Lesson Research Proposal

Date of lesson: 28 February 2018
School name: St. Flannan's College Ennis
Teacher giving lesson: Michael Ryan
Associate: Séamus Ó Conghaile
Lesson plan developed by: Aoife Naughton; Catherine O'Brien; Michael Ryan

1. Title of the Lesson: "2*x* or not 2*x*? That is the question!!"

2. Brief description of the lesson

The building blocks of $2x$, $x+2$ and x^2 are often mishandled and misinterpreted. This lesson provides a physical approach to the understanding of *'like'* and *'unlike'* terms. Students will work with these physical building blocks (using substitution, comparisons and combinations) to gain a better appreciation of $2x$, $x+2$ and x^2.

3. Research Theme

At St. Flannan's College we want students to:

- Grow as learners while contributing their opinions and experiences to class discussion with confidence.
- Be able to report on present and explain the process and outcome of learning activities to a highly competent level.

As mathematics teachers we will actively support the achievement of our goals through:

- Assessing all relevant aspects of students learning using both Assessment of Learning and Assessment for Learning techniques and through engaging in collaborative practices
- Our intention to share teaching strategies and resources within the Mathematics department initially and subsequently other departments.

'One of our main goals for SSE in St. Flannans's College is Assessment for Learning'

4. Background & Rationale

We have chosen this area of introduction to Algebra for our lesson study. We have chosen this area of study as because the students traditionally have difficulty in this area.
Students find the transition to the use of letters and abstract concepts within mathematics difficult.
We hope that students will be able to recognise the differences between like and unlike terms and that they will be able to recognise the differences between different terms such as x^2, $2x$ and $x+2$.
We have picked this area as we have noticed that students have a particular difficulty with these aspects of the topic.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes

Within the primary curriculum the following Exploring and using patterns Examine and discuss money affairs Identify and discuss simple formulae from other strands e.g. d = 2 x r Transfer word problems with a variable into number sentences e.g. a length of tape divided into 5 parts, each being 30 cm - find the total length.	Students will identify patterns and make the connection between these patterns and algebraic expressions Students will represent unknown numbers using letters Students should recognise like/unlike terms and use order of operations when simplifying expressions Students should be able to multiply out brackets using the array model Student should be able to find formulae examining algebraic relationships and using graphs	This is a fundamental building block of all aspects of algebra.

6. Goals of the Unit

The students will:

- Evaluate Algebraic expressions.
- Add and Subtract algebraic terms.
- Multiply algebraic terms and expressions.
- Students will develop problem solving skills and appreciate that there are multiple approaches to problem solving.
- Students will gain the value and respect of each other's work.
- Gain a greater confidence in their own mathematical ability in the area of algebra
- They will gain a better appreciation and understanding of the difference between algebraic expressions

7. Unit Plan

Lesson	Brief overview of lessons in unit
1.	Introduction to Patterns
2.	Patterns leading to algebraic expressions
3.	Introduction to language of algebra and like terms – understand the terminology including coefficient, variable, term, constant, expression, like and unlike terms

4.	Substitution and evaluation of algebraic expressions – find the value of an expression by substituting given number for letters
5.	**Research Lesson** - Comparative look at unlike terms and common misconceptions
6.	Multiplication of terms and brackets (x2)
7.	Multiplication of two expressions (x2)
8.	Revision and assessment (x2)

8. Goals of the Research Lesson:

a. Mathematical Goals - Students will:

- Understand and appreciate the difference between the fundamental building blocks ($2x$, $x+2$, x^2)
- Find the value of expressions by substituting in given numbers for letters
- Develop a greater understanding of the meaning of coefficient, variable, constant and expression in the context of algebra.

b. Key Skills & Statements of Learning

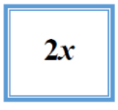
This lesson incorporates the following Junior Cycle Key Skills:

- Being Literate: Students will have the opportunity to develop their understanding the use of mathematical language.
- Being Numerate: It will develop a positive and confident feeling towards the abstract area of algebra
- Managing Myself: Students will have the opportunity to reflect on their own learning.
- Staying Well: Students will become confident and positive about learning.

This lesson will centers predominantly on the Junior Cycle Statements of Learning number 16 – The student describes, illustrates, interprets, predicts and explains patterns and relationships.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	**Teacher Support**	**Assessment**
Introduction (5 mins) Review terminology of term, coefficient, constant and 'like terms'	Teacher probes the class to evaluate and to consolidate their understanding.	Can students explain term, coefficient and like terms?
Posing the Task (5 mins) "Does $2x = x+2 = x^2$?" Task split into 2 parts (Investigation by substitution and evaluation):-		Correct evaluation of

1st Task: Each student is given a simple handout to fill out. 2nd Task: **(15 minutes)** The second part of the task is handed out.	Initial Investigation by substitution 	x	$2x$	$x+2$	x^2
---	---	---	---
1			
2			
3			
4			| Teacher shows first row (as per PowerPoint) done to show students what to do to encourage and get them on task and set everyone up for success Initial Investigation by substitution | x	2x	x + 2	x^2
---	---	---	---
1	2(1) = 2	(1) + 2 = 3	$(1)^2 = 1$
2			
3			
4				 Teacher reminds students of the importance of adherence to BIRDMAS rule Each student is presented with a pack of these blocks (terms) (laminated and cut-out version of blocks in attached word doc, 10 of each block) and working in pairs they must make as many different combinations of these blocks as possible in the allotted time ***(15 mins)***, record the combinations in the table, simplify the added expressions and evaluate each expression thereby leading to a clear understanding of the difference between these building blocks   	substituted terms in handout – thus ensuring competency in this technique and also seeing from the results that sometimes these terms evaluate to the same result while other times they do not. (Hence further investigation required!!) Can students create the different combinations? Can students create the expressions? Can students simplify the expressions? Can students evaluate the expressions?

Solution Number	Building Block 1	Building block 2	Building Block 3	Simplified expression	Value when x = 1	Value when x = -1
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

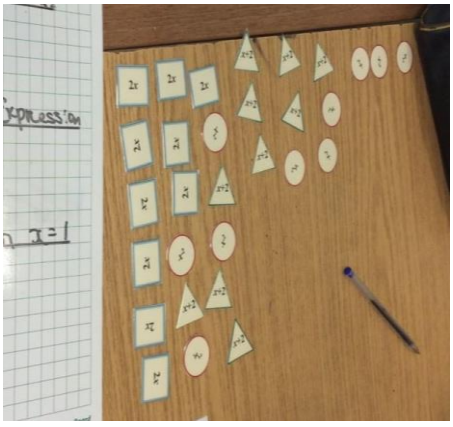
Teacher suggests for students who are struggling to look into other students' work near them.

Teachers remind students that the same block can be used more than once in a specific combination

Student Individual Work (within 15 mins above)

Solution Number	Building Block 1	Building block 2	Building Block 3	Simplified expression	Value when x = 1	Value when x = -1
1	$2x$	$2x$	$2x$	$6x$	6	-6
2	$2x$	$2x$	$x+2$	$5x+2$	7	-3
3	$2x$	$2x$	x^2	x^2+4x	5	-3
4	$2x$	$x+2$	$x+2$	$4x+4$	8	0
5	$2x$	$x+2$	x^2	x^2+3x+2	6	0
6	$2x$	x^2	x^2	$2x^2+2x$	4	0
7	$x+2$	$x+2$	$x+2$	$3x+6$	9	3
8	$x+2$	$x+2$	x^2	x^2+2x+4	7	3
9	$x+2$	x^2	x^2	$2x^2+x+2$	5	3
10	x^2	x^2	x^2	$3x^2$	3	3

An illustration of the building blocks in action.



Misconceptions:
$2x+2x+2x=6x^3$
$x^2+2x=4x$
writing $x^2=x2$

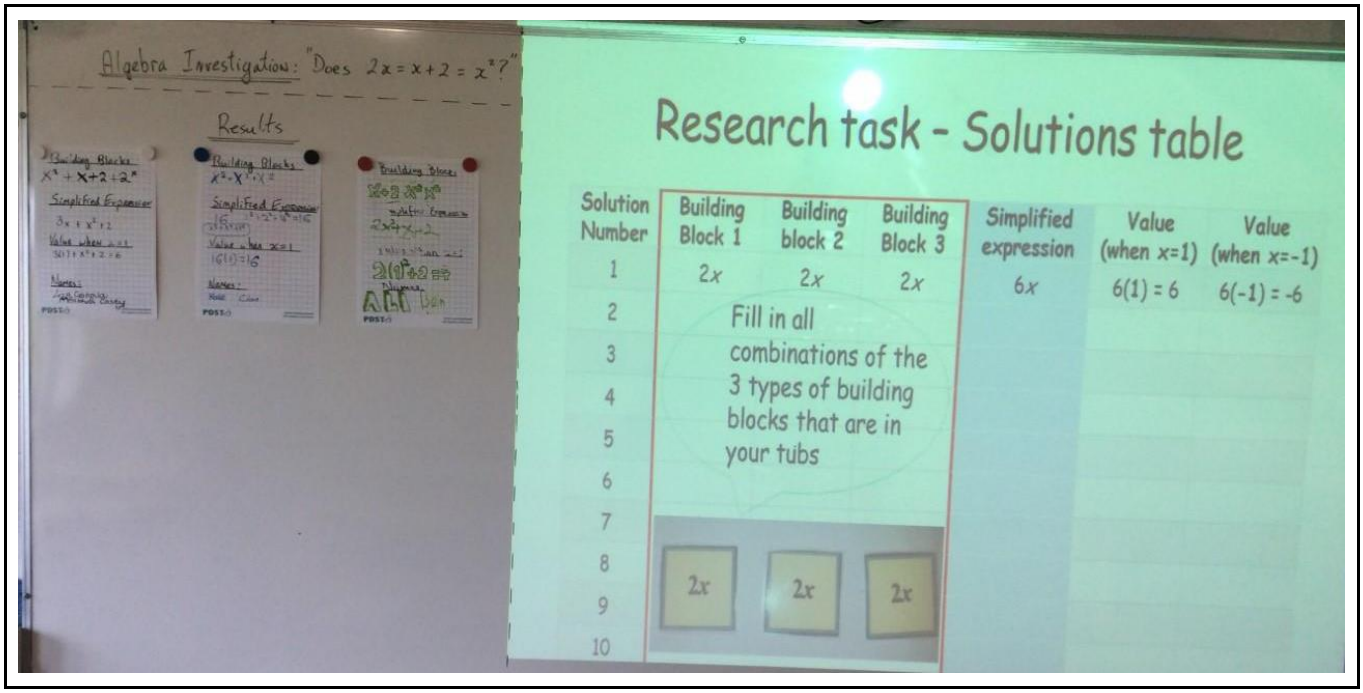
Observing and noting students work and interactions.

	$2x+2=4$	
Ceardaíocht /Comparing and Discussing Someone from each group (if possible) share a particular combination ***(10 mins)***	Teacher asks individual students to fill out/add to table of solutions on board. Teacher asks students to thoroughly explain their work through clever questioning. Type of questions used Where are you getting the 5x from the 2x and 2x? How come it's not 6x or 5x or $5x^2$ Why are you not getting $x^{4?}$ Are you sure? Could it be $x^3 + 6$? How come its '2'x^2 Q10. How come you got a positive answer with (-1) being subbed in? Full table is revealed	Could the solution added be made simpler? Could it be expressed differently? Do students learn from their peer's work and explanations? What did the student have to do to get the value suggested (e.g. using BIMDAS and rules for multiplying negatives etc.)?
Summing up & Reflection Students fill out an exit card with a section to put down their favourite expression/expression they found most challenging to evaluate etc. ***(5 mins)***	Teacher suggests the completion of the evaluation section of the table with the negative value if not already done so as a homework exercise. Students will be asked to fill in the reflection sheet and leave it behind. All students work should be left for assessment.	Can students evaluate 2x, x+2 and x^2? Do students know that 2x, x+2 and x^2 are **not** like terms? Could students verbalise the differences between 2x, x+2 and x^2?

10. Board Plan

Carefully plan the board work before the lesson takes place to decide on the order of the solutions and the links that will be made at the board. Put an image or a diagram of the pre-prepared board work here.

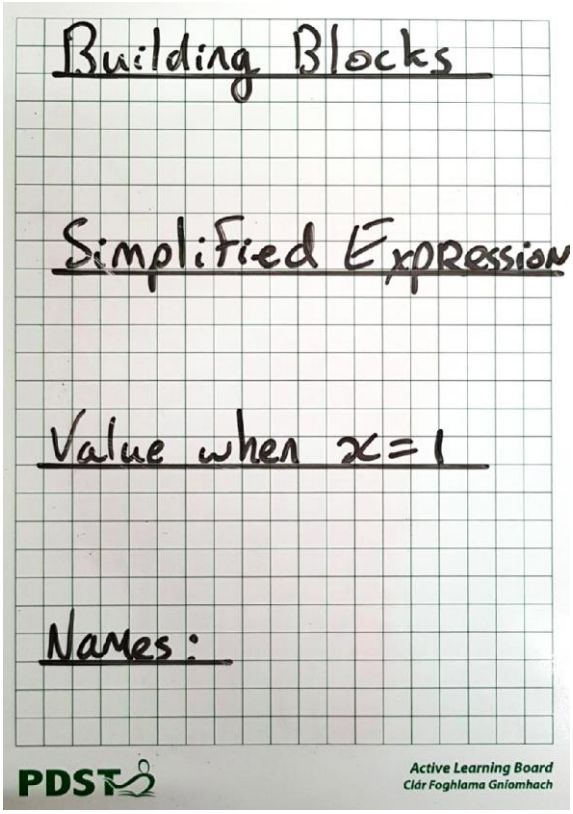
Board Plan



11. Evaluation and Reflection

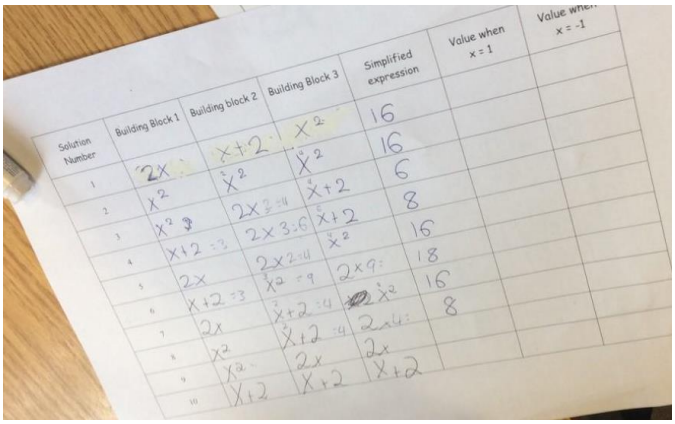
Mathematical discussion and debate was evident throughout the class whether on a peer to peer basis or on a whole class basis. It was observed that students' responses were being listened to / observed and used for learning.

Significant learning and understanding of the differences between 2x, x+2 and x squared had been expected in the class. Learning was observed throughout the class. The initial task gave students an insight into the differences between the three terms / expressions and they found this investigation very valuable. Some students struggled with the idea of creating the different combinations but once they received a helping hand from a colleague or a teacher they were able to do this part quickly. Students then created expressions and worked in pairs. They explained and questioned one another on a frequent, open and engaged basis. The use of the organized show me boards to display a groups finding to the class worked very well. Students were fully attentive and were witnessed self-correcting in the ceardaíocht. A misgiving was that there wasn't sufficient time in the single 40-minute lesson to fully develop the ceardaíocht. It was also observed that a student with SEN struggled with the lesson and while a significant number enjoyed the lesson, a number of students showed signs of anxiety during the lesson.



Misconceptions

A number of the misconceptions witnessed in class include $x+2+x^2+x^2=2x^2+2$ and when x =1 that $x^2 + x^2 + x^2 = 16$. On this occasion the student was thinking that 1^2 = 2 and the student was then substituting 2 into the second x^2 to get 4 and then substituted 4 into the third x^2 to get 16. After the ceardaíocht had been completed and the class had been dismissed the student who had this answer explained to the teacher that they were now convinced that $x^2 + x^2 + x^2 = 3$ when x =1. Another misconception was that 3^2 = 3.



Ideas for future study

1. Do the initial task in the previous lesson to allow for more time in the research lesson for the second task. It may be worth eliminating the first task entirely to see how students would deal with the task.
2. Do the second task with the combinations given as outlined in the picture below so that students

focus on creating and evaluating expressions (see below picture).

Research task – Solutions table

Solution Number	Building Block 1	Building block 2	Building Block 3	Simplified expression	Value (when x=1)	Value (when x=-1)
1	$2x$	$2x$	$2x$			
2	$2x$	$2x$	$x+2$			
3	$2x$	$2x$	x^2			
4	$2x$	$x+2$	$x+2$			
5	$2x$	$x+2$	x^2			
6	$2x$	x^2	x^2			
7	$x+2$	$x+2$	$x+2$			
8	$x+2$	$x+2$	x^2			
9	$x+2$	x^2	x^2			
10	x^2	x^2	x^2			

3. Spread the task over a number of lessons as there is significant amount of learning and revision to be derived from it.

Appendix of enlarged handouts / slides

Initial Investigation by substitution

x	$2x$	$x + 2$	x^2
1			
2			
3			
4			

-

Initial Investigation by substitution

x	$2x$	$x + 2$	x^2
1	$2(1) = 2$	$(1) + 2 = 3$	$(1)^2 = 1$
2			
3			
4			

Solution Number	Building Block 1	Building block 2	Building Block 3	Simplified expression	Value when x = 1	Value when x = -1
1						
2						
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6						
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10						

Solution Number	Building Block 1	Building block 2	Building Block 3	Simplified expression	Value when x = 1	Value when x = -1
1	$2x$	$2x$	$2x$	$6x$	6	-6
2	$2x$	$2x$	$x + 2$	$5x + 2$	7	-3
3	$2x$	$2x$	x^2	$x^2 + 4x$	5	-3
4	$2x$	$x + 2$	$x + 2$	$4x + 4$	8	0
5	$2x$	$x + 2$	x^2	$x^2 + 3x + 2$	6	0
6	$2x$	x^2	x^2	$2x^2 + 2x$	4	0
7	$x + 2$	$x + 2$	$x + 2$	$3x + 6$	9	3
8	$x + 2$	$x + 2$	x^2	$x^2 + 2x + 4$	7	3
9	$x + 2$	x^2	x^2	$2x^2 + x + 2$	5	3
10	x^2	x^2	x^2	$3x^2$	3	3