

# Lesson Research Proposal for 1st Year Common Level

Date of lesson: 13/02/2019

School name: Hartstown Community School

Teacher giving lesson: Rowan Webb

Associate: Brian O'Malley

Lesson plan developed by: Rowan Webb, Maeve Cunningham,  
Sinead McCarthy, Dawn Hutchings-Walsh

1. **Title of the Lesson:** A Pizza the (Fr)Action

2. **Brief description of the lesson**

Students are given a problem to consolidate their learning of adding fractions and to apply this to the addition of fractions with a variable.

3. **Research Theme**

At Hartstown Community School, we want students to:

- a) have an enquiring and open-minded attitude towards mathematics
- b) enjoy their learning, be motivated to learn, and expect to achieve as learners.

These goals reflect a focus on our school's SSE Priorities, which include student voice and learner engagement.

In order to achieve these goals in our classes, we:

- a) Use the PDST competency test to assess exactly what our students know coming in from primary school and in Transition Year to assess their learning from Junior Cycle. We then use this information to tailor our classes to this particular cohort of students
- b) Attempt to create problems that arouse the students' interest and motivation, and that have multiple solutions
- c) Use questioning and scaffolding techniques in order to try to show that there are multiple ways in which students can achieve.

4. **Background & Rationale**

Two sections to discuss:

Why we chose this topic?

This lesson is aimed at 1st Years. As a group we discussed how important it is for students to have a clear understanding of algebra from first year and how this can avoid misconceptions in other topics in maths later on. We discussed the difficulties students have with understanding algebra with many students struggling greatly with solving Algebra equations that involved fractions. These common misconceptions with fractions can be seen to lead into further issues at Senior Cycle, with many students not being able to solve Linear Equations with fractions or adding Algebraic Fractions.

**Research Findings:**

As a maths department we discussed the difficulties first year students had with fractions and address misconceptions of fractions developed by students from Primary School. As it stands , Algebraic fractions are not taught until a later stage in 2nd Year .We felt that if students were to link numeric fractions with Algebraic Fractions in 1st year , it would give students a better understanding of Algebraic fractions and also make students aware of the link between Algebra and other areas in Maths.

**5. Relationship of the Unit to the Syllabus**

Describe how this unit relates to the syllabus/learning outcomes from prior years, for this year and for future learning.

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Operations of addition, subtraction, multiplication and division Order of operations Simplifying, adding, subtracting, multiplying and dividing fractions Solving word problems involving natural numbers, integers and rational numbers	Students can simplify, add, subtract and multiply fractions Students can perform these operations with an algebraic numerator or denominator Students can use problem solving approaches and link their learning of rational numbers to their learning of algebra and variables.	<ul style="list-style-type: none"><li>● Factorising polynomials</li><li>● Calculus</li><li>● Trigonometry</li><li>● Functions</li><li>● Area and volume</li><li>● Statistics</li></ul>

**6. Goals of the Unit**

Describe the cognitive or emotional changes within the student. What students need to know or understand in order to....

- a) Students will learn how to simplify algebraic fractions, (specifically adding/subtracting and multiplying), with an emphasis on understanding the link between algebraic and numerical fractions
- b) Students will develop the skills to work both independently and collaboratively in a problem-solving context
- c) Through experiencing success in a structured learning environment, students will develop the motivation and willingness to attempt further problems throughout the unit
- d) Teachers will collaborate to develop common practice in methodologies

## 7. Unit Plan

How the research lesson fits into the larger unit plan, helping to show the bigger picture of the whole unit and the progression of learning. Clarify where the research lesson will be taught.

Lesson	Brief overview of lessons in unit
1	Simplifying Fractions
2	Add/Subtract Fractions
3	Research Lesson - Introduce the concept of the variable $x$
4	Basic Algebraic Fractions - Numeric Numerator
5	Basic Algebraic Fractions - Numeric Denominator
6	Algebraic Fractions 2 term numerator ( $x+1$ )
7	Algebraic Fractions - Different Algebraic Denominators

## 8. Goals of the Research Lesson:

Looking at the goals of the research lesson itself from two perspectives:

### a. Mathematical goals:

The goals of the research lesson is for students to be able to add fractions with a variable as the numerator.

By the end of the lesson, students will have used their prior knowledge of fractions and apply this to adding 2 fractions together with  $x$  as the variable of one of the numerators.

### b. Key Skills and Statements of Learning

In the planning and design of this lesson the Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will implement and promote JC Key Skills in the following ways:

1. Being Literate: Students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: It will develop a positive disposition towards problem solving.
3. Managing Myself: Student's will have the opportunity to reflect on their own learning.
4. Staying Well: Students' confidence and positive disposition to learning will be promoted.
5. Communicating: Students will present and discuss their mathematical thinking.
6. Being Creative: Students' will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with Others: Students will learn with and from each other.
8. Managing information and thinking: Students will be encouraged to think creatively and critically.

This lesson is also designed to meet the following JC Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.

15. The student recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning.

17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills

9. **Flow of the Research Lesson:**

<b>Steps, Learning Activities</b> <b>Teacher's Questions and Expected Student Reactions</b>	<b>Teacher Support</b>	<b>Assessment</b>
<p>This column shows the major events and flow of the lesson, including timings and what will go up on the board.</p>	<p>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</p>	<p>This column identifies (a) what the teacher will look for (formative assessment) that indicates it makes sense to continue with the lesson, and (b) what observers should look for to determine whether each segment of the lesson is having the intended effect.</p>
<p><b>Introduction</b></p> <p>Today we are going to use our mathematical knowledge to solve a problem. We're going to try to solve the problem by ourselves and then we're going to come together as a class and use all your knowledge to discover something new</p>		<p>Can we do it? Yes we can!</p>
<p><b>Posing the Task</b></p> <p>John and his friends are ordering pizza. They order a rectangular pizza and split the toppings: half pepperoni and half cheese.</p> <p>They cut the pepperoni side into 8 pieces and the cheese side into 12 pieces.</p> <p>Part 1: Ann takes 1 slice of pepperoni and 2 slices of cheese. How much of the total pizza has she eaten?</p>	<p>Distribute problem handout and explain the problem</p>	<p>Check for understanding and scaffold questions for groups who may not understand</p>

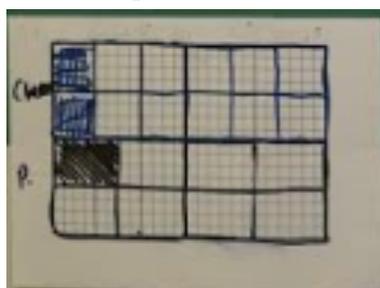
Part 2: Can you find an expression to show how much of the total pizza she has eaten if she takes 1 slice of the pepperoni and any number of the cheese?

**Student Individual Work**

**Part 1:**

**Student response 1**

Students colour in the laminated hand out to reflect the amount of pizza eaten.



**Student response 2**

Students use the laminates to divide each half of the pizza into 24, colour and count the pieces out of 24



**Student response 3**

Students use equivalent fractions to find a common denominator and simplify.

As the teacher makes class rounds, we look for examples that will lead the students to an understanding of the methods to solve the problem.

If students using the laminates to simply count 3 out of 20 slices, they will be encouraged to look at the size of the pieces. As we believe this may be a common misconception, a student may be selected to explain why this does not work.

For students who might have stopped after colouring, we can ask: "Could you do the above in a more efficient way".

Are the students able to handle this problem?

Are they using insight and trying different methods?

PEPPERONI                      CHEESE

$$\frac{1}{8} + \frac{2}{12}$$

$$\frac{1}{8} \times 3 = \frac{3}{24}$$

$$\frac{2}{12} \times 2 = \frac{4}{24}$$

$$\Rightarrow \frac{7}{24}$$

**Student response 4**

**MISCONCEPTION**

Students add  $\frac{2}{12}$  and  $\frac{1}{8}$  to get  $\frac{3}{20}$ .

$$\frac{2}{12} + \frac{1}{8} = \frac{3}{20}$$

**Part 2:**

**Student response 1**

Trial and error. Students may try a number of different slices of pizza to try to spot a pattern

$\frac{1}{8} + \frac{1}{12}$	$\frac{1}{8} + \frac{2}{12}$	$\frac{1}{8} + \frac{3}{12}$
$\frac{1}{8} \times 3 = \frac{3}{24}$	$\frac{1}{8} \times 3 = \frac{3}{24}$	$\frac{1}{8} \times 3 = \frac{3}{24}$
$\frac{1}{12} \times 2 = \frac{2}{24}$	$\frac{2}{12} \times 2 = \frac{4}{24}$	$\frac{3}{12} \times 2 = \frac{6}{24}$
$\frac{3}{24} + \frac{2}{24} = \frac{5}{24}$	$\frac{3}{24} + \frac{4}{24} = \frac{7}{24}$	$\frac{3}{24} + \frac{6}{24} = \frac{9}{24}$

**Student response 2**

Introducing variables. Students may write the

expression as the addition of 2 fractions, one of which with a variable as the numerator.

$$\frac{1}{8} + \frac{?}{12}$$

$$\frac{1}{8} + \frac{x}{12}$$

**Student response 3**

Algebraic fractions. Students will write the expression as the addition of two fractions, including a variable, and simplify.

$$\frac{1}{8} + \frac{x}{12}$$

$$\frac{1}{8} \times \frac{3}{3} + \frac{x}{12} \times \frac{2}{2}$$

$$\frac{3}{24} + \frac{2x}{24}$$

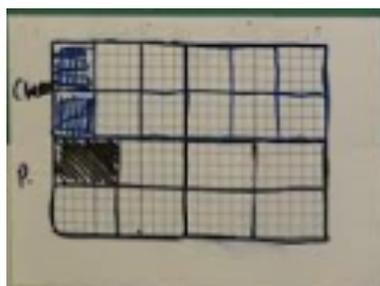
$$\frac{2x+3}{24}$$

**Ceardaíocht /Comparing and Discussing**

**Part 1:**

**Student response 1**

Students colour in the laminated hand out to reflect the amount of pizza eaten.



**Part 1:**

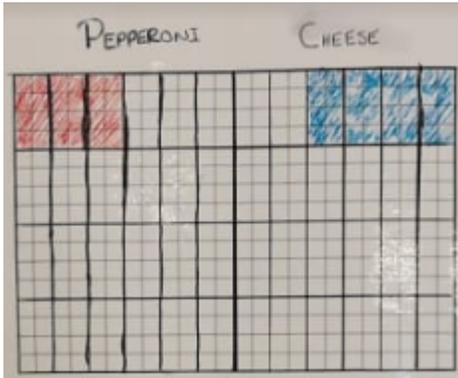
**Response 1:** Please raise your hand if you used this method. Did anyone else get the same method?

Are students able to discuss their solution? During this discussing, what is the level of interaction?

Are students looking at the merits of the various responses? Do students understand the concept of addition of fractions? Can they explain it in words?

### Student response 2

Students use the laminates to divide each half of the pizza into 24, colour and count the pieces out of 24



### Student response 3

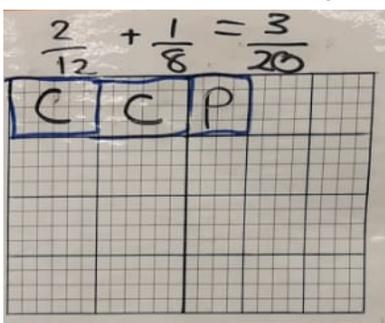
Students use equivalent fractions to find a common denominator and simplify.

$$\begin{array}{l} \text{PEPPERONI} \qquad \qquad \qquad \text{CHEESE} \\ \frac{1}{8} \qquad + \qquad \frac{2}{12} \\ \frac{1}{8} \times \frac{3}{3} = \frac{3}{24} \\ \frac{2}{12} \times \frac{2}{2} = \frac{4}{24} \\ \Rightarrow \frac{7}{24} \end{array}$$

### Student response 4

#### MISCONCEPTION

Students add  $\frac{2}{12}$  and  $\frac{1}{8}$  to get  $\frac{3}{20}$ .



**Response 2:** Would anybody like to talk about this solution? Why does this diagram represent the problem more clearly than this one? What are the advantages/disadvantages of using this method? Would you prefer it to the first method?

**Response 3:** Would anybody like to talk about this solution? Do you notice a pattern? What is the relationship between the fractions? Would you prefer this method rather than method 1 or method 2? If so, why? Can you now explain to me how this method works?

**Response 4:** Why does this method not work? Would you be happy to get a slice this size as opposed to a slice this size? Do they represent 20 equal slices?

**Part 2:**

**Student response 1**

Trial and error. Students may try a number of different slices of pizza to try to spot a pattern

$$\frac{1}{8} + \frac{1}{12}$$

$$\frac{1}{8} \times 3 = \frac{3}{24}$$

$$\frac{1}{12} \times 2 = \frac{2}{24}$$

$$\frac{3}{24} + \frac{2}{24} = \frac{5}{24}$$

$$\frac{1}{8} \times 3 = \frac{3}{24}$$

$$\frac{2}{12} \times 2 = \frac{4}{24}$$

$$\frac{3}{24} + \frac{4}{24} = \frac{7}{24}$$

$$\frac{1}{8} \times 3 = \frac{3}{24}$$

$$\frac{3}{12} \times 2 = \frac{6}{24}$$

$$\frac{3}{24} + \frac{6}{24} = \frac{9}{24}$$

**Student response 2**

Introducing variables. Students may write the expression as the addition of 2 fractions, one of which with a variable as the numerator.

$$\frac{1}{8} + \frac{?}{12}$$

$$\frac{1}{8} + \frac{x}{12}$$

**Student response 3**

Algebraic fractions. Students will write the expression as the addition of two fractions, including a variable, and simplify.

**Part 2:**

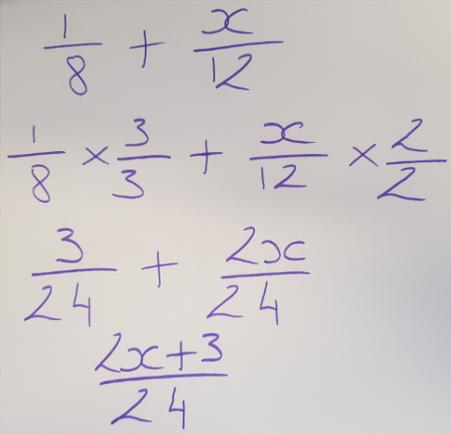
**Response 1:** Please raise your hand if you used this method. Did anyone else get the same method?

**Response 2:** Would anybody like to talk about this solution? What are the advantages/disadvantages of using this method? Would you prefer it to the first method?

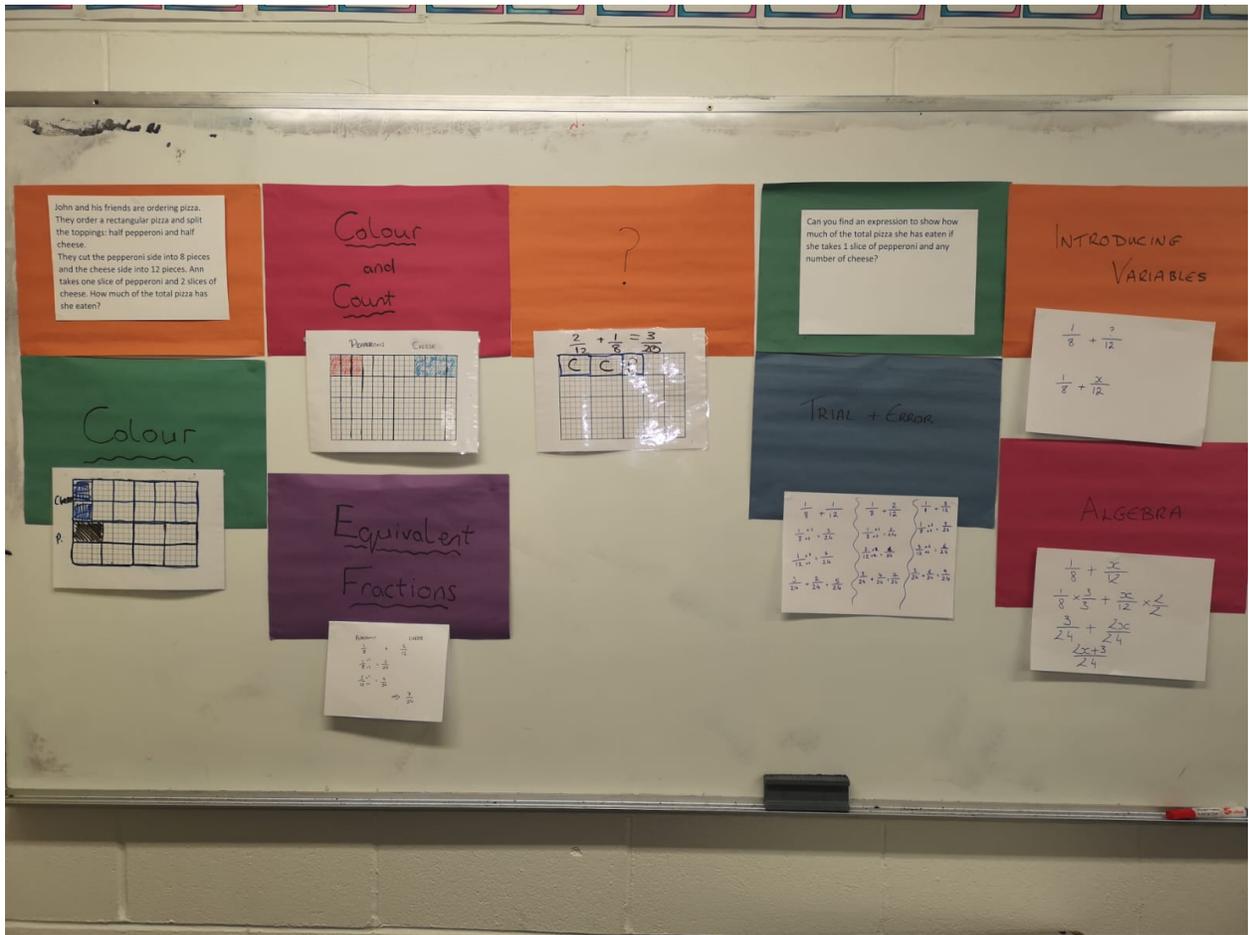
**Response 3:** Would anybody like to talk about this solution? Do you notice a pattern? What is happening to the numerator and denominator every time for the pepperoni side? What about the cheese side? Would you prefer this method rather than method 1 or method 2? If so, why? Can you now explain to me how this method works? What is different about this solution to the first

Are students able to discuss their solution? During this discussing, what is the level of interaction? Are students looking at the merits of the various responses?

Do students understand the concept of adding fractions with a variable? Can they explain it in words?

	<p>one? Do you see any similarities?</p>	
<p><b>Summing up &amp; Reflection</b></p> <p>We learned that:</p> <ol style="list-style-type: none"> <li>1) To add fractions, we need to find a common denominator</li> <li>2) We use equivalent fractions to find a common denominator</li> <li>3) We can use algebra and equivalent fraction to make an expression to represent an unknown amount</li> </ol> <p>Extension exercises will be provided for students to take home.</p>	<p>Use the layout of the board work to help provide students with a summary of the progression in their learning.</p>	<p>Do the students' reflections represent the teacher's view of the lesson?</p>

## 10. Board Plan



## 11. Evaluation

In terms of our evaluation, we focused on the following:

- Are all students able to access the content of the lesson at some level?
- Does the lesson motivate students to seek alternate answers and to think critically?
- Did the lesson successfully promote student-to-student discussion and enable student voice?
- Do students understand why it is necessary for both fractions to have the same denominator before we can add or subtract?
- Can students introduce a variable as the numerator of a fraction and apply their learning to add fractions with variables?

## 12. Reflection

We had hoped to observe students being motivated by the task so as to push them to keep trying alternate methods. We hoped they would see value in this and enjoy their learning. We wanted them to make

connections between their learning in number and in algebra. We also wanted to promote student voice and allow every student a chance to access the content.

Students were initially reluctant to work together as they are a quiet group, however the problem soon prompted them to start sharing their thoughts. By the second half of the lesson they were communicating their maths and motivating each other to try different things. Student voice was a big part of the lesson and we felt that they all got a chance to share their learning and contribute to group discussions.

Many students originally demonstrated the misconception that we had planned for in the first part of the problem, but after the discussion no students made this mistake in the second part of the problem. All students were able to correctly introduce a variable into their expression and most moved on to being able to find a common denominator and equivalent fractions. Some stopped at this point as they were unsure of how to write the numerator as an expression.

We think that the lesson went very well overall and that the lesson goals were met. We had extended the class by 20 mins (usually classes are 40 mins) in order to allow more thinking time. This worked well for the lesson as planned but in future we might divide this problem up over the space of 2 or 3 classes in order to maximise the learning that can be gained from student discussion.

From a logistical point of view, while students enjoyed the laminated sheets to draw on, it meant that they could erase their work easily if they thought it was wrong. If we were doing this again we would print multiple worksheets so that all student thinking can be captured and discussed. The use of the phrase “half of the pizza” was a sticky point for some students – they thought they had to find half of an eighth and this led them down the wrong path. We would keep the working to “two equal sides” and “one side...and the other side” in future. Students commented that they found the lesson enjoyable, challenging and that they would like to do it again. Some found the problem very challenging and said that they couldn’t do it, however this was not reflected in their work.

We learned that even our most quiet students can thrive in the environment of problem-based lessons with the right support and questions. We expected that some students would be reluctant to come up with more than one response, but we also learned that our students are capable of much more than we expect when the right planning has gone into the lesson.