

Lesson Proposal for 5th Year Ordinary Level Geometry

Date of lesson: 12th. Feb 2019

School name: St. Mary's Secondary School Charleville.

Teacher giving lesson: Martina O Dwyer

Lesson plan developed by: Martina O Dwyer, Lorraine Murphy & Lillian Waters

Lesson Study Associate: Lillian Waters

1. Title of the Lesson: Circling “Longitude”

2. Brief description of the lesson

Students will be presented with a real life scenario of a drone taking a photo at a music festival. Students will be tasked with finding the best position for the drone to capture the three main stages in the photo in as many ways as they can.

3. Research Theme

At this school we want students to:

- Benefit from collaborative work.
- Be engaged.
- Develop positive attitudes about their learning
- Enjoy and be challenged with doing their math.
- Explore the connections between geometry/coordinate geometry and the real world.
- Arouse student's motivation and to stimulate their mathematical interest.

As Maths teachers we wish to:

- Design suitable challenging problems which will encourage our students to think insightfully and creatively - providing our students with opportunities to come up with their own approaches to solving a problem - developing their confidence, competence and

communication skills through expressing their ideas to their peers in their own words which enables the comprehension of all students in the class.

- Engage in the collaborative Lesson Study process with a view to: - improving mathematics teaching and learning and students' experiences in the mathematics classroom - increasing the emphasis on the links between the strands of the mathematics syllabi - developing a consistent department-wide approach to the teaching of mathematics in our school.
- Focus on our School Self Evaluation and employ a number of Assessment for Learning techniques. For example, involving students in their own learning by assessing their own work and reflecting on their learning at the end of key lessons.

4. Background & Rationale

We chose to focus our lesson, based on constructions, coordinate geometry of the line and circle, at 5th Year Ordinary level students. In our experience, students don't see the link between coordinate geometry and constructions even though the topics are covered in close proximity to each other. They have all the Maths but aren't making the connections. The problem we chose relates these topics to a real life scenario. An appropriate balance needs to be struck between developing and consolidating candidates' basic skills on the one hand, and developing their capacity to apply, mathematize and reason in less familiar contexts on the other.

Following discussions with maths teachers in our school we concluded that students are challenged by spatial reasoning and particularly by geometry problems in an unusual context. It was felt that students have difficulty applying their knowledge and skills to solve problems in familiar and unfamiliar contexts. We recognise that the students need more understanding and appreciation of how Maths can be applied to real life situations. We recognise the difficulty of word based problems from previous Chief Examiner's reports.

Through discussions of members of the Maths Department we realise that our teaching of circumcentre was imbalanced towards the mechanical construction rather than its applications.

In designing the research lesson we believe it is important to engage students enthusiastically with

the subject matter. The lesson proposal tries to devise creative ways to make it easier to comprehend this concept by illustrating the problem and using suitable teaching aids. The approach depends on allotting students plenty of time to think about the problem and figure it out on their own. It is hoped that by developing a deeper understanding of the concept, students will retain the information for longer, make less mistakes and be flexible in their thinking.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<p>At Junior Cert, students become familiar with:</p> <ul style="list-style-type: none"> -Perpendicular bisector of a line segment construction. -Properties of lines and line segments including midpoint, slope, equation of the line. -Find the point of intersection of two lines 	<ul style="list-style-type: none"> -The instruments that are used to perform constructions with precision. -The significance of the point of intersection of two linear relationships. -Perform constructions on the leaving cert course how to apply these in real-life situations. -Solve problems involving slopes of lines (perpendicular) 	<ul style="list-style-type: none"> -Make them more aware of links between topics -Problem solving -Better visual representations of the line and circle when dealing with simultaneous equations.

6. Goals of the Unit

- Students will be able to apply their prior knowledge of the synthetic geometry
- Students may apply their prior knowledge of co-ordinate geometry.

- Students will be able to link to other areas of the mathematical Leaving Cert course.
- Students being able to work with different forms of geometry and recognizing how they relate to each other.
- Students can solve visual representation of a problem using mathematical methods.
- Students understand the characteristics of a geometric construction.
- Students gain confidence and familiarity in the use of scale, Cartesian plane, mathematical instruments. (Wellbeing)
- Improve students spatial awareness (realising the circle in the sky corresponds to the circle on our map)

7. Unit Plan

Lesson	Brief overview of lessons in unit
1	Revision of the Perpendicular bisector with some real examples.
2	Analysis on perpendicular bisector for deeper understanding
3	Connecting perpendicular bisector construction with coordinate geometry
4	Teach the construction of the circumcentre
5	Analysis of circumcentre meaning and some applications
.6.	Research lesson
7	Extend research problem to area, circumference and equation of circle.

8. Goals of the Research Lesson:

Mathematical Goals

- We want students to be able to perform the construction of the circumcenter and circumcircle of a triangle.
- We want students to use mathematical instruments with precision.
- We want students to use their knowledge of the construction to solve a problem.
- We want students to use their knowledge of coordinate geometry to solve a problem.
- The significance of the point of intersection of two linear relationships.

In this lesson:

- Through Ceardaiocht, students will have the opportunity to express their ideas clearly and accurately.
- By engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.
- Students will have the opportunity to reflect on their own learning when the teacher asks them to write a reflection at the end of the lesson.
- By engaging in tasks which are appropriate to their abilities, students' confidence and positive disposition to learning will be promoted.
- During Ceardaiocht, students will present and discuss their mathematical thinking.
- Students' will explore options and alternatives as they actively participate in the construction of knowledge.
- Students will learn with and from each other by discussing different approaches to solving problems.
- Students will be encouraged to think creatively and critically.

9. Flow of the Research Lesson:

Steps, Learning Activities, Teacher's Questions and Expected Student Reactions	Teacher Support	
<p>Introduction (5 minutes) (including video)</p> <p>In today's lesson, we will use our mathematical knowledge to solve a problem. The students will work initially in pairs and then individually and use their knowledge to come up with several ways to solve the problem.</p> <p>Prior Knowledge</p> <p>Check students understanding of relevant concepts through keywords poster.</p>	Link PowerPoint	<p>Are students engaged?</p> <p>Check their understanding, knowledge and learning.</p> <p>Note the quality of their understanding</p> <p>Also challenging their visual spatial intelligence</p>
<p>Posing the Task (3-5 mins)</p> <p>Task 1: (Paired Together) (5mins)</p> <p>The photographer wants a picture of each of the three stages (individually). What coordinates would you set for drone?</p>	  <p>A drone is rented to fly over longitude and capture aerial photographs. Will I Am is operating the drone and is given an aerial map of the festival with the best location for an aerial photograph of each stage marked with a large dot.</p>	

Go to the board with the solution (7 mins)

Expected solutions

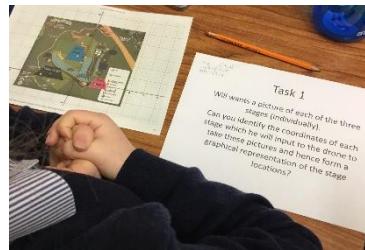
1. we expect that students will use the transparent grid to find the points
2. We envisage some students may try to draw axes and scale to try to identify to points.



Task 1

Will wants a picture of each of the three stages (individually).

Can you identify the coordinates of each stage which he will input to the drone to take these pictures and hence form a graphical representation of the stage locations?



Posing the Task (3mins)

Task 2: (countdown timer on board 15mins)

A drone needs to take a picture of a music festival, the picture must include the three stages in the one photo. What are the coordinates for the best location for the drone? (Drone is of equal distance from the 3 points) (as many solutions as possible)

Task 2

The organisers of the festival have text Will and requested the following:

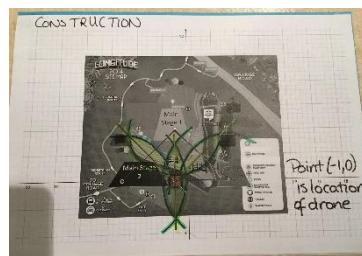
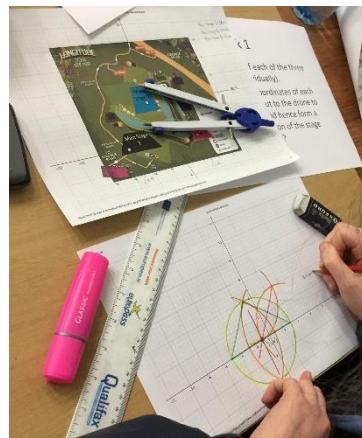


Find, in as many ways as you can, the co-ordinate he needs for the drone.

Student Individual Work (15 mins)

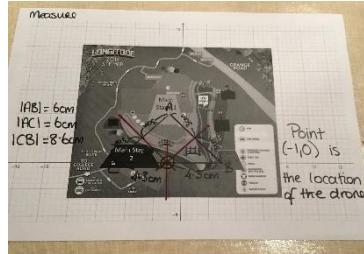
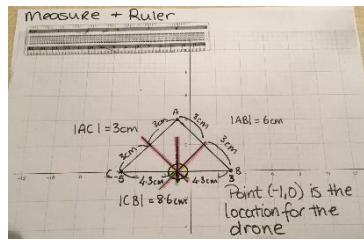
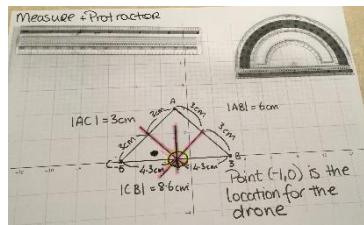
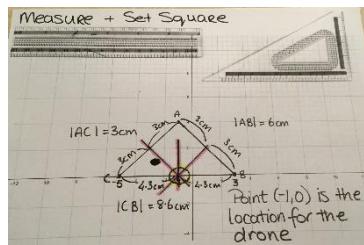
Solution 1:

We anticipate that the students will look at the points, identify the triangle and then construct the perpendicular bisector of the lines using the compass and straight edge which has been provided in their packs. We expect that in the next steps the students will construct the circumcentre and identify this as the location for the drone to be located and will identify it on the image/map.



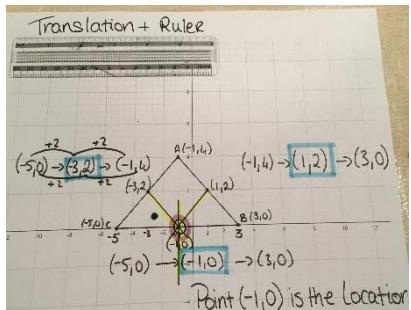
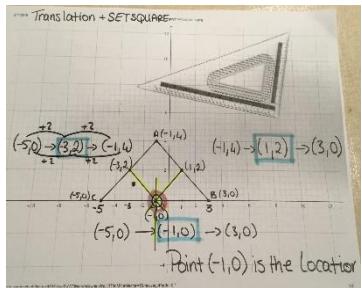
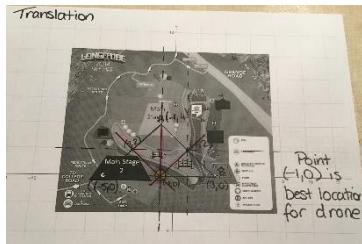
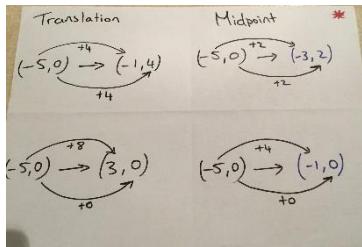
Solutions 2 ,3, 4

We anticipate that the students will find the midpoint using **measure** and using a **set square**, **or protractor or ruler** draw a perpendicular line through the midpoint. Students should identify the circumcentre of the triangle as the ideal position for the drone to capture the picture.



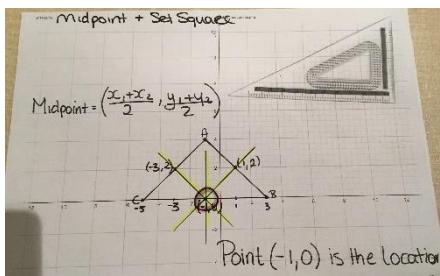
Solution 5 ,6, 7

Using translations to get the midpoint of two sides, get the slope of the same two sides, get the perpendicular slope from these, use **set square**, **ruler**, **protractor**, to get perpendicular bisectors then get the point of intersection of two lines to find the circumcentre.



Solution 8, 9,10:

We anticipate students will use the **midpoint formula** to identify the midpoint of each line segment and then using a **straight edge, protractor or set square** they will construct the perpendicular bisector and identify the point of intersection of the bisectors which should be recognised as the circumcentre of the triangle and therefore the ideal position for the drone to capture the picture of the three required stages at the festival.



Midpoint of $(-1, 4)$ and $(3, 0)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$
$$\left(\frac{-1 + 3}{2}, \frac{4 + 0}{2} \right)$$
$$\left(\frac{2}{2}, \frac{4}{2} \right)$$
$$(1, 2)$$

Midpoint of $(-5, 0)$ and $(-1, 4)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$
$$\left(\frac{-5 - 1}{2}, \frac{0 + 4}{2} \right)$$
$$\left(\frac{-6}{2}, \frac{4}{2} \right)$$
$$(-3, 2)$$

Midpoint of $(-5, 0)$ and $(3, 0)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$
$$\left(\frac{-5 + 3}{2}, \frac{0 + 0}{2} \right)$$
$$\left(\frac{-2}{2}, \frac{0}{2} \right)$$
$$(-1, 0)$$

Solution 11, 12:

Find the midpoint of two sides by any of the above ways, then get **the slope (formula)** of the same two sides, get the perpendicular slope from these, find the equation of the perpendicular bisectors from this, then get the point of intersection of two lines to find the circumcentre. This will inform the students of the ideal position for the drone to capture the picture of the three required stages at the festival.

$$\text{Slope } \frac{y_2 - y_1}{x_2 - x_1}$$

$$(-5, 0) (-1, 4) : \frac{4 - 0}{-1 - (-5)} = \frac{4}{4} = 1$$

$$\perp \text{slope} = -1$$

$$(-5, 0) (3, 0) : \frac{0 - 0}{3 - (-5)} = \frac{0}{8} = 0$$

$$\perp \text{slope} = 0$$

Equation of Perpendicular Bisectors.

$$y - y_1 = m(x - x_1)$$

$$(-3, 2) m = -1$$

$$y - 2 = -1(x - (-3))$$

$$y - 2 = -x - 3$$

$$x + y + 1 = 0$$

$$(-1, 0) m = 0$$

$$y - 0 = 0(x - (-1))$$

$$y = 0$$

Point of Intersection of Bisectors.

$$\begin{array}{l} x + y + 1 = 0 \\ \quad y = 0 \\ \hline x + 0 + 1 = 0 \\ \quad x = -1 \end{array} \qquad \begin{array}{l} -1 + y + 1 = 0 \\ \quad y = 0 \end{array}$$

$$\boxed{(-1, 0)}$$

Location of Drone

OR

$$\text{Slope } \frac{y_2 - y_1}{x_2 - x_1}$$

$$(-1, 4) (3, 0) : \frac{0 - 4}{3 - (-1)} = \frac{-4}{4} = -1$$

$$\perp \text{slope} = 1$$

$$(-5, 0) (-1, 4) : \frac{4 - 0}{-1 - (-5)} = \frac{4}{4} = 1$$

$$\perp \text{slope} = -1$$

Equation of Perpendicular Bisectors

$$y - y_1 = m(x - x_1)$$

$$(1, 2) m = 1$$

$$y - 2 = 1(x - 1)$$

$$y - 2 = x - 1$$

$$x - y + 1 = 0$$

$$(-3, 2) m = -1$$

$$y - 2 = -1(x - (-3))$$

$$y - 2 = -x - 3$$

$$x + y + 1 = 0$$

	<p>Point of Intersection of Bisectors.</p> $\begin{aligned} x-y+1 &= 0 \\ x+y+1 &= 0 \\ \hline 2x+2 &= 0 \\ 2x &= -2 \\ x &= -1 \end{aligned}$ $\begin{aligned} -1-y+1 &= 0 \\ -y &= 0 \\ y &= 0 \end{aligned}$ <p>$(-1, 0)$</p> <p>Location of Drone</p>	
Solution 13	<p>Students may use rise/run to get slopes of perpendicular bisectors, then form the equation of the bisectors and use simultaneous equations to find point of intersection of the bisectors.</p>	
Solution 14	<p>Fold tracing paper to find the circumcentre</p>	
Ceardaíocht /Comparing and Discussing (20 mins)	See Board Plan Below	

Summing up & Reflection (5 mins)

The teacher will summarise the main ideas of the lesson.

The students will be asked to answer a short evaluation sheet on the lesson.

Student Evaluation Name :

1. Circle the number that best indicates how clear the question was to you in the beginning.

Very Clear

Not clear at all

2. How many different ways did you find for answering the problem?

3. State one thing that you learned in the class from your own individual work?

4. State one thing that you learned in the class from your peer/class discussion?

5. How challenging did you find the task, once you were clear what you were being asked?

Very easy

Very hard

6. Explain why you chose your answer to Question 1.

7. Which method do you think was the best/ easiest?

8. Name something that you found helpful today for your learning of mathematics.

Homework

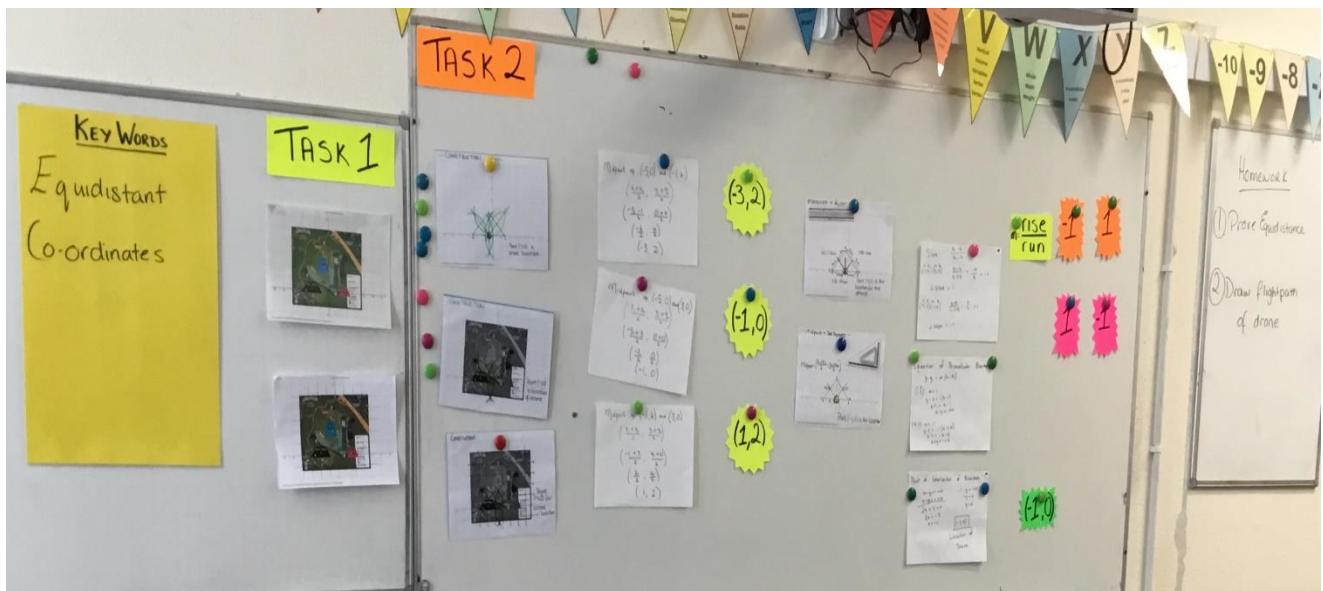
1. Draw the circular flight path of the drone incorporating the 3 stages
2. Calculate the circumference of flight path

Extension questions for next lesson

1. Equation of the circle
2. Area that it covers

10. Board Plan

Board Plan



11. Evaluation

Everybody felt the lesson was a success, and that the goals we hoped to achieve had been met. There was consensus that all students understood the concept of circumcentre by the end of the lesson regardless of how successful their own individual work had been. Students were engaged in the task from the outset. All the students responded positively to the tasks. The majority of student settled down very quickly to work on the initial task. All students were successful with Task 1. Students tended to spend most of the allocated time constructing circumcentre. Nearly all students recognised the need to construct the circumcentre. Once some students had completed the construction using a compass, they were challenged to think of alternative solutions. However, they persisted and almost all devised alternatives. Students enjoyed the opportunity to confer on their solutions and also the collaborative nature of Ceardaiocht. Students who had struggled to find more than one method grasped the idea when it came to Ceardaiocht. We noticed that the confidence of all students grew with peer consultation. We were very pleased that during Ceardaiocht the more complex algebraic approach was developed from the students' discussion. No student tried the paper folding approach and we feel this approach is certainly worth developing in a follow up class. Students enjoyed the opportunity to reflect and evaluate the lesson and their learning at the end of the class. For the most part students reported the lesson to be a very positive experience.

12. Reflection

Following the research lesson the team were very happy that students were able to apply their knowledge of constructions to real life scenarios. The team were also very pleased that students were able to establish connections across the Maths syllabus. Students remarked at the end of the lesson that a highlight was learning that “we have options”.

While reflecting on the lesson the team felt that students would benefit for more than 15 mins working on Task 2. This would have allowed students time to think beyond the mechanical construction of the circumcentre and to consider more solutions.

Students need to be conscious that the circumcentre is the point of intersection of two perpendicular bisectors rather than two medians. This misconception could certainly derail a student's learning in this lesson

We realised that student's confidence using mathematical language was key to this lesson. It was important for students to realise that they had the maths knowledge and were not afraid to think and use their prior knowledge. Following this research lesson, the team will be making a recommendation to the entire Maths Department that the teaching and learning of Geometry can be enhanced by taking into

consideration the following key points:

- When teaching constructions students need to be allowed to explore the many options and tools at their disposal to develop an in-depth understanding of each construction. It is essential for students to move beyond the physical construction of geometry.
- Paper folding should be included at a teaching methodology.
- There should be a concentrated focus on the mathematical language and meaning behind that language.
- Using formulae as a method for checking for accuracy should be encouraged.

This lesson has highlighted the importance of a deep understanding of constructions in order to problem solve with geometry. This will influence future lessons, from introducing constructions in first year right up to higher level leaving certificate synthetic and coordinate geometry.

Teachers' Reflections on their Lesson Study

The opportunity for professional dialogue during lesson study was seen as the most significant benefit for the team. Lesson Study resulted in a new depth in the nature of collaboration between teachers. All teachers felt that their content knowledge was developed giving them a new found confidence in their teaching.

The lesson study team feel their teaching has changed as a result of the process. Teachers feel they ask more questions, give more thinking time and allow their students more independence.

For the teachers involved this lesson was an exercise in self-reflection and self-assessment, allowing us to evaluate, make adjustments and thus improve the standard of Maths education we deliver.