# Lesson Research Proposal for 2nd Year Number Systems

Topic: Number

Target: Junior Cert Higher Level

For the lesson on 31st January 2018.

At St. Mary's Secondary School, Charleville.


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1. **Title of Lesson**: Who is being Irrational?

2. **Brief Description of Lesson.**

   After calculating some distances using Pythagoras Theorem, students are required to study the type of number they have calculated. They are then asked to represent the numbers both graphically and visually, as well as describing the numbers using words and symbols. Through this process it is envisaged that students will work towards the discovery and understanding of irrational numbers.

3. **Research Theme:**

   At this school we want students to:

   - To benefit from collaborative work.
   - To be engaged.
   - To develop positive attitudes about their learning

   Teachers are proactive in building collective expertise in the skills and approaches necessary to facilitate student learning for their futures.
   As mathematics teachers:

   - We want students to enjoy and be challenged with doing their math.
   - We want students to explore the language of mathematics.
   - We want to arouse student's motivation and to stimulate their mathematical interest.

   We will actively support the achievement of these goals by:

   - Developing a positive disposition towards investigating, reasoning and problem-solving.
   - Regularly giving opportunities to use prior learning to investigate problems.
   - Students are encouraged to form their own solution to the problems assigned in class. Students are also encouraged to discuss the merits of each other's solutions and appreciate the different approaches.
   - Students reflect on their own learning by discussing various solutions in class and also when asked to self-reflect on what they have learnt at the end of the lesson.
   - By presenting solutions at the boards and sharing them with the class, students have a sense of ownership of the solutions they have uncovered.

## 4. Background & Rational

Why we chose this topic?

This lesson is aimed at second year students. Over the past few years the approach to introducing students to irrational numbers has been as a fact or definition. Following discussions with maths teachers there is a general consensus that students do not gain any deep conceptual understanding of irrational numbers. Being fluent in the number systems is the foundation of maths. Students need to fully understand the different types of numbers if they are to work to any level of depth in maths. Using prior knowledge of number systems from first year we hope to bring them to a deeper level of understanding, progressing onto irrational number and surds. We see the need to link this to Inequalities and to exploring connections between Set Theory and the Number System.

Research findings

"Number sense reflects a deep understanding of mathematics, but it comes about through a mathematical mind-set that is focused on making sense of numbers and quantities." Jo Boaler. Mathematical Mind-sets. Jossey-Bass. 2016

Following discussions in our maths department, we have concluded that this area is important as students find it difficult and it's the starting point of maths. It is rare that students are given opportunities to understand the concept of irrational numbers and the types of situations in which it arises. They disassociate it from maths. We have found that students don't find this an integral part of maths. They don't have a visual of what they are doing. There is a lack of familiarity. There is confusion between rational and irrational numbers. Students struggle to see the relevance or any applications of irrational numbers. To address this we plan to introduce irrational numbers in the context of a problem, where students can apply their prior knowledge to discover a new type of number and the nature of these numbers. It is hoped that by developing a deeper understanding of the concept, students will retain the information for longer, make less mistakes and be flexible in their thinking to apply their learning to other areas (proof by contradiction in later years)

## 5. Relationship of the Unit to the Syllabus

| Related prior learning Outcomes | Learning outcomes for this unit | Related later learning outcomes |
|---|---|---|
| Through primary school, students are exposed to the natural numbers, fractions, decimals and an introduction to integers.<br><br>In first year students will revisit natural and rational numbers. They will also study integers in more detail:<br><br>The binary operations of addition, subtraction, multiplication and division and the relationships between these operations, beginning with whole numbers and integers. They explore some of the laws that govern these operations and use mathematical models to reinforce the algorithms they commonly use | For students to envisage where numbers are on the number line and in the real number system and the relationship between different numbers.<br><br>Operations in the context of rational numbers and irrational numbers (R/Q) and refine, revise and consolidate their ideas. | This topic will relate to complex numbers in senior cycle, -b formula in junior cycle and area in volume in 3rd year. The rationalization of surds later on and sequences and series in leaving cert.<br><br>In essence this lesson is an example of Proof by Contradiction which is on Leaving Cert Higher Level Course. |

## 6. Goals of the Unit

- Students will be able to apply their prior knowledge of the natural number, integers, rational numbers and the number line.
- Students may apply their prior knowledge of inequalities (primary school).
- Students may write any natural number and integer as a rational number. To carry this forward to finding perpendicular slopes.

- Students being able to work with different numbers and recognizing how they relate to each other.
- Students can solve problems using math's symbols and the number system.
- Students understand the characteristics of different types of numbers and inequalities.
- Students gain confidence and familiarity in the use of numbers. (Wellbeing)
- Students will be able to link to other areas of the mathematical junior cert course. E.g. sets
- They are developing the level of thinking, from concrete to abstract areas of math's to reach higher order thinking.
- Students make the link between their prior knowledge and the task.

## 7. Unit Plan

| Lesson | Learning goal(s) and tasks |
|---|---|
| 1 | Revision of prior knowledge from first year on Number |
| 2 | Plotting numbers on number line and inequality signs involving N (subset) and Z |
| 3 | Inequalities involving Real Numbers |
| 4<br>Research Lesson | Research lesson on Irrational Numbers |
| 5 | Continuing on with further classification of numbers in Number System including classification of square roots and surds. |

## 8. Goals of the Research Lesson:

Mathematical Goals

- Recognition that some numbers do not fall into N, Z and Q.
- Recognize what these numbers have in common.
- Use of prior knowledge of Number types/extended families
- Understand the word surd and distinguish between a surd and square root

Key Skills & Statements of Learning

In preparation for implementing the Junior Cycle Specification for Mathematics our maths department have begun to acknowledge key skills which historically we have tried to develop into our maths lessons.

This lesson will address the following key skills:

1. Being Literate: Through Ceardaiocht, students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: By engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.
3. Managing Myself: Students will have the opportunity to reflect on their own learning when the teacher asks them to write a reflection at the end of the lesson.
4. Staying Well: By engaging in tasks which are appropriate to their abilities, students' confidence and positive disposition to learning will be promoted.
5. Communicating: During Ceardaiocht, students will present and discuss their mathematical thinking.
6. Being Creative: Students' will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with Others: Students will; earn with and from each other by discussing different approaches to solving problems.
8. Managing information and thinking: Students will be encouraged to think creatively and critically.

This lesson also meets the following Junior Cycle Statements of Learning:

1. The student communicates effectively using a variety of means in a range of contexts.

15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.

17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

**9. Flow of the Research Lesson:**

| **Steps, Learning Activities**<br><br>**Teacher's Questions and Expected Student Reactions** | **Teacher Support** | **Assessment** |
|---|---|---|
| | | |
| **Introduction (5 mins)**<br><br>In today's lesson, we will use our mathematical knowledge to solve a problem. The students will work individually and then will collaborate and use their knowledge to come up with something new.<br><br>Prior Knowledge<br><br>Check students understanding of number system concepts through keywords poster.<br><br>Before we start today's problem, I want to quickly review some maths we learned a few weeks ago. "Can anyone remember Pythagoras Theorem and what it was used for? | | Are students engaged?<br><br>Make sure they ok with... checking their understanding… knowledge and learning. Checking for understanding.<br><br>Note the quality of their understanding<br><br>Also challenging their<br><br>their visual spatial intelligence |
| **Posing the Task (3mins)**<br><br>Mystery Venue<br><br>Buses are provided from six different stops. There are two buses from each stop. Buses are numbered as follows:<br><br>Bus 2 travels 2km<br><br>Bus 3 travels 3km<br><br>One bus goes directly to the party venue. The second bus does not go to the venue but meets another bus which will go the the mystery venue.<br><br>Please find a diagram depicting the 6 different stops on your desk. | | |

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| Your task is to find the number of the unnamed bus in each diagram.<br><br>An extension to this will be :<br>Having found the numbers :<br>Represent the numbers you found for the bus<br>Graphically & Visually<br>Describe the numbers you found for the bus<br>Using words & Using symbols<br><br>Clarifying the extension problem:<br>Students will find a visual of the the real number system, place cards with the bus numbers on them, empty number line, | | |
| **Student Individual Work (part 1)**<br>**(10 mins)**<br>Students are asked to find the bus numbers individually.<br>We expect the following response to be the completed response. It is very possible that students may not start in this order but we feel these will be the order they volunteer solutions they are satisfied with.<br><u>Student response 1</u><br>We expect that the first bus number found will be 5 in a 3, 4, 5 triangle. | | |

| | | |
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| Student response 2<br><br>We expect that the second solution the students will come up with is bus number 10 in the 6,8,10 triangle.<br><br>Student response 3<br><br>Now that all the natural numbers are dealt with we expect students to tackle the triangle with 4.5 and 6. We think students are more comfortable with .5 rather than other decimals. Bus number 7.5<br><br>Student response 4<br><br>Bus number 8.9 from triangle 3.9,8 and 8.9 is next completed solution.<br><br>Student response 5<br><br>Students will come up with the first surd answer ($\sqrt{2}$) from the isosceles triangle with units of 1.<br><br>Student response 6<br><br>Students will find last bus number. ($\sqrt{17}$) from the 1 and 4 triangle.<br><br>Misconception<br><br>Some students may find difficulty getting surd for bus number. | | |
| **Ceardaíocht (part 1)**<br><br>**(14 mins)**<br><br>We anticipate that we will share the reponses on the board in the order we have listed them above. We will push students to verbalise their answers and nudge them into classifying their answers,<br><br>**Student Individual Work on Extension**<br><br>**(10 mins)**<br><br>Student Response 1<br><br>Students will be able to place the natural numbers on numberline, on real number system | | |

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| and write description in their own words and using the symbol N. This may not all come from an individual student as we expect students will have a preference for working with either the number line, the real number system diagram or the written explanations.<br><br><u>Student response 2</u><br><br>We expect that students will not move towards the decimal answers. Again we expect to see some able to place on decimals on number line, real number system. We expect that students will recognise that the decimals are rational numbers. If necessary students will be pushed for an explanation of rational numbers and an ability to clearly define rational numbers. We expect students will be connect the letter Q with these decimals.<br><br><u>Student response 3</u><br><br>We will be looking out for students who use a compass to graph the surds. We will also be expecting students to try to evaluate the surds on their calculator. We expect that students will be able to use their knowledge of sets to place the cards with surds outside the Q set but inside the R set. We would really be delighted if students could use their knowledge of set notation to symbolise these surds.<br><br>In terms of words we expect students to verbalise what the surds are not.<br><br>**Ceardaiocht (part 2)**<br><br>**(15mins)**<br><br>We anticipate that we will share the responses on the board in the order we have listed them above. We aim to have a definition for irrational numbers on conclusion. | | |
| **Summing up & Reflection (3 mins)**<br><br>The teacher will summarise the main ideas of the lesson. At this point a homework task will | | |

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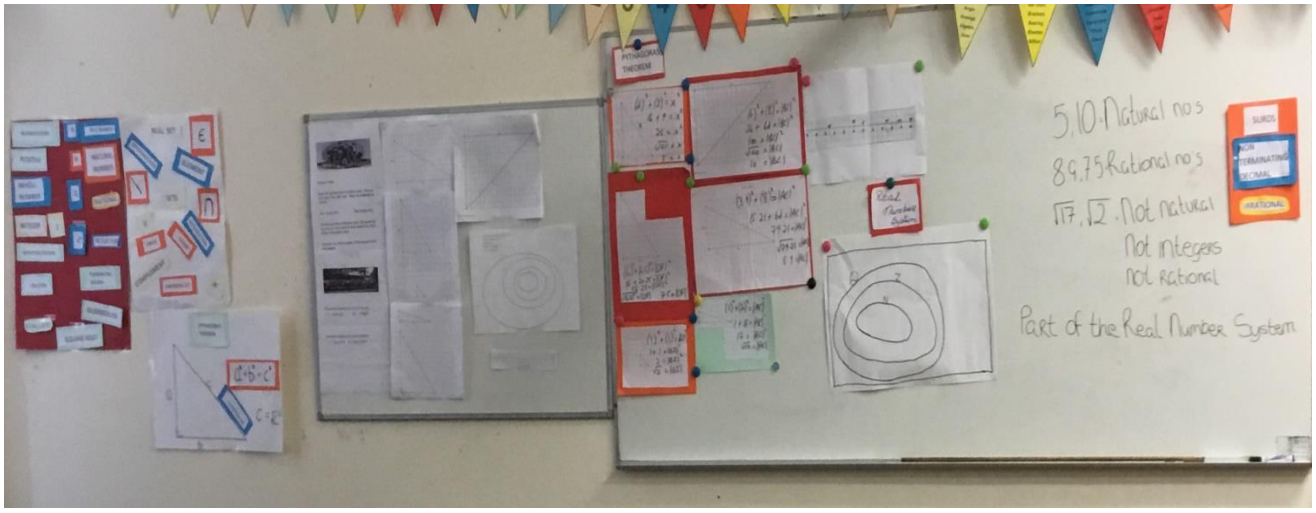
| be handed out. The homework will involve classifying some numbers.<br><br>The students will be asked to write up a short reflection on the lesson. | | |

**10. Board Plan**




**11. Evaluation**

Everybody felt the lesson was a success, and that the goals we hoped to achieve had been met. There was consensus that all students understood the concept of irrational numbers by the end of the lesson regardless of how successful their own individual work had been. Students were engaged in the task from the outset. All the students responded positively to the prior knowledge. The majority of student settled down very quickly to work on the initial task. All students were successful in finding at least one solution, with the majority finding all six solutions to the initial task. All students managed to successful complete some part of the extension task.
Students enjoyed the opportunity to confer on their solutions and also the collaborative nature of Ceardaiocht.

Some students really only grasped the idea when it came to Ceardaiocht. We noticed however that the confidence of these particular students grew with peer consultation.
The team were extremely happy that the students used the idea of what these number "were not" to realise that another class of number exists. This was a successful introduction to the idea of "proof by contradiction", which we had hoped for but were concerned that it may be a step too far.

## 12. Reflection

While reflecting on the lesson the team felt that getting students to represent the number both graphically and visually was too much work within the time frame. It was felt that graphing these numbers on the number line should be held over for the follow up lesson. This would have allowed for more time to be spent on the real number system and developing the goal of the lesson which was to discover the idea of real numbers that are not rational.

We realised that the prior knowledge was key to this lesson. Following this research lesson we feel the teaching of the prior knowledge in earlier classes should be amended as follow:

- When teaching Pythagoras there needs to be an emphasis on the squaring of the lengths and in turn finding the square root.
- It would be important that students could verbalise that the √ symbol is the square root as many did not have the right language.
- We realise that there seems to be a greater emphasis on fractions over decimals in the Common Introductory Course. The team will be making a recommendation to the full Maths Department that decimals get greater emphasis. For example it was vital that students could recognise repeating and terminating decimals. Students need to be conscious that the calculator is not the end of non-terminating decimals. This misconception could have derailed our lesson.

This lesson provides a starting point for future lessons, right up to higher level leaving cert.

For our school's maths department this lesson was an exercise in self-reflection and self-assessment, allowing us to evaluate, make adjustments and thus improve the standard of maths education we deliver.


Mystery Venue

Buses are provided from six different stops. There are two buses from each stop. Buses are numbered as follows:

Bus 2 travels 2Km Bus 3 travels 3Km

One bus goes direct to the party venue. The second bus does not go to the venue but meets another bus which will go to the mystery venue.

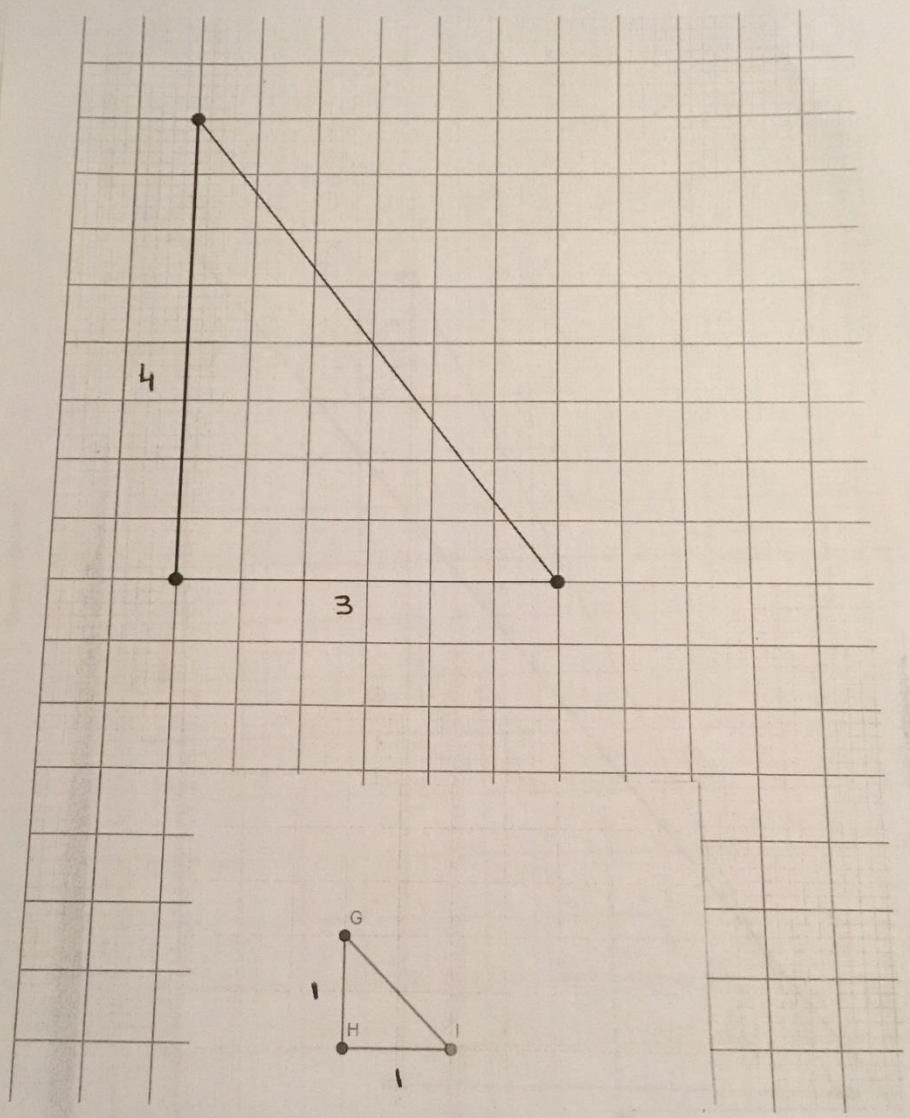
Your task is to find the number of the unnamed bus in each diagram.


Represent the numbers you found for the bus:

(i) Graphically (ii) Visually

Describe the numbers you found for the bus:

(i) Using words (ii) Using symbols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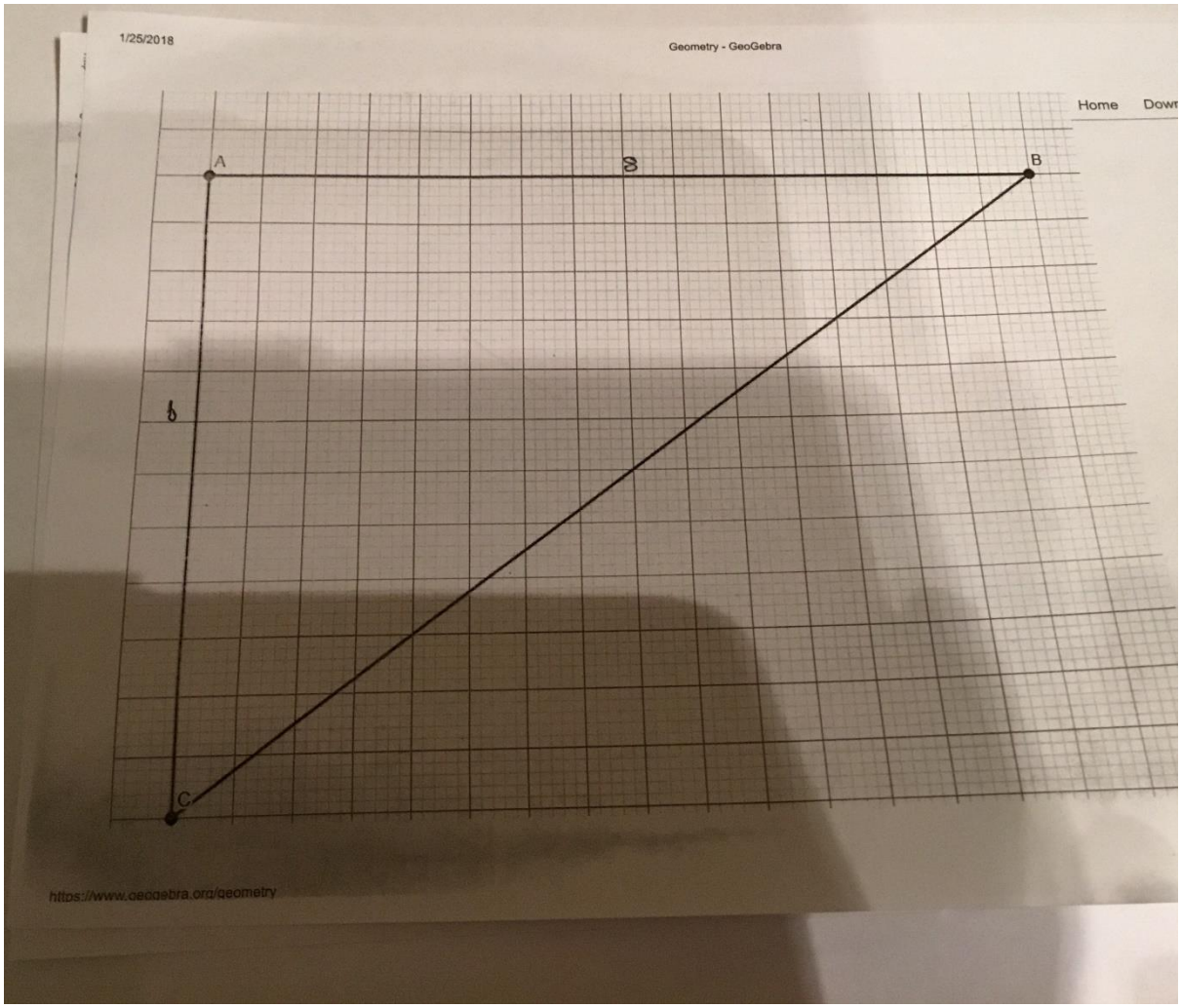
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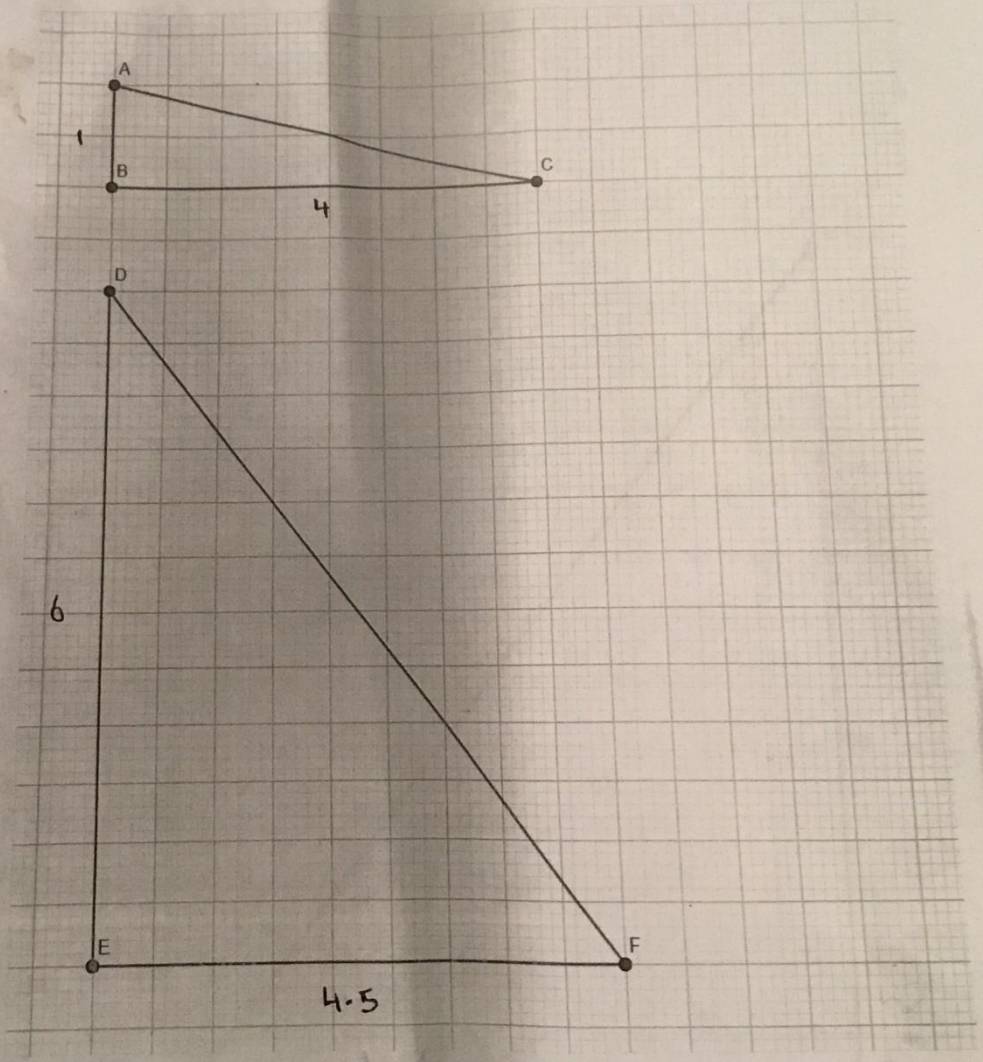

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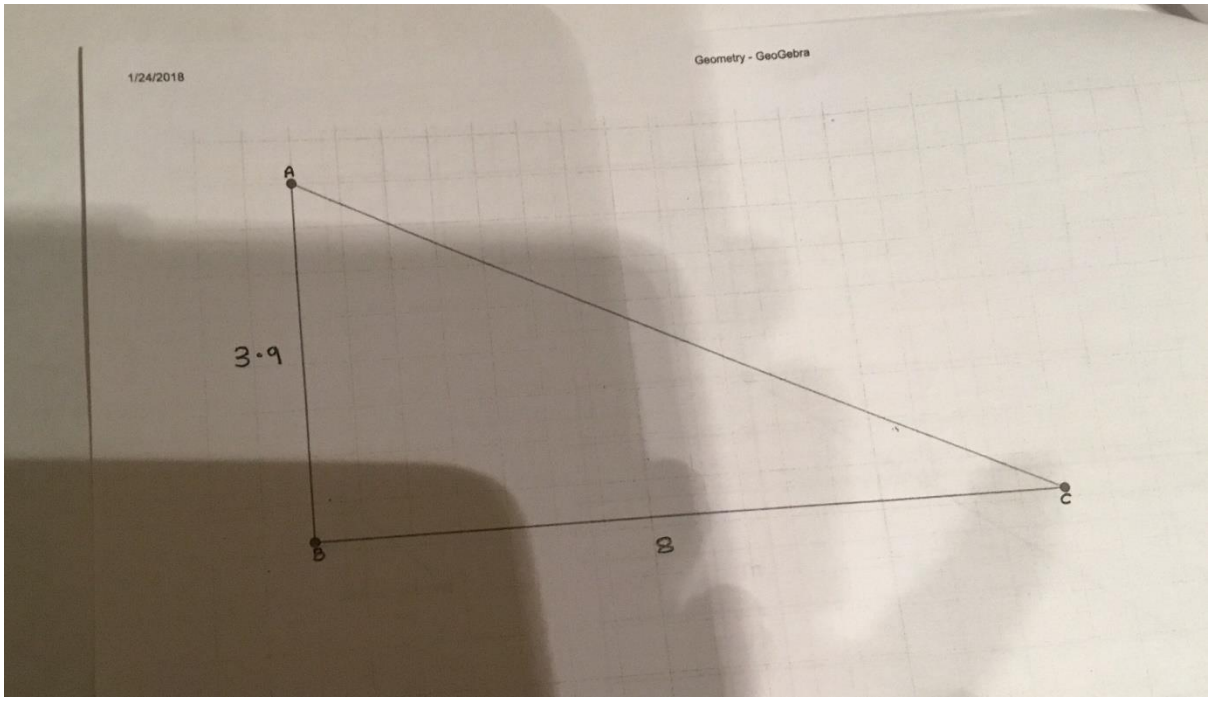
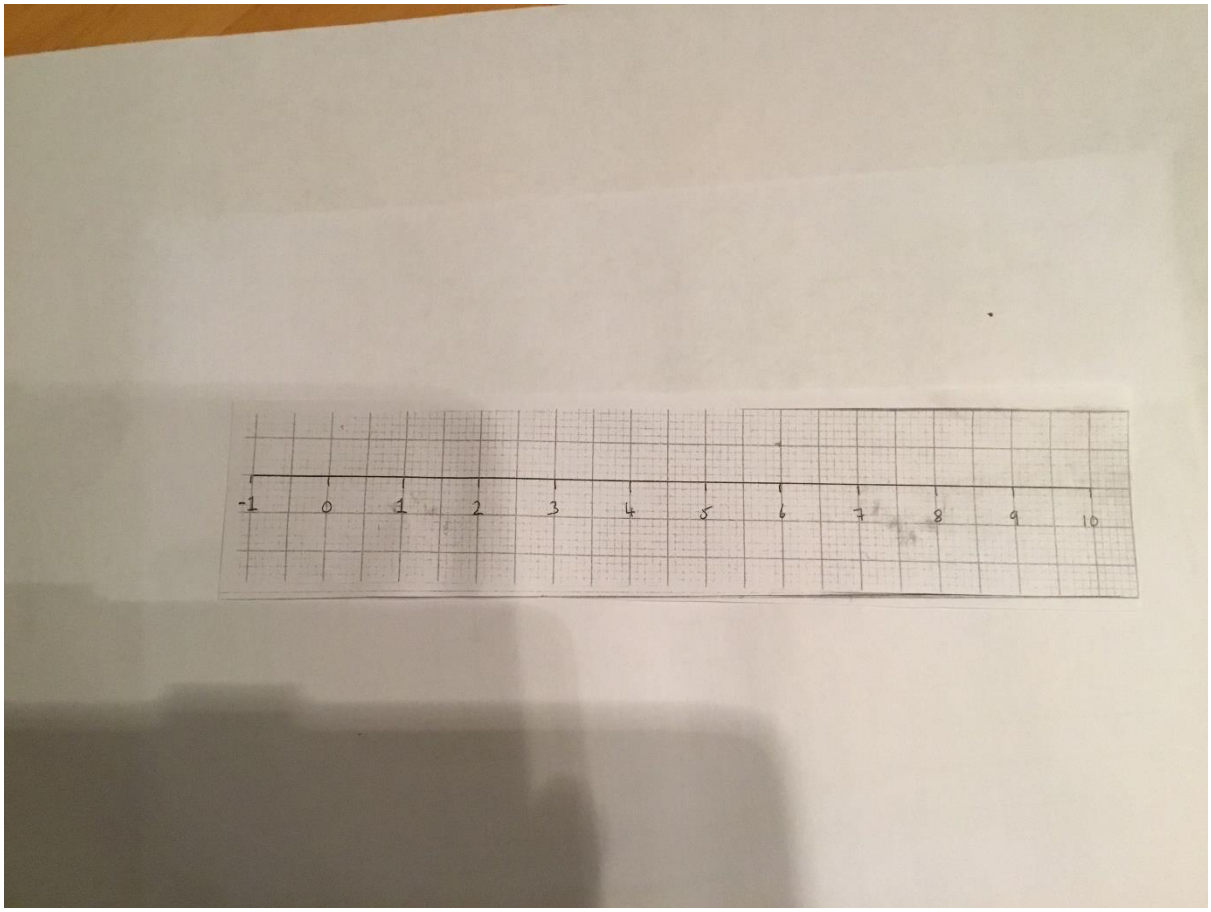
Geometry - GeoGebra



The diagram represents the sets:
Natural Numbers $\mathbb{N}$
Integers $\mathbb{Z}$
Rational Numbers $\mathbb{Q}$
Real Numbers $\mathbb{R}$