

Lesson Research Proposal for 2nd Year Functions & Algebra

Date of lesson: 6th March 2019

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Lesson plan developed by Margaret Barrett, Celine Buckley & Karen Wood

1. Title of the Lesson:

Disco Taxi

2. Brief description of the lesson

In this lesson students are presented with a graph of $f(x) = x^2$ and asked to come up with the algebraic representation of the function in as many ways as possible. Students will then experience two further problems that involve students sketching a shifted graph and then using their methods from the first question, come up with the algebraic representation for the functions in questions two and three. The learning will then be summarized through a GeoGebra file and use of students answers before the students take the challenge question for homework.

3. Research Theme

Blooms Taxonomy: One of our schools' priorities was to introduce Blooms Taxonomy to the students. Through the use of effective questioning we aim to link algebra, patterns and functions with emphasis on quadratics to allow the students to experience questions at the various levels of Blooms Taxonomy. We hope that students will take ownership of their learning come to their own conclusions whereby they will be able to make connections across the curriculum between patterns algebra and functions.

4. Background & Rationale

The Chief Examiners Report from 2015 stated 'functions should not be taught as a standalone topic'. As a group we find that when the chapters are studied separately the students are unable to make the links between them e.g. they are unable to match the shape of the graph to its quadratic.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<ul style="list-style-type: none">• Generating arithmetic expressions from repeating linear patterns• Representing linear situations with tables, diagrams and	<ul style="list-style-type: none">• Linking quadratic patterns, functions and algebra.• Generating quadratic expressions from repeating quadratic patterns• Representing	<ul style="list-style-type: none">• Transformational geometry• Link exponential patterns, functions and algebra.

<p>graphs.</p> <ul style="list-style-type: none"> • Examining linear relationships, constant rate of change. • Graph linear functions. • Explore linear patterns using real life situations 	<p>quadratic situations with tables, diagrams and graphs.</p> <ul style="list-style-type: none"> • Examining quadratic relationships, constant rate of change. • Graphing quadratic functions. • Explore linear patterns using real life situations 	
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6. Goals of the Unit

Our goals within this unit are:

- The student connects with what level they are on Blooms taxonomy.
- The students are apply knowledge to real life situations.
- The students are able to draw connections through identifying, questioning , comparing and analysing,
- The students are able to create their own problem by relating and integrating patterns, functions and algebra.

7. Unit Plan

Lesson	Brief overview of lessons in unit
1	Revision of Linear patterns and their link to functions and Algebra
2	Real life Linear problem
3	Revision of quadratic Algebra
4	RESEARCH LESSON

8. Goals of the Research Lesson:

a. Mathematical Goals:

Students will be able to create the algebraic representation of a function from its graph.
Students will be able to describe how a function has been translated based on its algebraic representation.

b. Key Skills

Problem solving, making connections, managing myself, literacy, numeracy and inclusive.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
This column shows the major events and flow of the lesson, including timings and what will go up on the board.	This column shows additional moves, questions, or statements that the teacher may need to make to help students.	This column identifies (a) what the teacher will look for (formative assessment) that indicates it makes sense to continue with the lesson, and (b) what observers should look for to determine whether each segment of the lesson is having the intended effect.
<p>Introduction Student's will be shown a quadratic graph and asked for whatever information pops into their head regarding the diagram. (4 mins)</p>	Probing questions to lead students to establish a word bank of terms associated with quadratic functions.	Effective questioning and assessing the content of the prior knowledge section.
<p>Posing the Task Students will be given out the worksheets 'Disco Taxi' and allowed individual time to read and interpret the problem. (2-3 mins)</p>	Teacher will bounce queries from one student to another regarding the prior knowledge to get the class comfortable with responding to one another and not the teacher.	The content in the prior knowledge section. The desired responses are in the board plan image under the heading 'Prior Knowledge on the right-hand side.
<p>Student Individual Work Student's will then be given time to work on the three problems by themselves on their sheets. Rulers will also be made available for the students. (20-25 mins)</p>	Teacher will circle the room directing students towards their prior knowledge section in the even they become stuck on an aspect of the problem.	
<p>Ceardaíocht /Comparing and Discussing Six students will be chosen during the individual work phase to come up to the board and explain their solutions to the class in the order outlined in the board plan. (15-20 mins)</p>	The following are the order of the solutions to be chosen and discussed: <u>Question 1:</u>	

① Repeated Roots

$$x=0 \quad x=0$$

$$f(x)=(x-0)(x-0)$$

$$f(x)=(x)(x)$$

$$f(x)=x^2$$

② Using Co-ordinates

$$f(x)=x^2+bx+c$$

(1,1) $f(x) \Rightarrow 1^2+b(1)+c=1$

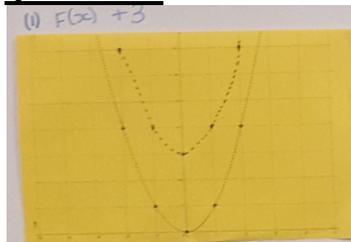
(-1,1) $f(x) \Rightarrow (-1)^2+b(-1)+c=1$

$$\begin{array}{r} b+c=0 \\ -b+c=0 \\ \hline 2c=0 \\ c=0 \end{array} \quad \left. \begin{array}{l} b=0 \\ f(x)=x^2+bx+c \\ f(x)=x^2+0x+0 \\ f(x)=x^2 \end{array} \right\}$$

Student will be asked to explain the relationship between factor and root here.

Student will be asked about why are they subbing in coordinates and what made them think of that approach.

Question 2:



① Repeated Roots

$$g(x) = f(x) + 3$$

$$f(x) = x^2$$

$$g(x) = x^2 + 3$$

Student will be asked how they knew to draw the new graph in this new location.

Student will be questioned on the idea of describing $g(x)$ in terms of $f(x)$.

② Moving Co-ordinates and $g(x) = x^2 + bx + c$

by (a) Using a Ruler
(b) Shifting each co-ordinate

$(-2, 4) \rightarrow (-2, 4+3)$
 $(-2, 7)$

$(2, 4) \rightarrow (2, 7)$

$$g(x) = x^2 + bx + c$$

$(-2, 7) \Rightarrow (-2)^2 + b(-2) + c = 7$

$$\begin{array}{r} 4 - 2b + c = 7 \\ -2b + c = 3 \end{array}$$

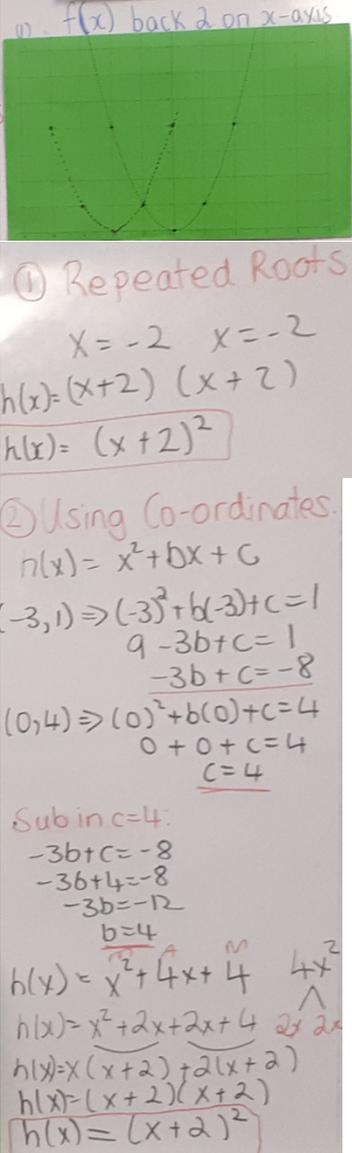
$(2, 7) \Rightarrow (2)^2 + b(2) + c = 7$

$$\begin{array}{r} 2b + c = 3 \end{array}$$

$$\begin{array}{r} 2b + c = 3 \\ -2b + c = 3 \\ \hline 2c = 6 \\ c = 3 \end{array} \quad \left. \begin{array}{l} \text{Sub in } c=3 \\ 2b+3=3 \\ 2b=0 \\ b=0 \end{array} \right\}$$

$$g(x) = x^2 + 0x + 3$$

Student will be questioned on why this approach looks different to the similar approach in question 1.

	<p>Question 3:</p>  <p>1) $f(x)$ back 2 on x-axis</p> <p>① Repeated Roots $x = -2 \quad x = -2$ $h(x) = (x+2)(x+2)$ $h(x) = (x+2)^2$</p> <p>② Using Co-ordinates. $h(x) = x^2 + bx + c$ $(-3, 1) \Rightarrow (-3)^2 + b(-3) + c = 1$ $9 - 3b + c = 1$ $-3b + c = -8$ $(0, 4) \Rightarrow (0)^2 + b(0) + c = 4$ $0 + 0 + c = 4$ $c = 4$</p> <p>Sub in $c = 4$: $-3b + c = -8$ $-3b + 4 = -8$ $-3b = -12$ $b = 4$</p> <p>$h(x) = x^2 + 4x + 4$ $h(x) = x^2 + 2x + 2x + 4$ $h(x) = x(x+2) + 2(x+2)$ $h(x) = (x+2)(x+2)$ $h(x) = (x+2)^2$</p>	<p>Student will be asked how they knew to draw the new graph in this new location.</p> <p>Student will be asked about the similarities/differences between this approach in Q1 and Q3</p> <p>Student will be questioned on why this approach looks different to the similar approach in question 1 and 2.</p>
<p>Summing up & Reflection Students will be asked to describe which was consider the different approaches and evaluate them (2-3 mins).</p>		<p>Student's will be asked to describe something they learned and which method they preferred from the lesson.</p>

10. Board Plan

Question 1: Find the equation of this second year curve.

Question 2: Many more 2nd years attend the Bandon Music Festival than the regular disco so there will need to be more taxis each hour. What do you think this graph will look like? Sketch it.

Find the Equation for this graph, $q(x)$.

Find the Equation for this graph, $h(x)$.

PRIOR KNOWLEDGE

- U-Shaped Graph
- Positive
- Factors
- Cuts y-axis
- Cuts x-axis
- Repeated Roots
- $x^2 + bx + c$
- Quadratic
- Co-ordinates
- Symmetrical
- Constant = 0 \Rightarrow cuts y-axis at 0

Method 1: Repeated Roots
 $x=0$ $x=0$
 $f(x) = (x-0)(x-0)$
 $f(x) = x^2$
 $f(x) = x^2$

Method 2: Moving Co-ordinates and Using a Ruler
 (a) Using a Ruler
 (b) Shifting each co-ordinate
 $(-2, 4) \Rightarrow (-2, 4+3)$
 $(2, 4) \Rightarrow (2, 7)$
 $q(x) = x^2 + bx + c$
 $(-2, 7) \Rightarrow (-2)^2 + b(-2) + c = 7$
 $4 - 2b + c = 7$
 $-2b + c = 3$
 $(2, 7) \Rightarrow (2)^2 + b(2) + c = 7$
 $4 + 2b + c = 7$
 $2b + c = 3$
 $2b + c = 3$
 $-2b + c = 3$
 $4c = 6$
 $c = 1.5$
 $2b + 1.5 = 3$
 $2b = 1.5$
 $b = 0.75$
 $q(x) = x^2 + 0.75x + 1.5$

Method 3: Shifting each co-ordinate up
 (a) $f(x)$ back 2 on x-axis
 $h(x) = (x+2)(x+2)$
 $h(x) = (x+2)^2$
 (b) Using Co-ordinates
 $h(x) = x^2 + bx + c$
 $(-3, 1) \Rightarrow (-3)^2 + b(-3) + c = 1$
 $9 - 3b + c = 1$
 $-3b + c = -8$
 $(0, 4) \Rightarrow (0)^2 + b(0) + c = 4$
 $0 + 0 + c = 4$
 $c = 4$
 $-3b + 4 = -8$
 $-3b = -12$
 $b = 4$
 $h(x) = x^2 + 4x + 4$
 $h(x) = x^2 + 2x + 2x + 4$
 $h(x) = (x+2) + 2(x+2)$
 $h(x) = (x+2)(x+2)$
 $h(x) = (x+2)^2$

11. Evaluation

- Were the students exposed to a significant variety of questions in light of Bloom's Taxonomy
- Did the students identify the link between the visual transformation of a quadratic graph and the algebraic representation?
- Is the ceardaiocht in the best position to aid with student progression? (After Q3?)
- What unforeseen misconceptions arose during the lesson?
- Did the students approach the question from the 3 methods we had thought of previously?

12. Reflection

We had hoped to see students attempting the question in all three methods that had been previously thought of, however, the students came up with a 4th way that combined two of our methods to make a more efficient method of solving (see image below).

One misconception that arose that was not forecast was the use of slope. In the prior knowledge section of the class a student said slope. While it was not added to the board under prior knowledge it was the feeling of the group that the student's latched onto this word as it is something that they are comfortable with using. Considering this many of the students went on to pursue the equation of a line formula to establish the equation for the curve. Therefore, they needed to calculate the value of m for their equation. They chose two points and used the slope formula. While showing us a possible misconception amongst the students it was the opinion of the group this would be a great place to start the next day's lesson. The teacher is planning to put the curve up and calculate the "slope" and ask the

students do they see a problem with this. The teacher will then mention that this is a problem that faced mathematicians many years ago (Newton & Leibniz) and resulted in the birth of calculus which is something they will see more of in fifth year. The group really enjoyed discussing the benefits of using a misconception like this when compared to dismissing the idea straight away.

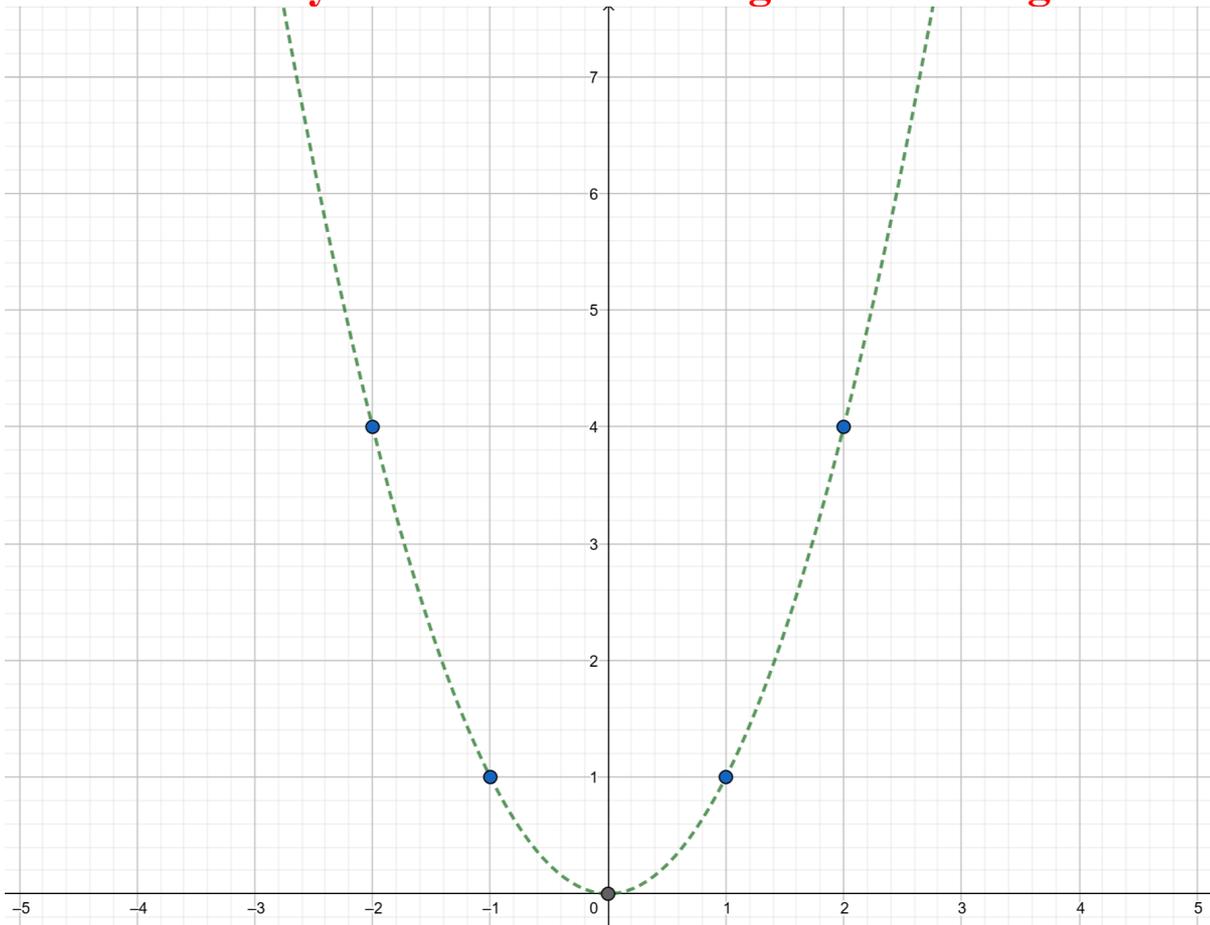
All teachers of the group remarked on when looking at the students' work, the learning and progression was obvious to see. Students struggled in some way with Q1 but by Q2 and Q3 there were much less errors or stumbles in establishing the equations of arguably more difficult functions. (See image below)

This learning was due in no small part to the decision of the teacher to deviate slightly from the plan regarding ceardaiocht. The teacher noticed during the lesson that a large number of students were struggling with Q1. This sparked the decision to do three mini ceardaiochts as opposed to one directly after Q3. We did a ceardaiocht after Q1 with a focus on establishing the equation followed by a ceardaiocht on Q2 focusing on sketching the curves. This meant that by Q3 almost all students had a firm grasp of the link between the graph and the algebra.

THE DISCO TAXI...



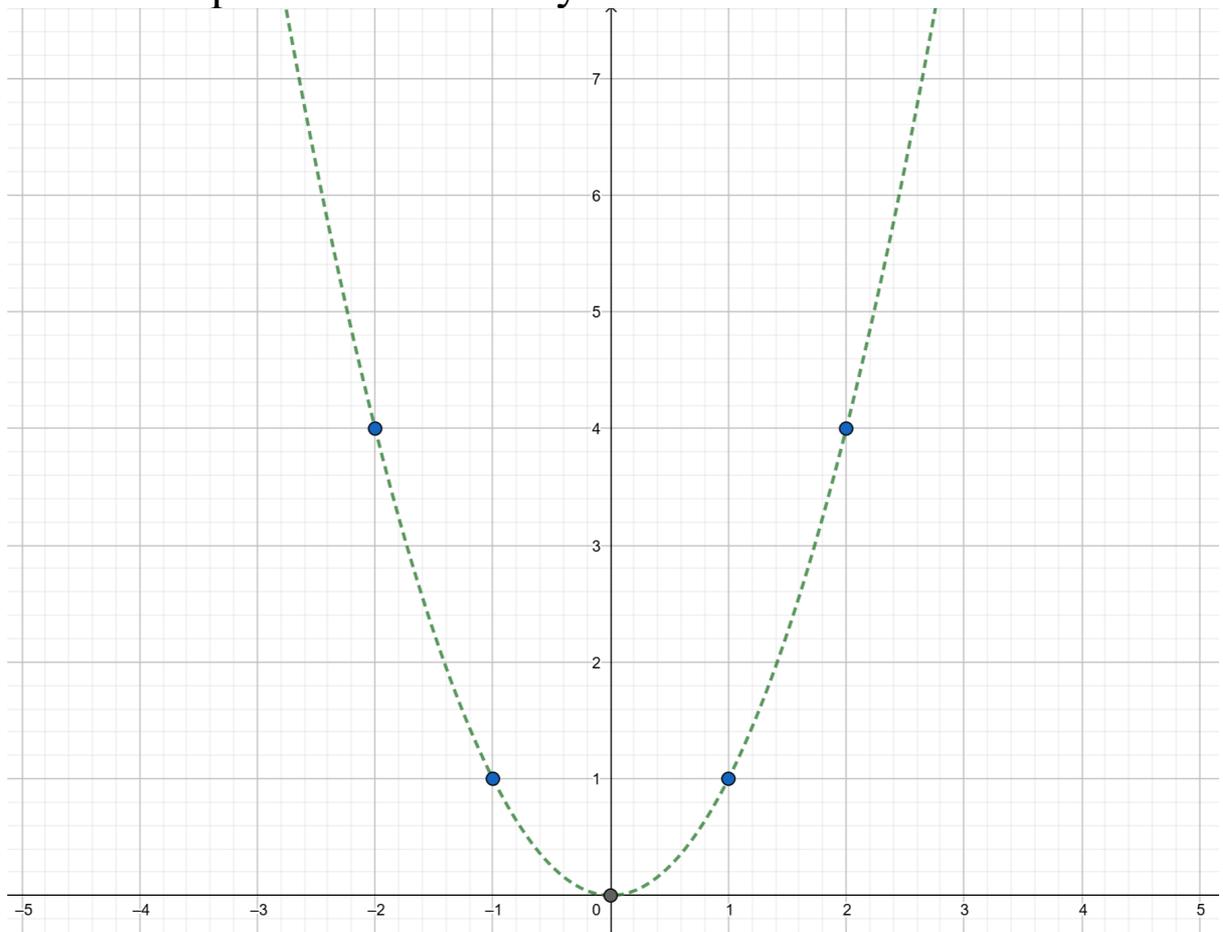
The following graph represents the number of taxis needed per hour to bring 2nd years home from a regular disco night.



- The x-axis represents time and the y-axis represents the number of taxis
- There are no taxis at 4 am
- There are 4 taxis at 6am

Question 1.

Find the equation of this 2nd year curve.



Rough work

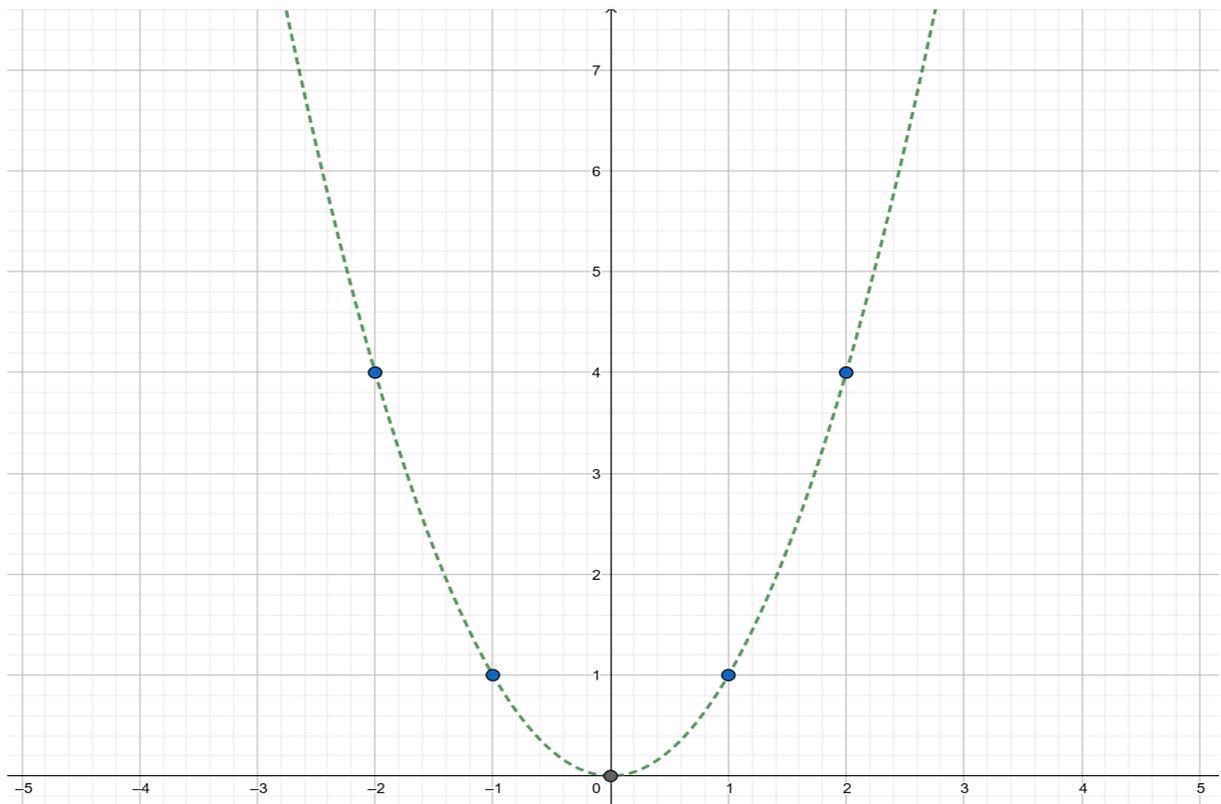


Question 2.

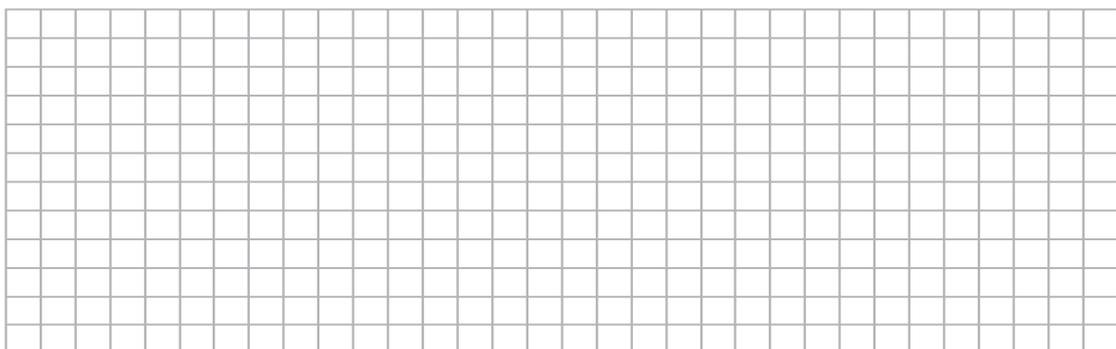
The 1st year disco finishes earlier so they need to go home 2 hours before the 2nd years. **The same number of taxis are needed but each taxi will be 2 hours earlier.**

What do you think this graph will look like?

Sketch it below using the same axes and scale

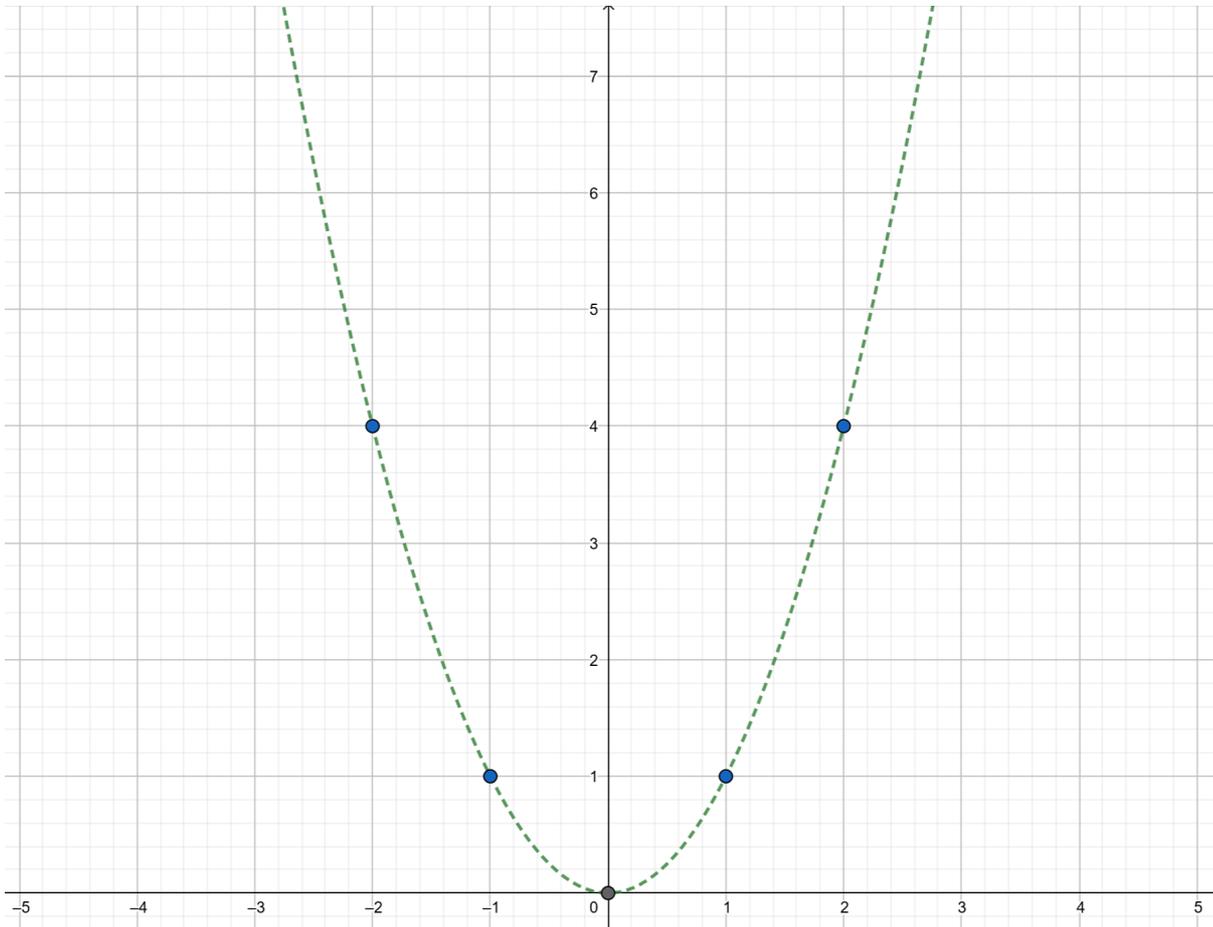


Find the equation for this Graph



Question 3.

Many more 2nd years attend the Bandon Music Festival than the regular disco so there will need to be 4 more taxis each hour. What do you think this graph will look like? Sketch it.



Find the Equation for this graph.

