, Joanne Kelly, Anna Carroll, Irene Stone

1. Title of the Lesson: "What's your angle?"

2. Brief description of the lesson

Through exploring relationships in triangles, students will discover that the exterior angle of a triangle is equal to the sum of the two interior opposite angles.

3. Research Theme

At St. Mark's CS, we want our students to…

- **a)** …engage purposefully in meaningful learning activities
- **b)** …reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning.

As teachers we will support the achievement of these goals by...

- **a)** Purposeful engagement in an active lesson.
  Students will be encouraged to form their own solutions to the problem assigned in class.
  Students will discuss the merits of each other's' solutions and come to a consensus on which solution is most effective.
- **b)** Reflection on progress as learners.
  By discussing various solutions in class, students reflect on their own learning and are given constructive feedback from their peers. Students are also asked to reflect on what they have learned at the end of the lesson.
- **c)** Sense of ownership of and responsibility for learning.
  By presenting solutions at the board and sharing them with the class, students develop a sense of ownership of the solutions that they have discovered. Students take responsibility for their own learning by developing their own solutions to problems, and take responsibility for the learning of fellow students by presenting solutions to them.

4. Background & Rationale

This lesson is aimed at 1st year students. The Chief Examiner Report, 2015[1] described how students still struggle with proving geometric theorems. We have found this in our own classes also. When teaching geometry; we, traditionally, give the theorem statement at the start of the lesson. We as teachers then work through its proof. However, we have discovered that students have difficulty proving theorem 6; the learning of this has traditionally been done through rote. We give the students the list of steps which they then learn off. We find that this approach to teaching geometry can leave students disengaged and knowledge weary. We are going to approach this topic differently; we will let them investigate the theorem and come up with the words of the theorem themselves. We will focus on formal proof later in 2nd or 3rd year. We hope that an investigative approach to geometry will impact more upon students' retention of knowledge and their enjoyment of the lesson.

[1] Chief Examiner's Report JC Maths 2015: https://www.examinations.ie/misc-doc/EN-EN-25073660.pdf

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
By **fifth class**, students will be familiar with angles and triangles: They will have explored the sum of the angles in a triangle through… …cutting off the three corners of a paper triangle and putting them together to make 180°. …measuring the angles in a variety of triangles using a protractor. …calculating and recording their sum. …examining and discussing results. They will have used angle and line properties to classify and describe triangles and quadrilaterals. In **sixth class**, these ideas are reinforced: They can recognise, classify and describe angles and relate angles to shape. They learn to identify types of angles in the environment. They estimate, measure and construct angles in degrees.	In **first year**, the concepts introduced within the study of synthetic geometry are: **Axioms:** Students will be introduced to concepts of plane, points, line, line segments, straight angles, parallel line, protractor axiom. **Theorems:** Students will learn about Theorems 1-6 through a discovery and investigative approach. **1**. Vertically opposite angles are equal in measure **2**. In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.) **3**. If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse). **4**. The angles in any triangle add to 180˚ **5**. That two lines are parallel if and only if, for any transversal, the corresponding angles are equal. **6**. That each exterior angle of a triangle is equal to the sum of the interior opposite angles **Constructions**: Students will learn constructions 1-6	Theorems 1-6 will be revisited in **2nd and 3rd year** when formal proofs will be looked at. The following theorems will also be covered in 2nd and 3rd year: **9.** In a parallelogram, opposite sides are equal and opposite angles are equal (& converses). **10.** The diagonals of a parallelogram bisect each other. **11**. If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal. **12**. Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio (and converse). **13**. If two triangles are similar, then their sides are proportional, in order (and converse). **14**. [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides. **15.** If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. **19**. The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc. **Constructions:** Students will learn constructions 7 -15

6. Goals of the Unit

- Students will be able to identify and name points, lines, line segments, rays, and angles. They will understand what a plane is. They will examine the axioms on points, plane, lines, line segments, straight angles and parallel lines. This knowledge will be applied to explore the fact that vertically opposite angles are equal in measure.
- Students will know how to use a protractor to measure angles. This knowledge will be applied to explore the construction "how to bisect an angle using a compass". This will reinforce their skills in using a protractor to measure angles.
- Students will understand that the angles on a straight line add up to 180°.
- Students will know that the three angles in a triangle add up to 180°.
- Students will understand that each exterior angle is equal in measure to the sum of the interior opposite angles.
- Students will identify and understand the properties of an isosceles triangle by discovery and investigation
- Students will understand what parallel lines and transversals are. This will enable them to identify and measure the angles around the transversal (e.g. corresponding, alternate)
- Students will explore parallel lines and understand that lines are parallel, if and only if, for any transversal, the corresponding angles are equal.
- Students will be able to construct parallel lines and be able to divide a line segment into 3 equal parts.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Introduce geometric terminology (Line, line segment, ray, points, plane, angle, acute, obtuse, horizontal, vertical, vertex, parallel, perpendicular, transversal, corresponding, alternate, triangle, interior angle, exterior angles) **Axiom 1.** [Two points axiom] There is exactly one line through any two given points. **Axiom 2**. [Ruler axiom] The properties of the distance between points.
2	**Axiom 3**. [Protractor Axiom] The properties of the degree measure of an angle. Use of a protractor. Measurement of a variety of angles (0°/360°, 180°, acute, obtuse, reflex).
3	Discovery and investigation (through measuring) of **Theorem 1**: Vertically opposite angles are equal in measure.
4	**Construction 1:** Bisector of an angle. Students will learn how to bisect an angle and use a protractor to measure and check it's bisected correctly. This construction will reinforce Axiom 3. **Construction 2:** Perpendicular bisector of a Line Segment. This construction will reinforce Axiom 2.
5	Discovery and investigation (through measuring) of **Theorem 4:** The angles in any triangle add to 180˚. (Proof later in 2nd year)
The Research Lesson	Discovery and investigation (through measuring) of **Theorem 6:** Each exterior angle of a triangle is equal to the sum of the interior opposite angles.
6	Solving problems involving exterior angles. Leading to solving more challenging problems involving many relationships; straight, triangle, opposite and exterior angles.
7	Discovery and investigation (through measuring) of **Theorem 2:** In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.
8	**Axiom 5**. [Axiom of Parallels] Given any line l and a point P, there is exactly one line through P that is parallel to l. Discovery and investigation (through measuring) of **Theorem 3:** If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse).
9	Discovery and investigation (through measuring) of **Theorem 5:** Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.
10	**Construction 5**: Line parallel to a given line, through a given point. This construction will reinforce Theorems 3 and 5.
11	**Construction 6:** Division of a line segment into 2 or 3 equal segments, without measuring it. This construction will reinforce parallel lines and also axiom 2.

8. Goals of the Research Lesson:

a) Mathematical Goals

Students will…

...understand that there are different ways to find the measure of an angle, e.g. through measuring or through using their previous knowledge of relationships of straight angle and angles in a triangle.

...realise that there is a relationship between the interior angles of a triangle and the exterior angle and understand the nature of this relationship.

b) Key Skills and Statements of Learning

Being Numerate: Students will see patterns, trends and relationships.

Managing information and thinking: Students will reflect on and evaluate their learning.

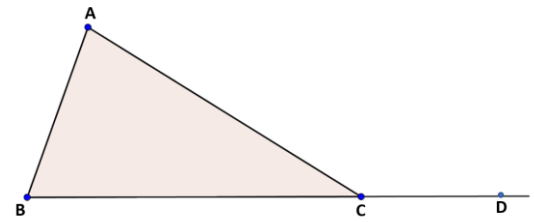
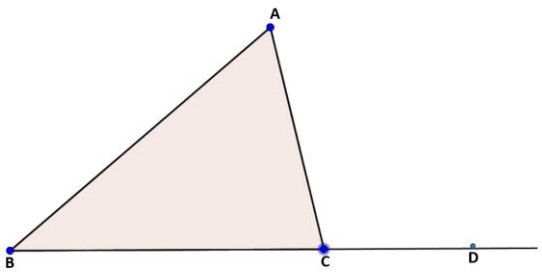
Communicating: Students will discuss and debate, listen and express ideas through presenting their findings.

Working with Others: Students will learn with and from each other by discussing different ways to find angles.

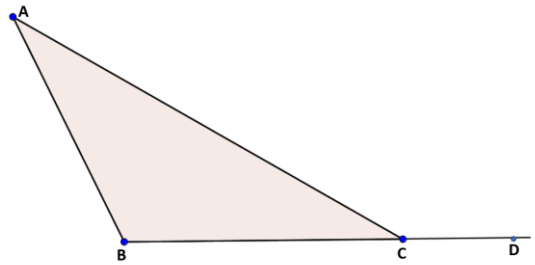
This lesson is also designed to meet the following JC Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The students describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

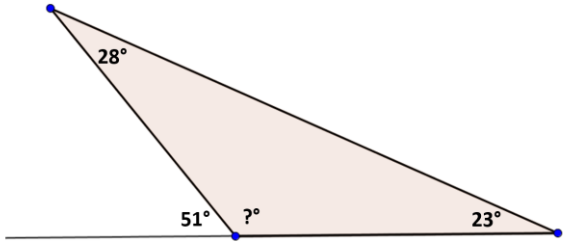
9. Flow of the Research Lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Today we are going to use our previous learning to investigate angles in and outside of a triangle. We will try to solve problems individually and as a group based on angles. Discuss prior knowledge of how to correctly name an angle using letters. Reminding students about the vertex and how to use a protractor.	A picture of an angle will be displayed and students will be presented with naming of angle 3 different ways. A picture of a triangle will be on the board with the vertices labelled.	Are students motivated? Can students name an angle using letters? Will they remember that the vertex must be in the middle when naming angle? Do they recognise the vertex of an angle?
Posing the Task **Measure all the missing angles in the triangles.** Today you will investigate angles in and outside of a triangle. Use the protractor to measure the angles in triangles 1, 2 and 3 on the handout. Use your previous learning to work out the missing angle in the triangles 4 and 5, without using a protractor. This work should be done individually. **1.** Measure all the angles with a protractor  **2.** Measure all the angles with a protractor  	The triangles will be stuck onto the board as a visual aid. Each student will receive a handout containing 5 triangles featuring the interior and exterior angles. See Appendix A. Students will be allowed to use protractors for the first 3 triangles (as described on the worksheet).	

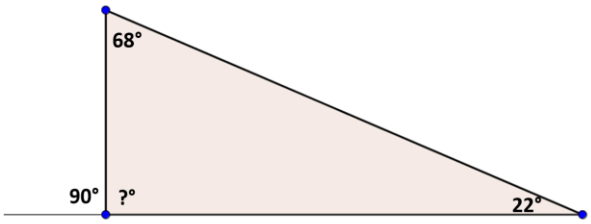
3. Measure all the angles with a protractor



4. Work out the missing angle without a protractor



5. Work out the missing angle without a protractor

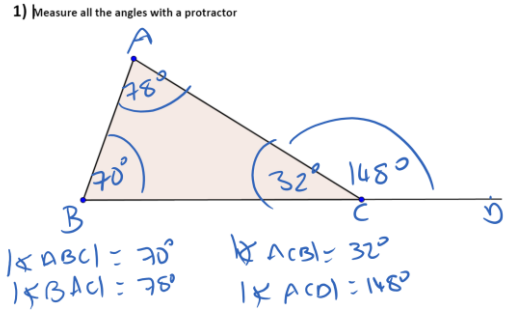


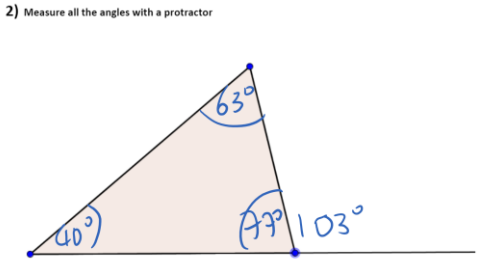
In triangles 4 and 5, students will have to work out one angle in the triangle, given an exterior angle and the opposite interior angles.

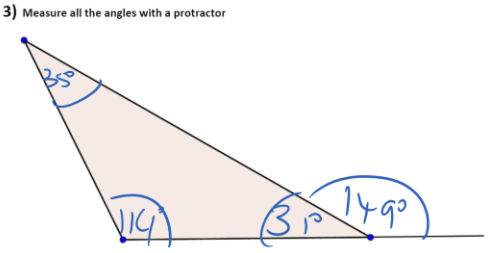
Teacher might ask students:
"Can we all remember how to use protractor?
"Where should we measure the angle?"

Student Individual Work

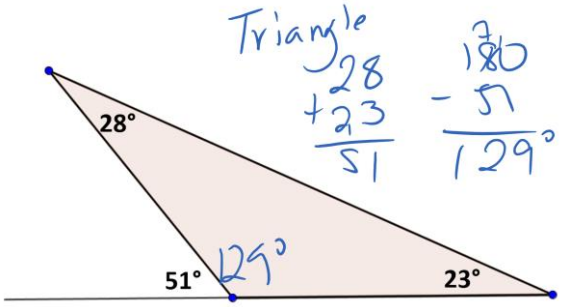
Three correct solutions of measuring angles using protractors, from three separate students.

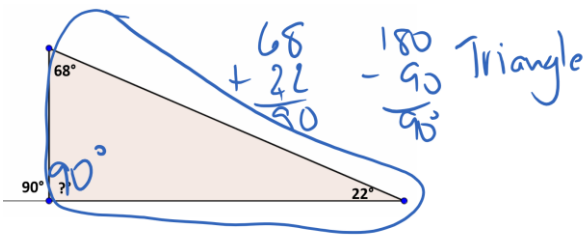






For triangles involving finding missing angles without using a protractor. Two solutions using the relationship **the sum of angles in a triangle is 180°**.





Teacher circulates room and checks students are measuring angles correctly. Any incorrect measurement, teacher can ask *"Show me how you are measuring the angle?"*

"Where is the vertex on the protractor?"

"Where is the vertex of the angle you are measuring?"

"Where are the 2 rays of the angle you are measuring?"

For students who don't describe the angles using letters:
"Can you describe the angle using letters?"

For students who may line up the protractor correctly but are reading it incorrectly
"Does that look like an acute or an obtuse angle?"
"Does the measurement you have written down there match your estimation?"

Some students may measure the angle using a protractor. They can be prompted:

"Try finding the angle without the protractor".

"Think of relationships you might have learned".

Are all students using a protractor correctly?

Can they all identify the vertex of an angle?

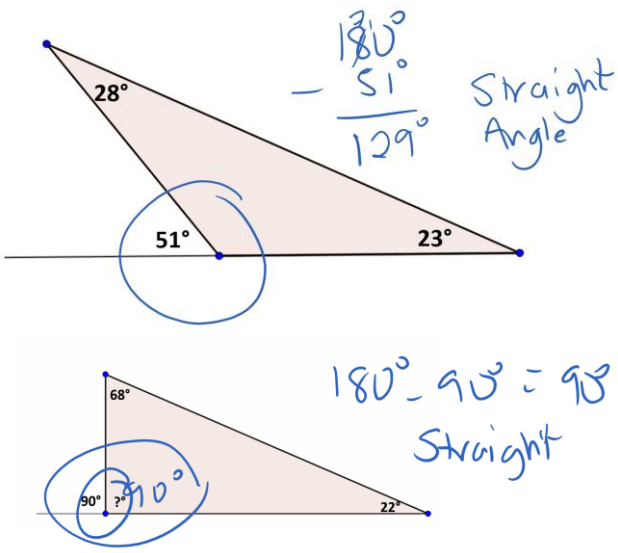
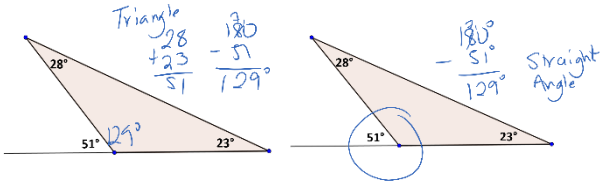
Can all students use letters to name an angle?

Are their measurements accurate?

Are they eager to write the answers? Are they engaged?

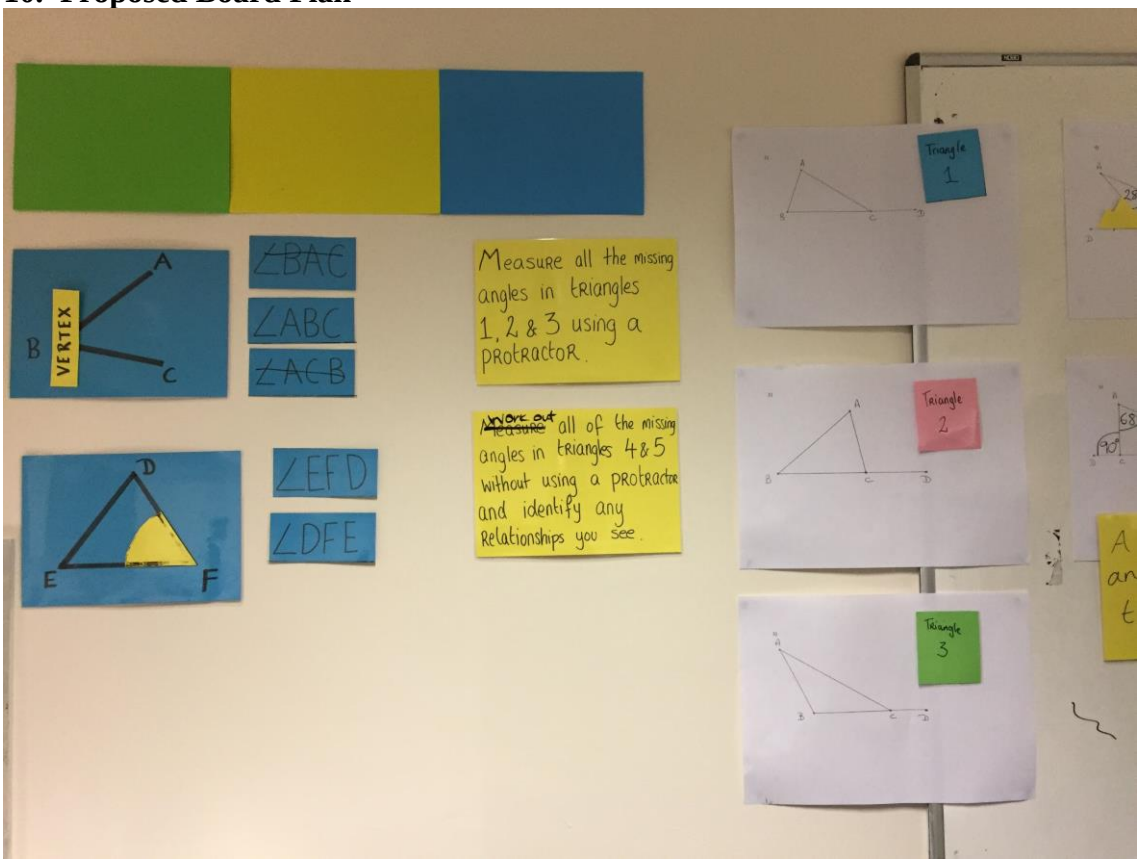
Are they writing down the relationships?

Has any student discovered the "new" relationship?

Two solutions to finding missing angle, using the relationship **the angles on a straight angle add to 180°**.  	Further hints may be given: *"What type of shape is this?"* *"Can you remember anything about the angles in a triangle?"* Some students may forget to write down their reasons *"How did you get 129°?"* *"What calculation did you do?"* *"Can you explain/write down why you did that?"* Are you using any relationships to identify these angles? If so, write them down.	
Ceardaíocht /Comparing and Discussing All solutions will be put up on the board in order. For example, for triangle 4, the solutions will be placed as follows beside each other.  This will allow for the relationships (prior knowledge) to be compared. A discussion will take place around the names of the angles. Teacher will also put up a blank table containing all angles (columns) and triangles (rows). Beside the exterior angle, there will be a column containing the sum of the opposite interior angles. Students will be called up to fill in table (one student per row).	During responses: *"What do you think"?* (ask another student(s) other than the presenter) *"Why is that"?* (Looking for evidence). *"Did anyone else solve it the same way? Can you explain this method?"* Asking a student who didn't use letters *"Can you describe this angle using letters?"* Triangle 4: Asking a student who used angles in a triangle relationship to work out the missing angle: *"Can you describe what you did here?"* *Asking another student who used different way:* *"Can you describe what X did?"* *"Did she get same answer you did?"* Asking a student who used the straight angle relationship to work out the missing angle: *"Describe what you did and how you did it differently?"*	Are students engaged in discussion? Are students able to defend their ideas? Are students willing to question and challenge ideas they don't agree with or understand? Will students come up with the word interior and exterior.

	Once table is filled in: *"Can you see a relationship between any of the angles?"* *"There is a 3rd relationship we haven't seen before… can anybody see it?"* *"What's a good name for these angles inside the triangle?"* *"What's a good name for this angle here on the outside?"*	
Summing up & Reflection We were reminded how to use a protractor to measure angles. We noticed that we can find angles through measuring. We can also use our previous knowledge of relationships to find angles such as: - the angles in a triangle add to 180 degrees - angles on a straight line add to 180 degrees. We came up with names for the angles inside and outside the triangle; interior and exterior. We noticed that there are two ways to find this interior angle of a triangle (straight angle and triangle relationships) We came up with a new relationship. We concluded that the angle on the outside is equal to the adding of the two angles inside on the opposite side. Possible extension exercises include asking students to identify relationships in diagram using different exterior angles.		

10. Proposed Board Plan



11. Evaluation

The classroom will be divided up in 3 sections, approximately 9 students per observer, see Appendix B. One observer will use the LessonNote[2] app. Observers will take note of student interactions, engagement etc. keeping in mind the goals of the lesson. Student worksheets will be collected and photographed to be reflected on later. The completed board will be photographed to be reflected on later. A post-lesson meeting will take place immediately after the lesson for reflection to take place. When observing the lesson, the following questions will be kept in mind:

- Were students engaged in the lesson? Were they on task at all times?
- Was there any downtime?
- Did students feel a sense of ownership of their learning?
- Were students giving constructive feedback to each other?
- Do students understand that there are different ways to find the measure of an angle?
- Do students understand the nature of the relationship between the interior angles of a triangle and the exterior angle?
- Did anything "not work"?
- Were there any surprises?

12. Reflection

What had we hoped before the lesson:

Before the lesson, we had hoped to observe the following:

- Student interaction
- Student engagement
- Students would see that there are different ways to find the measure of an angle, e.g. through measuring or through using their previous knowledge of relationships (straight angle and triangle)
- Students would discover the "new" relationship (e.g. the exterior angle is the sum of the two interior opposites).
- Evidence of key skills being embedded in student learning.
- Furthermore, we hoped that the experience of lesson study would help inform our teaching and help us recognise the value of collaboration and reflection.

[2] Designed for the collaborative improvement for teaching and learning http://lessonnote.com/

What was observed during the lesson:

During the lesson, students were fully engaged and were on task at all times. We noted that particular focus and student attention were evident when a student was presenting at the board. The students were clearly focused as they were encouraged to form their own solutions. Some students were finished earlier than others on the task. However, we observed that they were checking their work as they waited for the next instruction.

Every student had the opportunity to contribute to the learning. The teacher made sure to ask every student in the class a question while also looking for students to volunteer responses to questions. Students took great care in their work; e.g. they were diligent about labelling angles correctly.

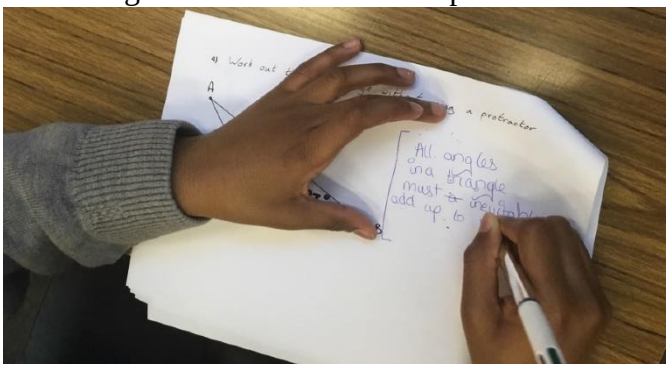
When at the board, they were confident in presenting their work. The teacher asked other students to reinforce the ideas that students presented at the board. This was a great way to assess their understanding. Students had a sense of ownership as they presented and explained their work to the class. When the teacher asked students to come to the board; there was a big emphasis on other students reinforcing the ideas that were presented. Sense of ownership was also evident as students came up with the words of the new relationship themselves e.g. the words exterior and interior.

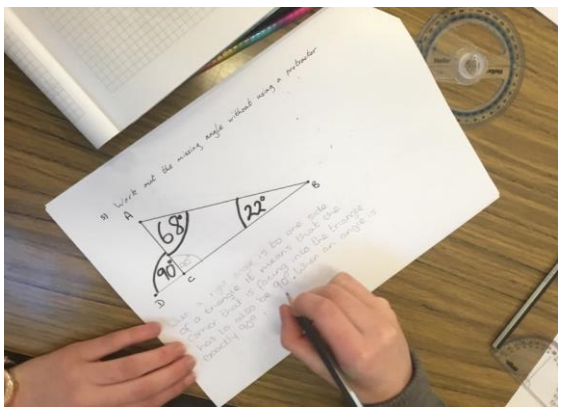
Key Skills were evident in the lesson. Students were numerate as they described the relationships in the triangles (e.g. straight angle and angles in a triangle). They looked at patterns when viewing the table at the end of the lesson. There was clear communication among students as they presented their work at the board and discussed their findings with each other and the teacher. Students were communicating through their diagrams, words and mathematical "rough work", while also communicating vocally when at the board. It was observed during the lesson that students were very quiet when doing the individual tasks.

Major points raised during the post-lesson discussion, and the team's own opinions;

The first 5 minutes of the lesson was spent on prior knowledge, e.g. naming angle with letters, emphasising the keyword vertex and naming the angles in triangles. This was an important part of the lesson as the teacher was not the main class teacher. This really put students at ease as their confidence increased answering questions about content they were familiar with. It was clear from the start of the lesson that some students were nervous. The teacher very easily put them at ease and it was clear that students became less aware of the adults in the room observing them.

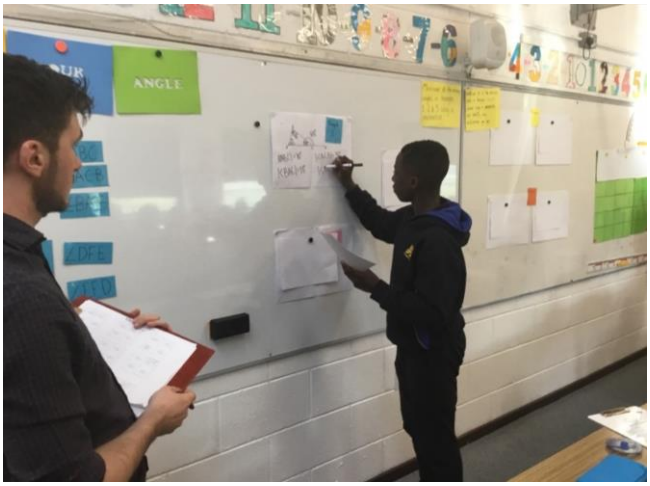
On reflection, however, there was too much time dedicated to individual student work (20 minutes). Students were being so careful and diligent about measuring and naming all the angles accurately that it took longer than expected for the 5 triangle worksheets to be completed.





Student Individual work

This diligence and attention to detail was also evident when presenting their work at the board as students carefully took the time to name the angles (e.g. <BAC). While it was important to obtain these angle measurements so as to fill out the table at the end of the lesson, it may not have been absolutely necessary to get students to write out all the angles using letters. On reflection, we should have prepared A3 worksheets so students would have just had to stick up their solutions as opposed to writing them out again. It was also difficult for students at the back of the room to see work that was presented on the board by their peers.



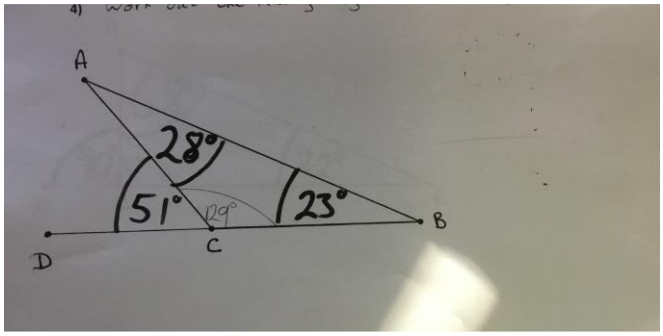
Evidence of key skills: students presenting at board and communicating with others

It was 30 minutes into the lesson before ceardaíocht began. This included 10 minutes spent discussing and presenting students' solutions in triangles 1, 2 and 3. It was 40 minutes into the lesson that true discussion started around triangles 4 and 5. This was where the ceardaíocht should have been focused. Triangles 1, 2 and 3 were reinforcing previous learning. We hadn't anticipated that this would have taken so long.

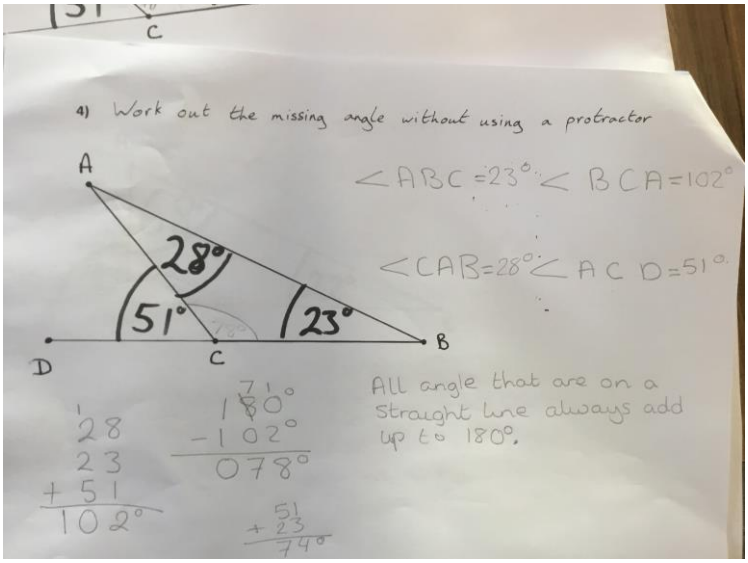
The end of the lesson felt rushed as a result. While the tasks for triangles 4 and 5 clearly showed there were different ways to find the missing interior angle, one student spotted the new relationship. On reflection this would have been the time to call this student up to the board to explain the "new relationship". This was a missed opportunity.

It was left to the end of the lesson when filling out the table to then call on this student who had recognised the new relationship. However, as the lesson was approaching the end, not enough time was taken to tease out the thinking here. The table, on reflection, should have been used to consolidate the understanding of the "new" relationship (exterior angle equals sum of interior opposites). Instead, the table was used as the time to discover the new relationship.

After the lesson when we observed the students' worksheets, we noted that while 24/27 students correctly worked out missing angles in triangles 4 and 5; many had not written down the relationships (e.g. straight angle or triangle). More emphasis on explaining their thinking was needed.

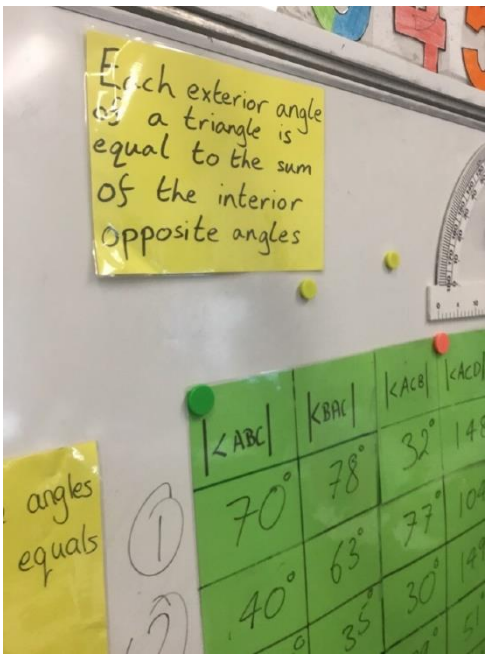


Student clearly worked out missing angle but failed to explain her thinking



Student shows how she worked out the missing angle and explains the relationship (straight angle)

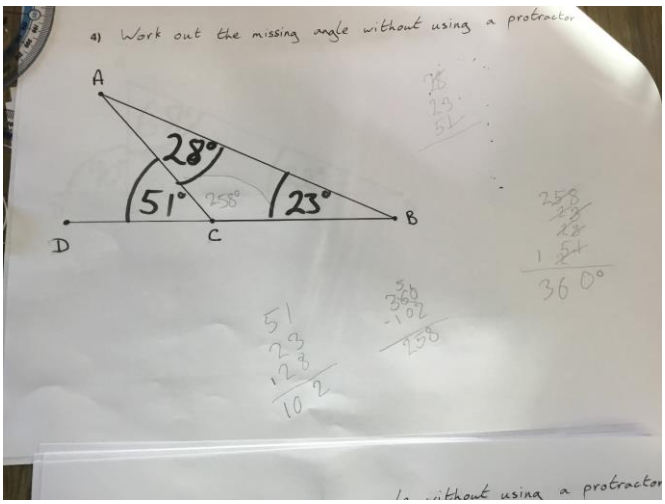
The teacher was very careful not to display the words of the new relationship straight away. The words "interior" and "exterior" were extracted carefully from students with questions asked such as *"What do we call someone who designs the inside of a house?" "What do you think would be a good word to describe this outside angle?"*. This was a great example of students taking ownership of their own learning.



Students came up with "words" themselves. Teacher had relationship ready to stick on board after students 'came up with the words themselves'.

"New learning" was not adequately assessed due to the time constraints at the end of the lesson. This was an unfortunate consequence. From looking at the reflection sheets at the end of the lesson, half of the students commented that they learned that the "exterior angle equals the sum of the two interior opposites". However, we would only have been able to conclude this as true if they had been given a triangle to solve that involves using the new relationship. The assessment of this fact will be done in the next lesson. It was difficult, therefore, to quantify the number of students who understood the new relationship.

After the lesson, when we observed the worksheets, we spotted that 3 students had misconceptions - they added the 2 interior and 1 exterior angle. This was not picked up on during the lesson by the teacher, however it was not possible for the teacher to observe all 27 student worksheets during the lesson. The teacher also commented that it was very difficult to look at all students work as they were flicking their pages moving onto 2nd, 3rd worksheets etc. These misconceptions will be addressed in the next lesson by the class teacher.



Misconception: Student adds two interior and exterior angles

One student needed reinforcement about the straight angle relationship in triangles 4 and 5. She became disengaged after being at the board herself for a different task. She missed the point another student made and then asked a question based on the straight angle relationship. This was then explained to her.

Students did understand that there were different ways to find the angle (e.g. using straight angle and angles in a triangle). This was a very positive effect of the process. Some students who didn't spot one way to find angle clearly recognised an alternative way when the work was presented at the board. There were a few "Ah ha" moments as students saw this unfold.

Ideas for future study:

If teaching this lesson again, less time would be spent on students presenting work for triangles 1, 2 and 3. In fact, it would be worth considering just looking at triangles 4 and 5. The main surprise for us all was the amount of time the whole process took. We had not anticipated the lesson taking so long.

Another important point would be to make the worksheets bigger and use these to stick to the board rather than ask students to do work again.

When doing research lesson again, we would have numbers in front of students or get them to wear labels with their names/numbers. This would have made it easier for observers to take notes on individual students, especially as two of the observers were from a different school.

We would leave more time at the end of the lesson for assessment. The group reflected on the timings and we could see how ambitious we were in expecting the tasks to be carried out in 10 minutes. While we still believe our mathematical goals of the lesson are achievable in a one-hour lesson, we would approach the tasks differently.

What did we as teachers learn from process?

It was evident that students really enjoyed coming up to the board. In particular, we noted how engaged the rest of the class were when this happened. We will aim to adopt this practice more in our classes.

While the students were working individually, we could see how much they valued this time to think. We will allow students more time to come up with solutions on their own. Normally, we encourage students to work with others in class and we commented that our classes are normally "nosier" with students discussing a Maths problem(s) among themselves. We will try in the future to find a balance between students working quietly individually and working with others.

We can see the value in allowing students up to the board in a predetermined order and also the importance of allowing others to verbally reinforce the learning that occurred on the board. This approach meant that the teacher was spending less time with the students who were struggling with concepts as their misconceptions were being addressed at the board by other students.

The most valuable feedback and critique of the lesson resulted from the conversations that we had as a group after the lesson. We will aim, as teachers, to bring this powerful message back to our schools. We were amazed at how much we picked up from the lesson when we were discussing it with each other *after* the lesson.

Summary:

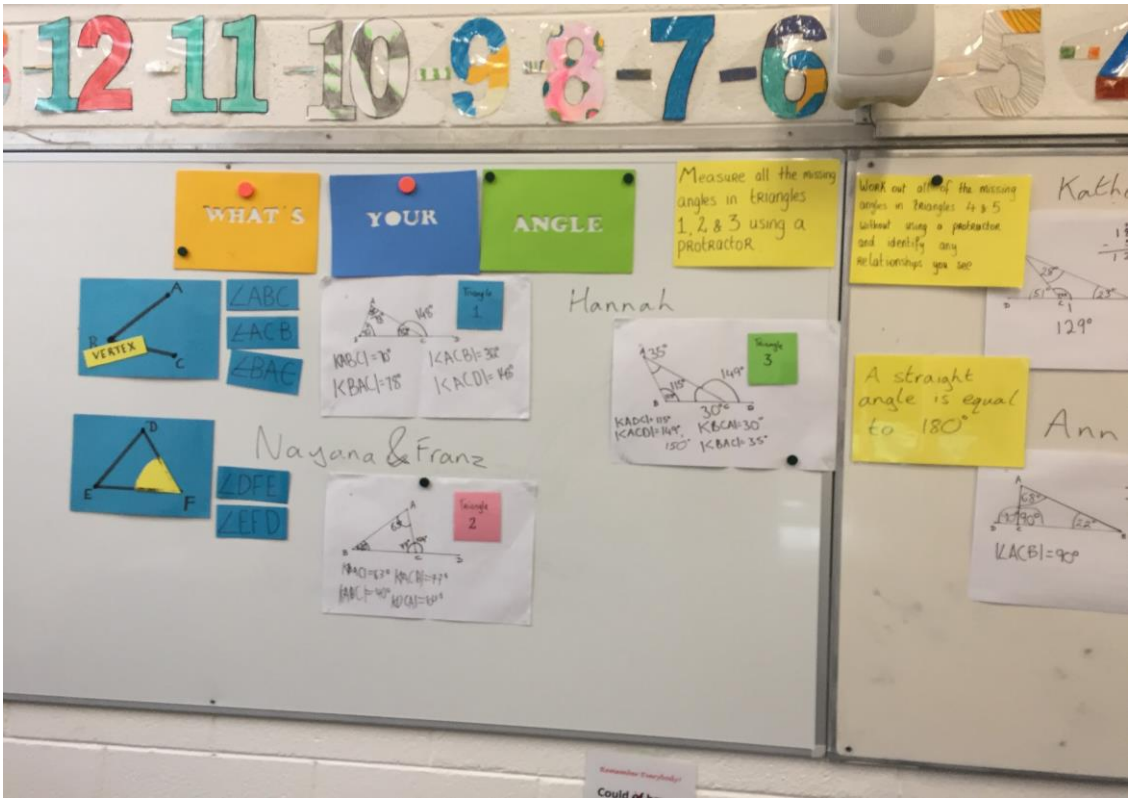
The timings of the lesson were clearly down to the design of the research lesson. The teacher delivered the lesson in a way that was engaging and inclusive. The teacher could not have delivered the lesson in any way that would have sped up the process more. Students were at ease and clearly enjoyed the methodologies that were used. This was evident in their reflections.

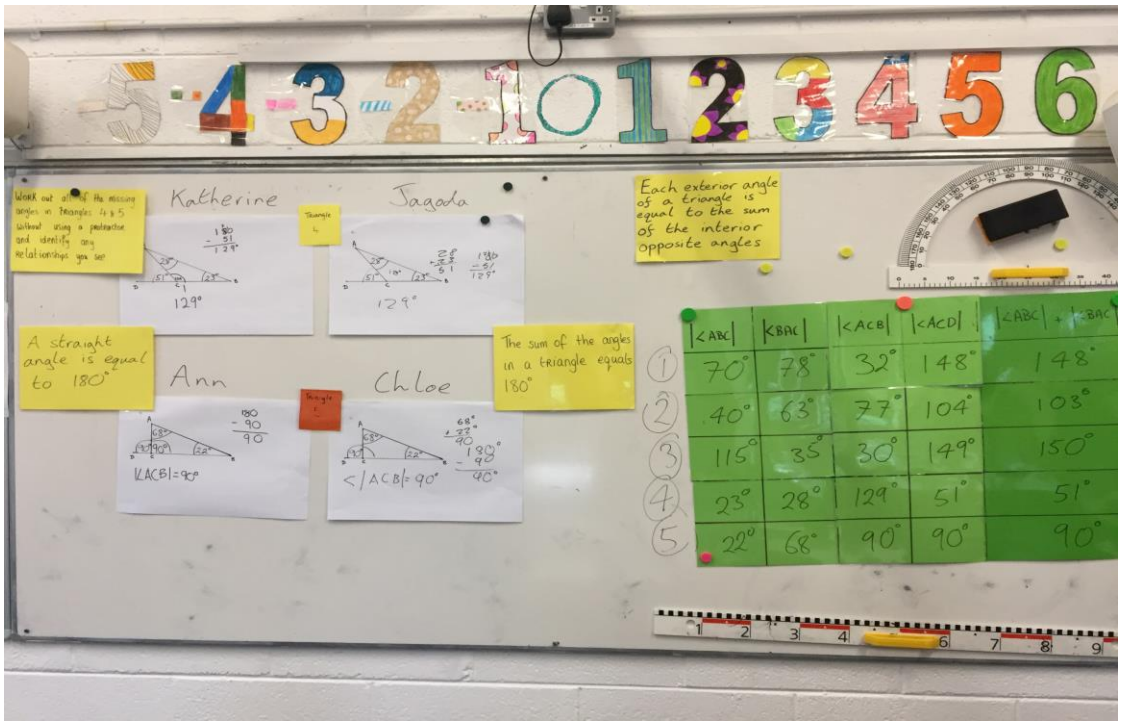
There were two mathematical goals of the lesson. It was clear that the first goal of the lesson was achieved. Students did understand that there are different ways to find the measure of an angle, e.g. through measuring or through using their previous knowledge. We can't conclude if the board-work led to further development of new learning for *all* students, therefore, we can't say if students had fully understood the *new* relationship between the interior angles of a triangle and the exterior angle.

The other goals of the lesson were clearly met. Key skills of numeracy, managing information and thinking, communicating and working with others were evident in the lesson. Students were fully engaged and took ownership of their learning.

Finally, we have learned so much through just focusing on one single lesson. The process of Lesson Study has provided a structure where we collaborated, observed and reflected on a lesson. We have gained insights from working with teachers from other schools. We feel we have a better understanding of how students think and will look at making changes in our own classrooms to reflect what we learned.

Board Work


WHAT'S
YOUR
ANGLE
Measure all the missing angles in triangles 1, 2 & 3 using a protractor
Work out all of the missing angles in Triangles 4 & 5 without using a protractor and identify any relationships you see
A straight angle is equal to 180°
Hannah
Nayana & Franz
Ann
VERTEX
∠ABC
∠ACB
∠BAC
∠DFE
∠EFD

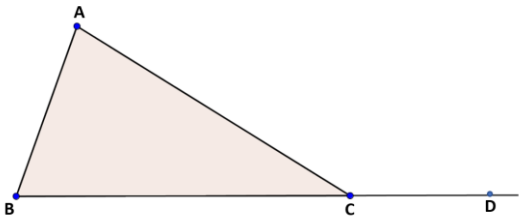

Katherine
Jagoda
Ann
Chloe
Each exterior angle of a triangle is equal to the sum of the interior opposite angles
A straight angle is equal to 180°
The sum of the angles in a triangle equals 180°
|∠ABC| |KBAC| |∠ACB| |∠ACD| |∠ABC| + |∠BAC|
1 70° 78° 32° 148° 148°
2 40° 63° 77° 104° 103°
3 115° 35° 30° 149° 150°
4 23° 28° 129° 51° 51°
5 22° 68° 90° 90° 90°

WHAT'S YOUR ANGLE
Measure all the missing angles in triangles 1, 2 & 3 using a protractor
Each exterior angle of a triangle is equal to the sum of the interior opposite angles
The sum of the angles in a triangle equals 180°
A straight angle is equal to 180°
Katherine
Tayda
Ann
Chloe
Hannah
Nayana & Franz

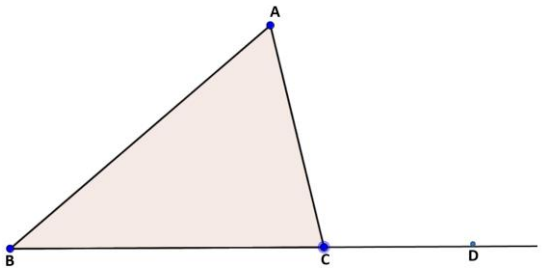
Irene Stone (Lesson Study Associate), Seán Gunnigan (St. Marks CS), Joanne Kelly (Old Bawn CS) and Anna Carroll (Old Bawn CS)

Appendix A: Worksheets

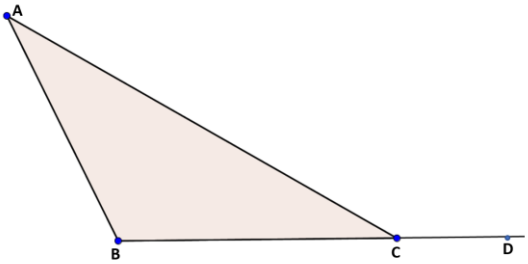
1. Measure all the angles with a protractor



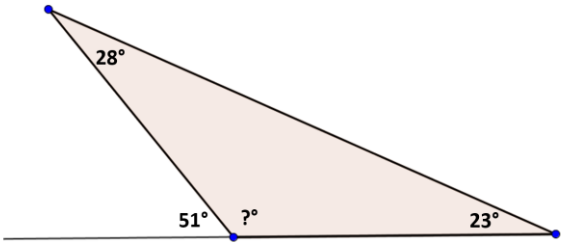
2. Measure all the angles with a protractor



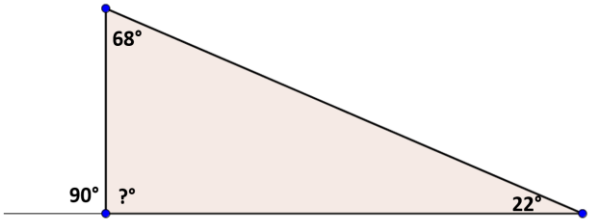
3. Measure all the angles with a protractor



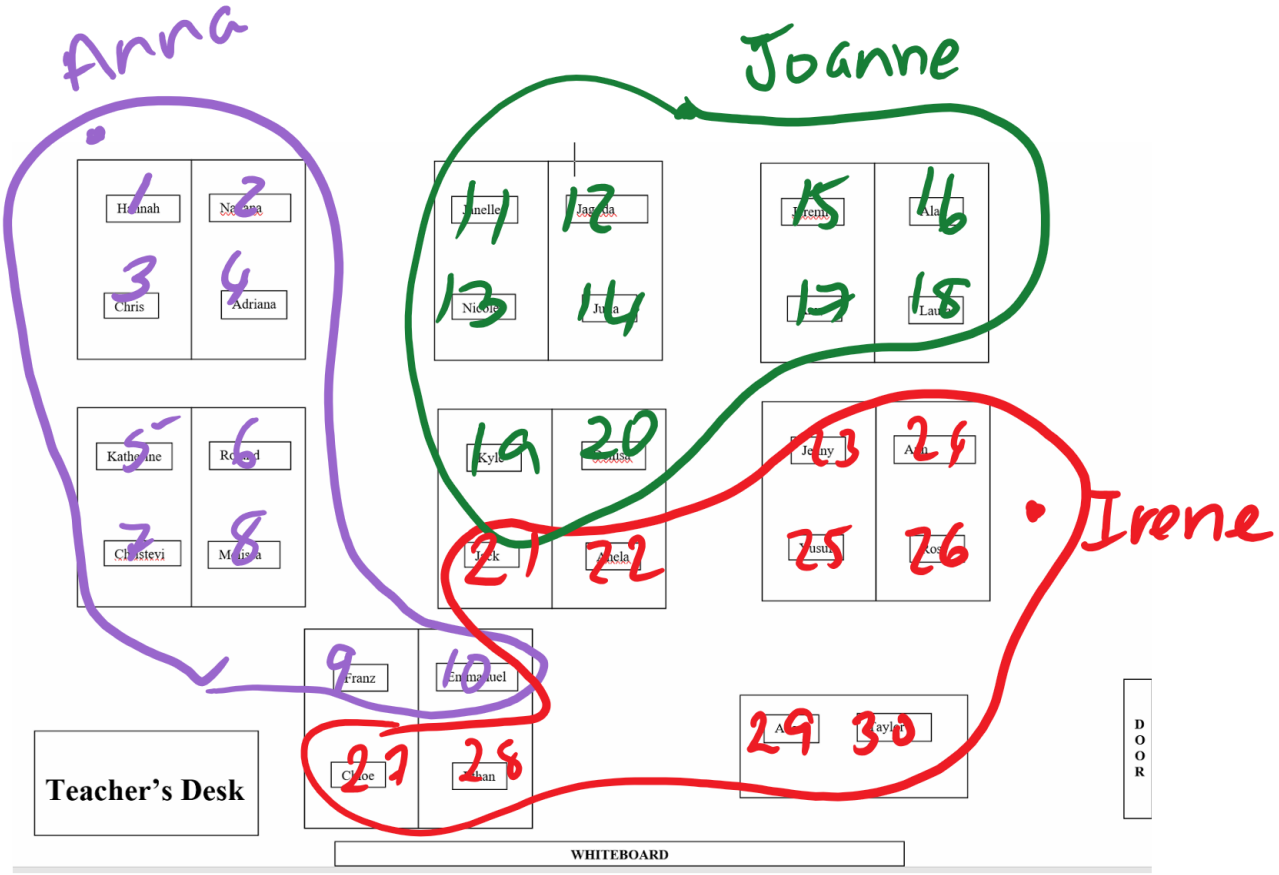
4. Work out the missing angle without a protractor



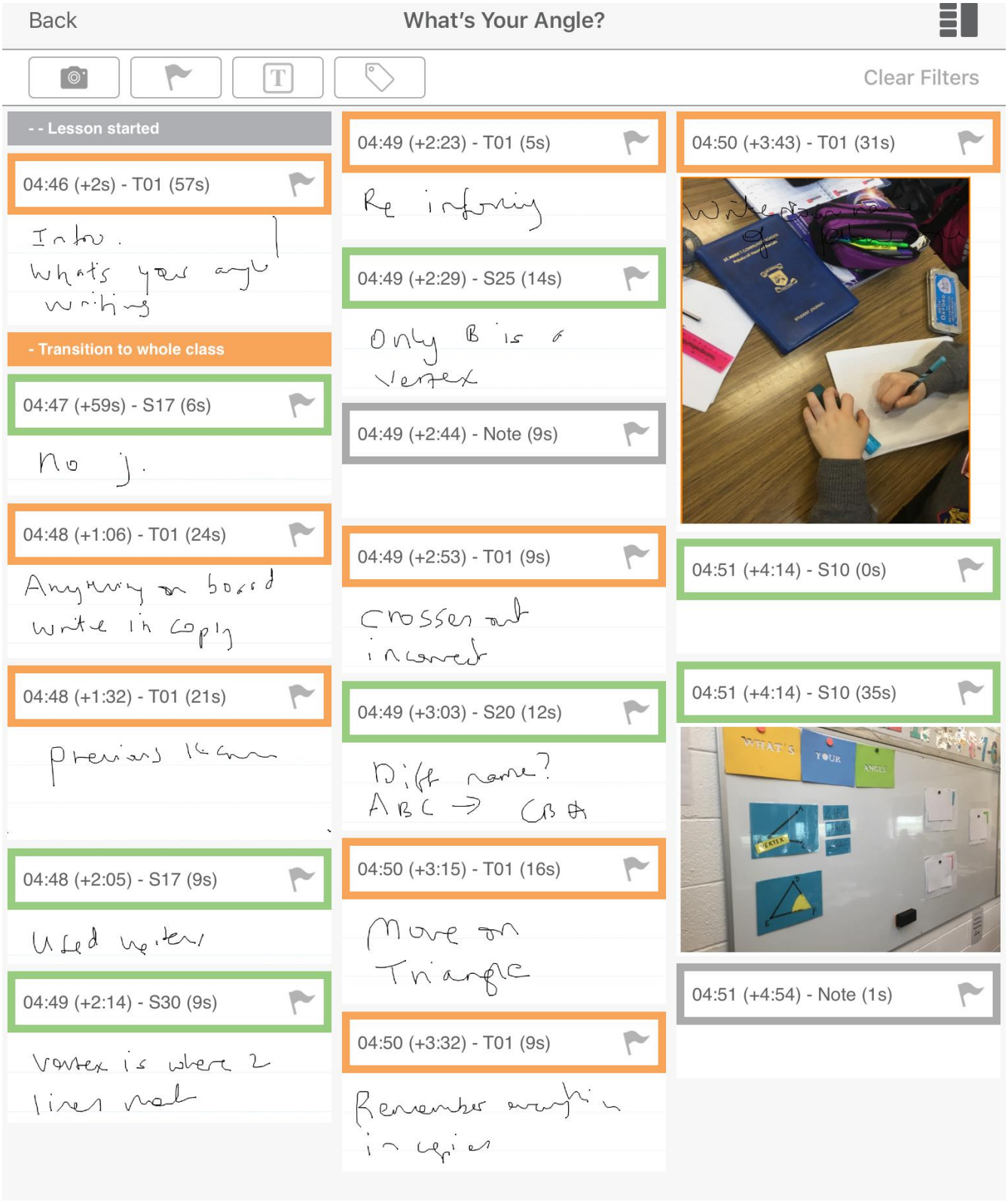
5. Work out the missing angle without a protractor



Appendix B: Classroom layout



Appendix C: Sample of notes taken on Lesson Note during observation



Back

What's Your Angle?

Clear Filters

05:28 (+41:10) - S13 (5s)

At board

05:28 (+41:24) - T01 (7s)

Talk us through

05:28 (+41:34) - S13 (14s)

Straight line Talking

05:28 (+41:50) - S13 (16s)

180 - 51 = 129°

05:29 (+42:16) - T01 » S27 (5s)

What did she do

05:29 (+42:23) - S27 (16s)

She took away from 180°

05:29 (+42:42) - S29 (1s)

05:29 (+42:43) - S22 (8s)

Is D there for a reason

05:29 (+42:59) - S30 (3s)

"Aha"

05:29 (+43:04) - T01 (5s)

Well can back

05:30 (+43:13) - S06 (3s)

05:30 (+43:16) - T01 » S06 (6s)

What did she do

05:30 (+43:24) - S06 (16s)

I added 25 + 23

05:30 (+43:41) - S06 (15s)

I then 180 - 51

05:30 (+43:58) - S06 (12s)

05:31 (+44:29) - T01 » S21 (6s)

What did I do!

05:31 (+44:37) - S21 (7s)

05:31 (+44:45) - S01 (11s)

Can you not take away from 180°

05:31 (+44:58) - T01 (31s)

Yes that's what I did.

Back

What's Your Angle?

Clear Filters

05:32 (+45:34) - S24 (18s)

Board
Ann (5) Straight

05:33 (+46:18) - S23 (5s)

Sit down

05:33 (+46:24) - S28 (4s)

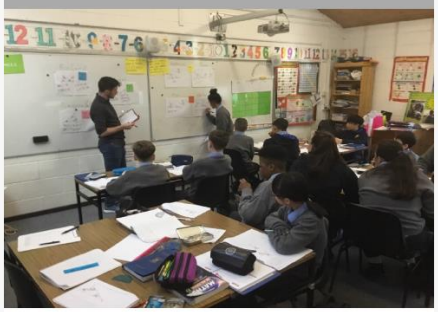
05:33 (+46:28) - S28 (13s)

She took 90° from 180° straight

05:33 (+46:42) - S26 (14s)

Board Triangle
Why

05:33 (+47:04) - Note (12s)



05:34 (+47:30) - T01 » S07 (1…

(7) She added & took away 180°

05:34 (+47:48) - S11 (18s)

Should 149° + 30° not be 180°

05:35 (+48:08) - S17 (11s)

Should be 150° because of 30°

05:35 (+48:29) - T01 (12s)

Re emphasising Measurement errors

05:35 (+48:48) - T01 (9s)

We might expect 150°

05:35 (+49:00) - S22 (10s)

Why should they calculate this

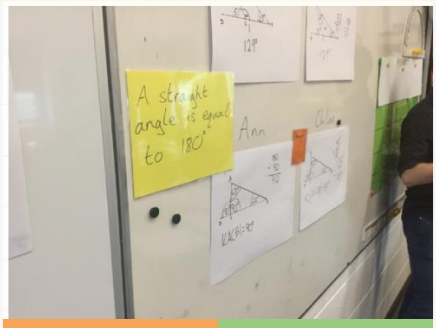
05:36 (+49:11) - S17 (8s)

Bec. straight line

05:36 (+49:21) - T01 (10s)

Emphasis straight

05:36 (+49:33) - T01 (12s)



05:36 (+49:49) - T01 » S14 (1…

What did J & Chloe use

05:36 (+50:00) - S14 (5s)

They used △ 180°