Lesson Research Proposal for Second Year- equations and functions

For the lesson on 15 and 22 /1/18
At John Scottus/Della Salle
Classes Y2 (and Y3)
Instructor: EvaMcCarthy (and Maria Creaney)
Lesson plan developed by: Eva MacCarthy, Maria Creaney, and Ned Pender.

1. **Title of the Lesson:** "When Area is 64", from rectangles to functions.

2. **Brief description of the lesson**

In this research lesson, students will be lead to see that a function can succinctly encapsulate particular types of relationships-in this case length and width of all the rectangles of a given area, by looking at different ways of recording these rectangles. A related contextual problem (set as homework) will help students contrast continuous and discrete functions. There will be many spin offs for future work in Junior Cert and extending to leaving cert maths and science- using graphs for finding outputs (inputs) given inputs (outputs) and comparing accuracy of this with using the function (formula); inverse proportion (speed-time; current-resistance; Boyle's Law), statistical data, domains and codomains to name but a few.

3. **Research Theme**

Learner outcomes:

1) Enjoy their learning, are motivated to learn, and expect to achieve as learners and reflect on this in evaluation.

2) Learners will benefit from consistent, oral and written feedback from the teacher, indicating how their understanding of a given topic meets syllabus expectations. This will be underpinned by the collective review of students' work by the Maths department and an analysis of how well students learn from each other in lesson study.

Learner experiences:

Engage purposefully in meaningful learning activities. Communicate effectively using a variety of means in a range of contexts: to peers, teachers and others verbally, and in written form using diagrams, graphs (including via geogebra), tables and mathematical symbols..

Teachers' individual practice: Teachers design and prepare in advance a sequence of learning tasks and activities using a tried and tested approach which encourages depth of mathematical knowledge and how best to educate students.

Teachers' collaboration: Teachers view collaboration as a means to improve students learning and enhance their own professional development. Learning together is given central place.

4. **Background & Rationale**

"Student understanding of the meaning of letters in algebra, and how they use letters to express mathematical relationships are at the root of algebraic development". Nuffield Report

We chose this topic because, in our experience in the maths classroom, it is difficult for students to go from the specific to the general (concrete to abstract). Students in primary school get familiar with numbers after much manipulation of concrete objects from which they abstract the concept of "Threeness" etc., and begin to grasp the operations of addition and subtraction.

They are introduced to empty squares to be filled with an unknown that they have to work out (See section 5, "Related Prior Learning"). So far it is sensible enough and can be done like a puzzle, as with the patterns that continue through the later primary years.

From first year they continue to generalise and explain patterns in words in the common introductory course and go on to solve first degree equations in one variable.

Somewhere in the development of their algebraic knowledge by second year, students get lost with the use of letters and can't get to grips with solving equations involving x and y, nor transfer their skills to recognise and use algebraic representations of real world problems. It seems that, because they have worked so hard to learn the alphabet and its basis in building words and sentences, and have been rewarded with exciting story books, there is disgust with the treachery of hijacking these same letters for their unfamiliar use as unknowns or variables. "What's wrong with staying with numbers? Aren't there enough of these to satisfy the maths that I need?", is a regular refrain in our maths classes.

Before the introduction of what was called "Project Maths" we mainly taught students to solve linear or quadratic equations through rules and demonstrations, where they worked with the letter relationships with some sense of the letters standing for one, or at most two values. Then functions were introduced as a separate topic in second year, where now the letters are placeholders that can have many values. When they go back to equations they are totally confused
We wanted to bring letters into mathematical usage as meaningfully as when they learned to use them in words. As "*Algebra Through the Lens of Functions, Part 1*"[1], states," It might be better for students to understand why a skill is required before learning it"... They need to see a connection between functions and equations and come to see letter use in maths in contexts that are at least enticing and manageable enough for them to invest their time and be intrinsically rewarded at least to a fraction of that they gained by learning to read.
Our

Facilitating understanding of the role algebra, relations and equations (in particular through functions) and their representations, in the natural world, makes it meaningful to students. If they can be hooked by a problem, there is a better chance of them sticking with it and going on to what we as teachers want them to ultimately learn. It's not a case of sugaring the pill but intrinsically eliciting their curiosity and puzzle solving tendencies. If we can foster an expectation that they can always succeed to some extent and then gain proficiency from exchanging with others, they will have a "bring it on" mentality rather than "but we haven't done this before" or the oft repeated myth of, "I'm just not good at maths, like Johnny or Mathilda". Therefore, we want problems that are accessible to every student in the class-that lend themselves to informal as well as formal means of attack, This is one reason why we picked the problem in this study (and those we will further develop in the unit and beyond). An opening of "How many ways can you…...?" allows for naïve and immediate listing/sketching etc., all the way up to the

[1] Accessed at http://www.projectmaths.ie/documents/pdf/AlgebraThroughTheLensOfFunctions.pdf [November 24th, 2017]

more challenging formal approaches that allow the students to see the beauty and efficiency of functions. But, along the way, every method, and by extension, every student, is prized.

The notation for functions, such as f(x) can cause confusion for students who can have the misconception that it means f times x. In this lesson we want the students to see that W(l) means the width when the rectangle of area 64 has a length l. We are aware of the danger of using first letters to stand for quantities in algebra, e.g. a for apple, but for functions it makes more sense to students to use meaningful letters such as W(*l*+2) for a function when wanting to get the width given a length *l* plus two more units.

4. Relationship of the Unit to the Syllabus

Getting to algebra through functions

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
understand the use of a frame to show the presence of an unknown number: 3 + 5 = • 2 + • = 6. (1st class) 24 + 6 = • 14 + • = 20, 2 + 4 + • = 12. (2nd class) solve one-step number sentences and equations: 75 - 43 = • ; 3.5 x 3 = •: 14 25% of • = 1 (5th class) –3 + +6 = • 10 x • = 8 x 5 (6th class) translate word problems with a variable into number sentences (6th class) 6th class:	To be able to use the specific values of functions to solve equations. Extend the recognition and use of letters to stand for constants, unknowns and variables in formulae. To create a relationship between graphs, functions and equations. To have a better grasp of inverse/indirect proportion and how it contrasts with direct proportion. Appreciate the power of functions to encapsulate much information in precise mathematical language.	Using graphs to solve problems with quadratics Awareness that there are other relations/graphs/functions besides linear, quadractic, cubic and exponential Understanding and using function notation. (For example, even up to leaving cert use of the log function, students will not make the mistake of seeing $\ln(x^2 + x)$ the same as $\ln x^2 + \ln x$. Recognising the importance of domain and Codomain in defining functions Discrete and continuous functions and data

• explore variable in context of simple patterns, tables and simple formulae and substitute values for variables Identify and discuss simple formulae from other strands e.g. d = 2 3 r; a = l 3 w substitution into formulae and symbolic rules developed from number patterns. (Plotting points CIC) Tables diagrams and graphs as a tool for analyzing situations. Patterns and Relations.		

5. Goals of the Unit

Students will engage in problems that allow them to "investigate number patterns, and use ratio and proportionality to solve a variety of problems in numerous contexts." (JC mathematics draft document)

They will come to appreciate:

1) The value of using letters as efficient symbols for short hand;

2) How a formula allows for infinite solutions when trial and error or other methods are limited.

6. Unit Plan

Lesson	Learning goal(s) and tasks
1	Direct proportion relationship, where length is constant and area is varied with width. Function notation A(w): Function/graph used for currency exchange-direct proportion; constant factor (i.e.slope) is exchange rate. £ to € and \$ to \$ and inverse. What if, for example, the euro was the same value as sterling?
2	Research Lesson.
3	Perimeter constant, varying length and width. $2x + 2y = 250$ Reference : Patterns and Relations Approach to Algebra (PMDT 2011)
4	Following on: perimeter remains constant as the area and width changes: Quadratic.

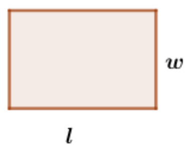
7. Goals of the Research Lesson:

Apply the use of functions to solve word problems.
Understand the concept of discrete and continuous functions with reference to their outputs.
Have a better understanding of constants and variables in functions.
Locate input (output) from a graph when given output (input)
Understand indirect proportion.
Contrast between direct and indirect proportion.

a) Key Skills and Statements of Learning:

1) **Working with others**: Students will learn from each other by observing how their peers solve problems. Students will give and receive feedback to further enhance their understanding.
2) **Staying Well**: Students are given the opportunity to stretch their comfort zone by demonstrating their ideas in front of their peers.
3) **Being Literate**: Students will be given the opportunity to develop their literacy skills by expressing their ideas clearly and concisely to the class. Students will also be exposed to new vocabulary throughout the course of the lesson.
4) **Communicating**: Students will provide feedback when elicited by teachers during discussion (during Ceardaoicht).
5) **Being creative**: Students will learn to be flexible in approaching problems and trying out different ways to solve problems.
6) **Managing myself**: Students will be constructive in how they accept feedback.
7) **Managing Information and thinking**: Students will evaluate different methods of solving a problem and decide if any of the methods is more efficient than the others or if it depends on the context of the problem.
8) **Being Numerate**: Looking at the role of numbers and letters in functions.

8. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	**Teacher Support**	**Assessment**
Introduction (5 mins) Today we are going to work on a problem. The aim is to find as many ways as possible to solve the problem. The problem involves area. Can anyone tell me what do we mean by area of a shape? Look at the diagram on the board. What shape is it? How would you find the area of that shape? Write a formula on your show me boards? (20 secs) Hold up your show me boards?	Present on the board $w(l) =$  $Area = l \times w$ There are some materials on the table you might find helpful to do the task.	Check that students can write a formula for area Area = l x w by observing the show boards as they display their answers to you.
Posing the Task (2 mins)	Project the problem onto the board. Leave graph paper with labelled axes, squared paper, pencils, rulers and blank	What do students choose to do first? Do they hesitate?

<table>
<tr>
<td>

It is grandad’s 64th Birthday. You are going to give him a present of a T-Shirt. You need to put a rectangular colour transfer onto his T-Shirt of Area 64 square units.

How many different ways can you make an area of 64 square units?</td>
<td>tables - see Appendix</td>
<td></td>
</tr>
<tr>
<td>**Student Individual Work (20 mins)**

Figure Diagrams of Rectangles

Drawing rectangles on squared paper. Each rectangle has an area of 64
● randomly
● in order

Listing factors:
● randomly
● In order

Using a table:
<table><tr><td>L</td><td>1</td><td>2</td><td>4</td><td>8</td><td>16</td></tr><tr><td>W</td><td>64</td><td>32</td><td>16</td><td>8</td><td>4</td></tr></table></td>
<td>If students have drawn rectangles on the squared paper, where the area is not 64, then ask them to count the squares. Remind them that the area is constant.
As you go around the class select students who have the methods we are looking for. Also note any different approach.
Ask students are there any faster methods for listing the rectangles with an area of 64.

Are there any other factors?

Take note of students who have incorrect factors so as you can address it in the Ceardaoicht

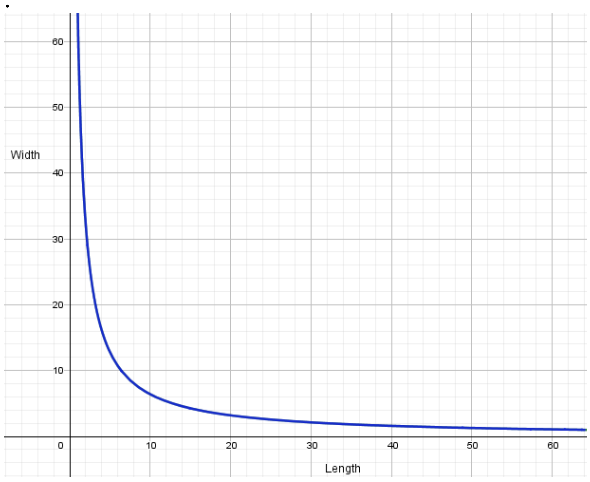
Check the order is correct in the table.

If students have stopped with only one approach ask them if they could represent/show their answers in a different way.

Students might need encouragement to use the labelled axes e.g. “Would the axes be useful for a different way to show your results?</td>
<td>Are students able to tackle the problem?
Are they using the materials effectively?
Are they engaging with the problem.(Actively working on the problem)

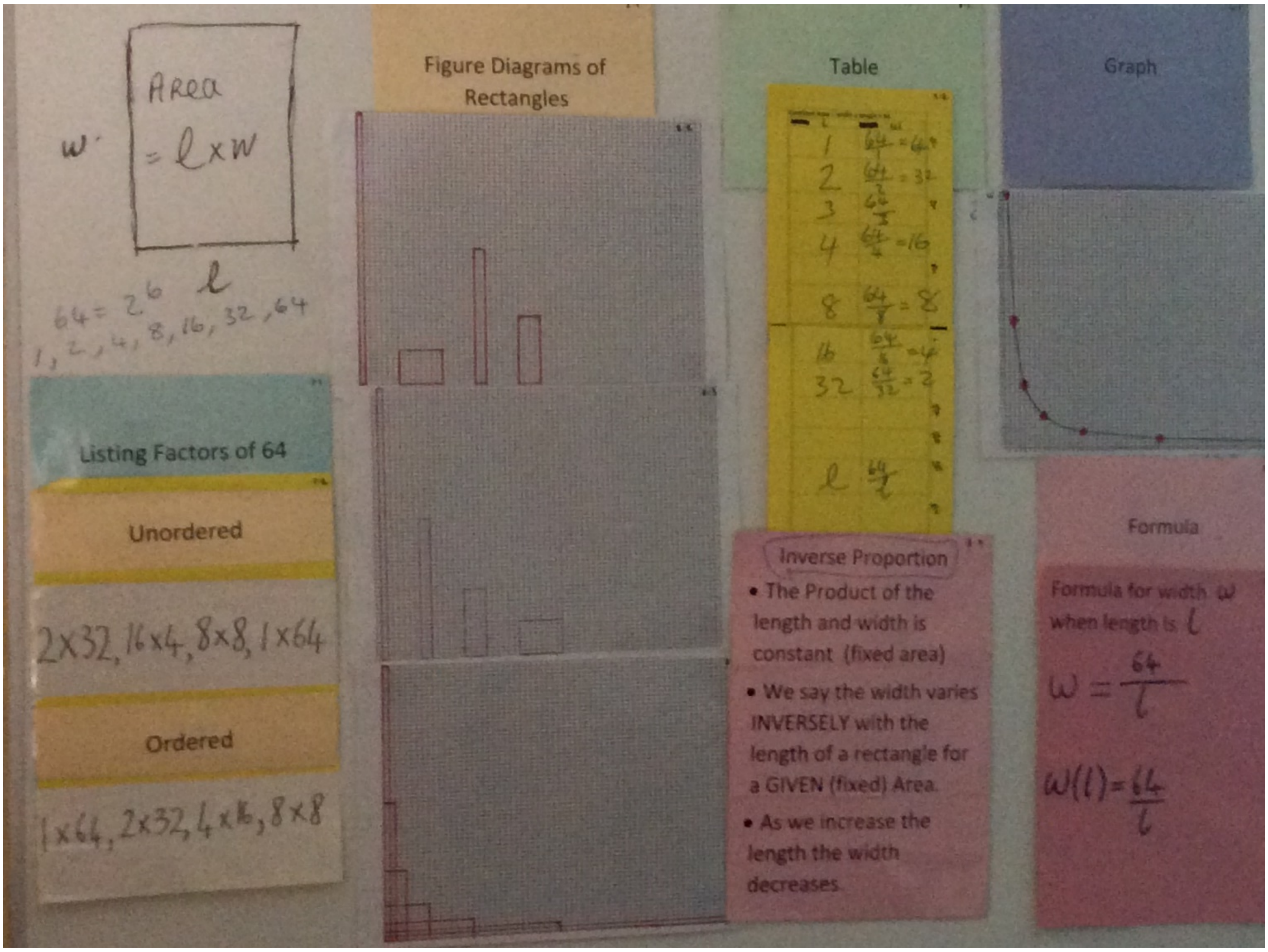
Check students diagrams of rectangles have area of 64.</td>
</tr>
</table>

Description in words: e.g. “You divide 64 by any chosen length to get the related width”. Graphical solution Formula w=64/l w(l) = 64/l	How do you know this will give all possible rectangles? Students who have an ordered table could be prompted to come up with a rule relating variables. (The teacher is conscious of saving the students’ energy by leading them away from dead ends/time by these directive hints, while not obstructing them from doing the work for themselves. In the Ceardaíocht we can address how this notation does not mean w times l.	
Ceardaíocht /Comparing and Discussing) (20 mins) **Student Response 1**	Who used the grid paper to draw rectangles with an area of 64? To demonstrating student: “How did you get the area in this rectangle?” “How many different rectangles were you able to make with area 64” Who put the rectangles in order? How did you choose the order?	Notice how students defend their way of ordering the rectangles. Resist the urge to take over or explain on behalf of students-ask them questions to elicit their explanations.
Response 2 Using a table	Teacher Support Who used a table to get the different dimensions? Here we track (in a systematic way) how the length and width vary. • When the length is 1 the width is 64. • When the length is 2 the width is 32. • When the length is doubled what do we divide the area (64) by to get the width? • When the length is 4 the width is 16. • When the length is multiplied by 4, what do we divide the area (64) by to get the width? • Continue filling in the table emphasizing multiplying the length and dividing the area (64) to get the width.	Are students recognizing the pattern between width and length Do they see what is staying the same and what is varying?

Response 3. Formula	• When the length is l what do we divide the area (64) by to get the width$w$? • Has anyone come up with a formula that we can use for any length? • Say $w = \frac{64}{l}$ • Say $w(l) = \frac{64}{l}$ (Width when length is l) • Have we seen this type of relationship before? • Is it Linear? • Is it Quadratic? • (not for second years - Is it exponential?) • This type of relationship is one of inverse proportion. The product of the variables, length and width, is constant. The greater the length the smaller the width. We say that the width varies inversely with length for a rectangle of GIVEN area.	
Response 4. Graph  	Has anyone drawn a graph showing how the width varies with length? · To draw a graph we need points, where will we get the points? · What are the variables? · We will put the length on the horizontal axis and the width on the vertical axis. · All the graph is in the first quadrant? Why is this? · Can you see the shape of the inverse relationship? · When the length is very, very small, say 1/100 of a unit, what happens the width? · Will the graphs ever touch the x and y axis? · Do you think that the input values of the length have to be whole numbers?	Can students see how the pattern that they recognised in the table is being represented in the graph? Watch the use of language here and the discussion that takes place. Could address how dividing by zero makes no sense i.e. can't have a rectangle with zero length-how is this demonstrated by the graph?
Summing up & Reflection (3 mins)	Distribute evaluation sheet. I	

.Homework (2 mins): Mary is organizing a concert and has 64 tickets which she is giving to charity as prizes. They are to be put into equal bundles. How many ways can she make the bundles? In next class relate the student's solutions to the area activity to compare and contrast.	In next class relate the students' solutions to the area activity to compare and contrast.	Next class check homework to see how students have handled the choices.

10.Board Plan



11. Evaluation

We tried the lesson with two classes-18 Y3 higher level and a class of mixed ability Y2. Overall the higher level students in both classes (Y2 and Y3) were most capable with using the formula explicitly (e.g. inputting decimals or π into the formula, in order to get width from length, rather than just remembering pairs of factors of 64 from times tables.)

Nobody came up with the function notation W(l), but in the Y2 class, when the teacher showed them the final form w(l) = 64/l and asked, "What do we usually use instead of w?" one student in higher said "f", which indicated he had remembered the notation and could connect it to a different context and letter use.

Students also did not easily see that 64 was to be divided by *l* rather than w when the final notation was presented. They had no problem when the table was being presented by a student in the Ceardaoicht, seeing what to input as it was clearly on the left column. In general understanding that the notation f(*x*) means the output when input is *x* can be tricky, though when it is connected to “y-value” and “output”, they can use it, at least in the moment!

(One Y2 student was called to the board because he created a table starting with 16 x 4 and going on to 32 x 2, 64 x 1, 128 x ½, which we had not anticipated but he explained that he was halving the first while doubling the second and so his explanation helped the class to see the inverse pattern and the use of a constant answer to generate couples.

We would have liked to relate this to another student’s table where she went from 1 x 64, 2 x 32, 3 x 21.3, etc going up in consecutive natural numbers, and the pattern was more noticeably non-linear, but she was in the Y3 class and no one there had done it by halving and doubling).

Feedback on whole class:

Most people were on task and seemed to enjoy drawing rectangles and filling tables, though some had to be encouraged to move away from drawing after they had found all the rectangles (they thought) possible with this method (i.e. Whole number lengths and widths).

8 went straight to the preformed table. We had debated beforehand whether or not to label the table so that students got the order right for the final formula, that is, length as a function of width. However it seems, from this evidence that it lead the students straight to using the tables rather than exploring other methods first (e.g. using diagrams or unordered pairs of factors of 64).We might avoid presenting tables at all the next time, just to see if the students would come up with their own without a preformed prompt.

We may have been too ambitious in our goals and perhaps could have emphasized inverse proportion as the major aim rather than culminating in the function notation in the one lesson. The lesson was very effective in introducing inverse proportion via creating a pattern from disorder to order and then in the graph which was a visual way of showing this relationship. Perhaps function notation could be emphasized in a follow-on lesson.

Constructive student discussion took place about 2x32 being the same as 32x2 (indirect observance of the commutative law) but that each order would give a different point on the graph and so two separate solutions to the formula.

However, at first most students stopped when drawing isolated rectangles at 4x16 rather than going on to 16x4 so the table and graph elicited awareness of these as distinct couples in a function.

Nobody had used the “coordinates” to draw a graph, without prompting. Why not (considering that in exam questions a graph is asked for after a table is filled in)? Exam questions prompt them to do so and they do not have the ability/effective knowledge to independently spot the connection/see the graph as a more concise and complete representation of the data. Thus the Flow of the Lesson was hindered by lack of graphs, a gap between table and graph. (Maybe this could be rectified by previous lesson on direct proportion, emphasizing this connection more with a few exam questions for consolidation).

The importance of choosing order in preparation for lesson:

A good board plan helped to give students a connected view at the end of the class.

Did students' apply use of functions to solve problem?:

Yes, some students used fractions and decimals and calculator with the equation for area-inputting L to get W, though not going as far as function notation.

One Student used pi to divide into 64, already going beyond whole numbers as dimensions for the rectangles and heading towards the continuity of a graph.

Drawing the rectangle to start with

Make it clearly an oblong to avoid discussion at this stage of square vs rectangle. Some students left out 8 x 8 because it was a square. It seems they were stuck in a belief that a shape cannot be a member of two sets of shapes at once.
Reflect aims more clearly in lesson flow: distinguish between constant vs variable, discrete and continuous etc.

Content:

Do they have a better understanding of the content of the lesson now? They Saw that area was constant (64) etc emphasized in Ceardaoicht, (especially by careful teacher questioning of students at board). More emphasis could be made to indirect proportion by connecting to previous lesson (as a contrast) when introducing this lesson. Indirect vs inverse.
Key skills met: e.g. being numerate and being creative with coming up with own ways; Learning from others and managing self when explaining and checking understanding.

12. Reflection

Did we observe what we hoped for?

Students used all methods (at least one student in each of the two class groups). Superimposed rectangles (bottom left vertex set at the origin for each) by 2 students allowed flow on to the graph in the Ceardaoicht. Table to graph directly needed prompting (did not use the (ordered) table to plot a graph otherwise).
One student in evaluation said we did not have enough discussion-we need to clarify whether or not group work was allowed during the task, as this was not one of our goals.

While there is much scope for improvement, we were satisfied that the class was learning, as were we, and that lesson study is a rich medium for deepening our teaching of maths and for working together in ways we have not in the past.

The whole class gained a better understanding of:

1) Distinction between constants and variables- students recognised that area was constant at 64 square

units while the length and width varied.

2) Discrete and continuous in the context of tables and graphs. As the lesson progressed, students extended their solutions to decimals and then, from discussion, saw that there is “an infinite number of solutions”. Students who had dots on the graph went on to join them after other students demonstrated their continuous graphs.
(5 out of twenty students (Y2, de la Salle) got as far as graphs with dots. And two of these got to the continuous graph.)
The lesson did allow flexibility and creativity and a chance to try out different ways to solve the problem.
Feedback from each other on their methods helped the classes to see that some methods were more efficient and complete e.g. the concept of infinite solutions arose from seeing the graph-“it never crosses the x or y axis” and another “it never went negative”- some connected this to seeing that you cannot have a zero width or length.

The evaluation sheets showed that the students enjoyed the lesson and that they learnt at least one new thing e.g. “I learnt that there are an infinite number of rectangles of area 64”; “I learnt that a graph is the best way to show all the rectangles possible”.

Overall the lesson provides a springboard for many future lessons, right up to the higher level leaving cert, and can be recalled for this purpose to show continuity of learning (or is that discrete?).

RESOURCES FOR LESSON:

Posing the problem

__T-Shirt for Grandad's 64Th Birthday__

It is grandad's 64th Birthday. You are going to give him a present of a T-Shirt. You need to put a rectangular colour transfer onto his T-Shirt of Area 64 square units.

How many different ways can you make an area of 64 square units?

Come up with as many methods as possible to show how to get the different ways.

Homework

Bundles of kindness

Mary is organizing a concert and is donating 64 of the tickets to be raffled as prizes for a charity.

How many choices of equal sized bundles can she make of the 64 tickets?

See how many different ways you can find to present her possible choices.

STUDENT EVALUATION OF LESSON STUDY

Name ________________________

1. Circle the number that best indicates how clear the question was to you at the beginning.

Very clear				Not clear at all
1	2	3	4	5

2. How many different ways did you find for representing the rectangles?

3. State one thing that you learned in the class:

a) From your own individual work:

b) From your peers/class discussion.

4. How challenging did you find the task, once you were clear what you were being asked to do? (Circle the appropriate number)

Very easy				very hard
1	2	3	4	5

5. Name something that you found helpful today for your learning of mathematics.

6. Name one thing that could be improved, in this lesson, for your learning of mathematics.

Thank you!