

# Lesson Research Proposal for Introduction to Algebra

Common Level, First Years

Date of lesson: 11/02/2019

School name: Glenstal Abbey

Teacher giving lesson: Trevor Fitzgerald

Associate: Bairbre Ní Mháille

Lesson plan developed by: Trevor Fitzgerald, Enda Bell, Bairbre Ní Mháille

## 1. Title of the Lesson: *Express my point*

## 2. Brief description of the lesson

Students will be introduced to the concept of writing mathematical expressions and exploring values for a fixed variable in two league tables. Students will be asked how teams built their points in a league table and to write as many possible ways to express this.

## 3. Research Theme

Referring to *Looking at our schools 2016- a quality Framework for Post-Primary Schools* we identified the following goals as a priority for the improvement of teaching and learning:

- a) Students engage purposefully in meaningful learning activities and grow as learners through respectful interactions and experiences that are challenging and supportive.
- b) As teachers, we understand the value of engagement in professional development and professional collaboration and we want to work together to devise learning opportunities for students across and beyond the curriculum.

As math teachers we will address these goals by:

- a) - designing/selecting meaningfully differentiate content and activities which will encourage our students to think insightfully and creatively and will ensure that all students are challenged.
  - providing our students with opportunities to come up with their own approaches to solving a problem, and hence take ownership of their own learning.
  - developing their confidence, competence and communication skills through expressing their ideas to their peers in their own words which enables the comprehension of all students in the class (U.13 Junior Cycle Mathematics Learning Outcomes).
- b) - engaging in the collaborative Lesson Study process with a view to:
  - improving mathematics teaching and learning and students' experiences in the mathematics classroom
  - promoting a consistent department-wide approach to the teaching of mathematics in our school with a view to improving mathematics teaching and learning and student's experiences in the mathematics classroom

## 4. Background & Rationale

### a) Why we chose the topic

The lesson is aimed at first year students. We chose Algebra as it is a problematic area from 1<sup>st</sup> years all the way up to 6<sup>th</sup> year. We find that there is a fear or stigma around algebra with many students finding it difficult to generalize and explain patterns and relationships in words and numbers. Algebra is a key subject area in mathematics and while there are stand-alone questions on the topic, it crops up in all

aspects of the curriculum. Therefore, we have decided to make equations the focus of our research topic. The skill of solving equations is a fundamental skill needed by students.

Our current approach to introducing algebra is closely related to the content of the textbook that we use. We introduce letters to represent unknown values. In this project we hope to focus more on the meaning and purpose of variables, in particular, how variables can help us identify patterns and deal with algebraic expressions. We would like our students to understand that a variable can represent more than one value within the same context, and that the same letter can be used in different contexts.

## **b) Our research findings**

We have found that students often struggle with this especially when equations are taken out of their usual context. Students also have difficulty converting word problems to mathematical statements and thus we are going to work on this by forming equations from word and number problems. According to the Chief Inspectors report students ‘mastered routine procedures and skills, struggled to apply these in unfamiliar contexts and situations’ (Chief Inspectors Report 2010-2012). By aiming this lesson at first year students we hope to rectify these problems early on and prevent them from continuing in later years.

*Chief Examiner’s reports [24] have identified algebra as an area of weakness in national examinations over the past number of years. According to these reports, Irish student performance in algebra has shown little or no progress in the last ten years. Questions related to algebra are often the lowest scoring questions or continually avoided. On such evidence, it is clear that although algebra has long enjoyed a place of distinction in the mathematics curriculum many students have difficulty in understanding and applying even its most basic concepts. Pedagogical Framework Promoting Interest in Algebra*

*Mark Prendergast and John O’Donoghue*

From our professional discussions we feel students shy away from math with any mention of Algebra and following the New Junior Cycle Key Skills we want students to develop a positive disposition towards investigating, reasoning and problem solving in all areas of the curriculum. Also tying in with the skills document we want student to be confident and positive about their learning and be creative in their work exploring all options and alternatives.

In 1996, Kieran developed a model for conceptualising algebraic activity that identified three important components of school algebra; namely,

- Generational activities: These are activities where students generate their own rules, expressions, or equations from given situations.
- Transformational activities: These are often referred to as rule-based activities and require an appreciation of the need to adhere to well-defined rules and procedures.
- Global /meta level activities: These activities apply to all of mathematics and are not exclusive to algebra. For example, finding the mathematical structure underlying a situation, being aware of the constraints of a problem situation and explaining and justifying. Kieran, C., 1996, ‘The changing face of school Algebra’ in 8th international congress on mathematical education selected lectures, 271 – 286. S.A.E.M. ‘THALES’

## 5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<p>In primary school, students are introduced to variables and equations.</p> <p>In fifth class, students translate number sentences with a frame into word problems and vice versa.</p> <p>In sixth class students</p> <ul style="list-style-type: none"> <li>a) explore the concept of a variable in the context of simple patterns, tables and simple formulae and substitute values for variables.</li> <li>b) translate word problems with a variable into number sentences</li> <li>c) solve one-step number sentences and equations</li> </ul> <p>This is continued on in first year where students are reintroduced to variables and constants through patterns and conclude by solving given linear equations.</p>	<p><b>According to the Junior Cycle Mathematics Learning Outcomes</b></p> <p>AF.2 investigate situations in which letters stand for quantities that are variable so that they can:</p> <ul style="list-style-type: none"> <li>a. generate and interpret expressions in which letters stand for numbers</li> <li>b. find the value of expressions given the value of the variables</li> <li>c. use the concept of equality to generate and interpret expressions</li> </ul> <p><b>According to JC Mathematics Syllabus students should be able to:</b></p> <ul style="list-style-type: none"> <li>• Solve linear equations in one variable</li> </ul>	<p><b>According to the Junior Cycle Mathematics Learning Outcomes</b></p> <p>AF.1 investigate patterns and relationships (linear, quadratic, doubling and tripling) in number, spatial patterns and real-world phenomena involving change so that they can:</p> <ul style="list-style-type: none"> <li>a. represent these patterns and relationships in tables and graphs</li> <li>b. generate a generalised expression for linear and quadratic patterns in words and algebraic expressions and fluently convert between each representation</li> <li>c. categorise patterns as linear, non-linear, quadratic, and exponential (doubling and tripling) using their defining characteristics as they appear in the different representations</li> </ul> <p><b>According to JC Mathematics Syllabus students should be able to</b></p> <ul style="list-style-type: none"> <li>• Find the underlying formula algebraically for quadratic relations (HL)</li> <li>• Show that relations have features (constant rate of change in linear and non-constant rate of change in quadratic relations) that can be represented in a variety of ways</li> </ul>

## 6. Goals of the Unit

Our students will:

- (a) Generate and interpret expressions in which letters stand for numbers
- (b) Find the value of expressions given the value of the variables
- (c) Use the concept of equality to generate and interpret expressions
- (d) Students should be to write their own expressions through varying the inputs.
- (e) Able to write an expression a number of different ways.
- (f) Understand the meaning of variable and constant.
- (g) Identify algebraic expressions from worded problems using letters as variables.
- (h) Write algebraic expressions omitting the multiplication sign (e.g.  $2 \times a = 2a$ ,  $1 \times a = 1a = a$ ,  $(-1) \times a = -1a = -a$ ,  $(-5) \times a = -5a$ ) and using exponents
- (i) See a purpose of substitution (problems in context can be used, e.g. an expression for the circumference/area of a circle and in the case of the lesson, finding the points of each team based on win/draw/loss).

## 7. Unit Plan

How the research lesson fits into the larger unit plan, helping to show the bigger picture of the whole unit and the progression of learning. Clarify where the research lesson will be taught.

Lesson	Brief overview of lessons in unit
1 (Research lesson)	<p><i>Introduction to writing mathematical expressions</i> Students will be introduced to the concept of writing mathematical expressions and exploring values for a fixed variable in two league tables.</p> <ul style="list-style-type: none"> <li>a) We will ask them a question in how the teams built their points and to write as many ways as possible to express this. The first table will be one that the students will be familiar with (premier league top 4 teams), and should have prior knowledge of the value of a win/lose/draw in table and how these were used to build points.</li> <li>b) The second table will be a table that will be created by the teachers that will have a different value for a win/draw/loss. Again we will ask the students the question of how the teams built their points and to write as many ways as possible to express this.</li> <li>c) An extension of higher learning is the idea that students might hopefully suggest using different letters to represent a WIN in league table 1 vs the WIN in league table 2.</li> </ul>
2	<p><i>Introduction to substitution</i> Students will be asked to calculate the points for a given teams based on assigned values.</p>
3	<p><i>Practicing substitution</i></p> <ul style="list-style-type: none"> <li>a) Students substitute numerical values for the variable in algebraic expressions, including finding the values of <math>-a</math>, <math>a^2</math>, <math>-a^2</math>, <math>(-a)^2</math> when <math>a</math> is negative.</li> <li>b) Students understand that the same rules apply as per “bimdas”, the order of operations.</li> </ul>
4	<p><i>Multiplication</i></p> <ul style="list-style-type: none"> <li>a) Students write algebraic expressions omitting the multiplication symbol, e.g. <math>1 + 3 \times x = 1 + 3x</math>, <math>a \times 2 = 2a</math>, <math>1 \times a = 1a = a</math>, <math>(-1) \times a = -1a = -a</math>, <math>(-5) \times a = -5a</math></li> </ul>

	<p>b) Students identify the commutative properties in simple multiplication of algebraic expressions, e.g. <math>(5a)(4) = 5 \times a \times 4 = 5 \times 4 \times a = 20a</math>. A misconception <math>3(7a) = 3(7) \times 3a</math> is addressed.</p> <p>c) Use exponents (“How can we use algebraic expressions to represent an area of a square that has <math>a</math> cm sides, or a volume of a cube that has <math>a</math> cm long edges?”) d) Students distinguish between <math>2a</math> and <math>a^2</math></p>
5	<i>Addition of like terms</i>
6.	<p><i>Writing their own expressions</i></p> <p>Students will be asked to create their own tables and from that write their own expressions. This will be used as a form of assessment.</p>

### 8. Goals of the Research Lesson:

Looking at the goals of the research lesson itself from two perspectives:

a. Mathematical goals

Students prior to this lesson should know:

- How to investigate the representation of numbers and arithmetic so that they can represent the operations of addition, subtraction, multiplication and division in  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ , using models including the number line, decomposition, and accumulating groups of equal size. (N.1 a, JCT LO)

Students at the end lesson should be able to:

- Write a mathematical expression in a number of different ways.
- See that certain rules or relations can be found among the numbers in the tables.
- Can use a letter to assign a value and that the letter can have a different value depending on the context it is been used in.
- Can study tables and see the value of sample spaces in real life.
- See the benefit of using formulas to calculate large quantities.
- Should give a good introduction into the value of substitution.
- Build from the lesson, the knowledge needed in order to write their own expressions.
- Analyse numerical patterns in different ways, including making out tables and continue such patterns (N.4, JCT LO)

b. Key Skills and Statements of Learning

- **Being Numerate:** By engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.
- **Managing information and thinking:** Students will be encouraged to think creatively and critically and record their results.
- **Being Creative:** Students will explore options and alternatives as they actively participate in the construction of knowledge.
- **Communicating:** During the lesson, students will present and discuss their mathematical thinking.

- Working with Others: Students will learn with and from each other by discussing different approaches to solving the problem.
- Staying Well: By engaging in tasks which are appropriate to their abilities, students' confidence and positive disposition to learning will be promoted.

This lesson also meets the following JC Statements of Learning:

- 15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
- 16. The students describes, illustrates, interprets, predicts and explains patterns and relationships.
- 17. The student's devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills

## 9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment														
<p><b>Introduction (10 minutes)</b></p> <ul style="list-style-type: none"> <li>● Review of natural numbers and integers</li> <li>● Two-way tables -discussion around 2 way tables, what use they have and where they are used.</li> <li>● Letters – use of letters instead of a word and what it donates.</li> </ul>	<p>The teacher asks the students of how we can represent the results of a coin toss and rolling a die, thus encouraging students to recall a two-way table and its uses.</p> <p>Using previous work in set, the students will be asked how to effectively use letters instead of words to represent numbers.</p>	<p>Students show the results of a two-way table on the board.</p> <p>Students are asked to show how to express English to Math language.</p>														
<p><b>Posing the Task (5 minutes)</b></p> <p>Show the problem and table on the board and also distribute out the problem and table to the students.</p> <p>Task 1:</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>How many ways can you explain and write down how the points were calculated for Munster?</p> </div> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Pool 2</th> <th>P</th> <th>W</th> <th>L</th> <th>D</th> <th>B</th> <th>PTS</th> </tr> </thead> <tbody> <tr> <td>Munster Rugby</td> <td>6</td> <td>4</td> <td>1</td> <td>1</td> <td>3</td> <td>21</td> </tr> </tbody> </table>	Pool 2	P	W	L	D	B	PTS	Munster Rugby	6	4	1	1	3	21	<p>A prepared worksheet along with the tables will be distributed to the students</p>	<p>Students will be asked if they understand the task at hand. The teacher will address any misconceptions the students may have.</p>
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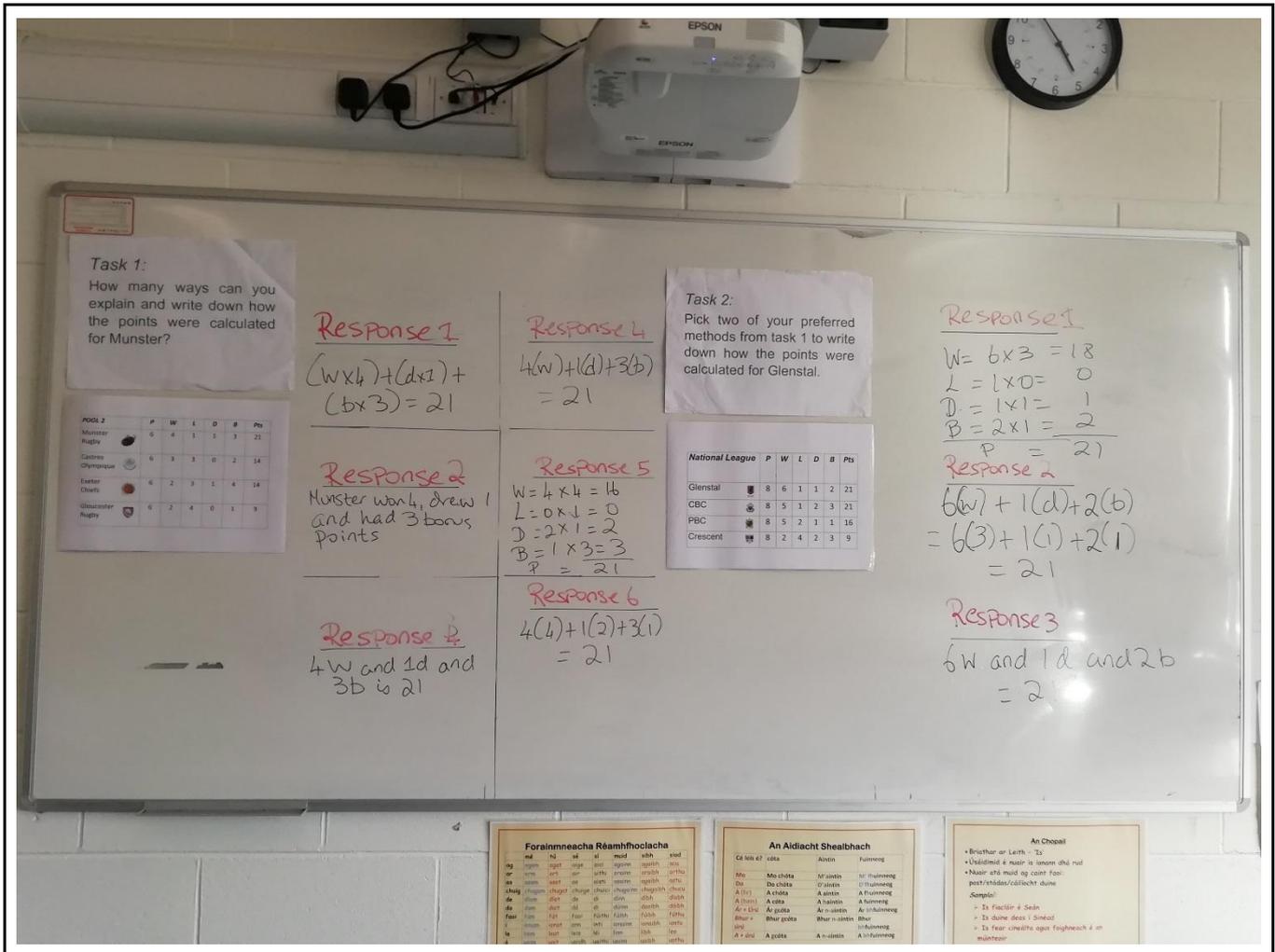
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<p><b>Student Individual Work (10 Minutes)</b> <b>(Task 1)</b></p> <p><i>Anticipated responses</i></p> <p>Response 1: <math>(w \times 4) + (d \times 1) + (b \times 3) = 21</math></p> <p>Response 2: Munster won 4, drew 1 and had 3 bonus points</p> <p>Response 3: 4w and 1d and 3b is 21</p> <p>Response 4: <math>4(w)+1(d)+3(b)=21</math></p> <p>Response 5: <math>W = 4 \times 4 = 16</math> <math>L = 0 \times 1 = 0</math> <math>D = 2 \times 1 = 2</math> <math>B = 1 \times 3 = 3</math> <math>P = 21</math></p> <p>Response 6: <math>4(4)+1(2)+3(1)=21</math></p> <p>Response 7: <math>w+w+w+w+d+b+b+b=21</math></p> <p>Response 8: <math>w=4/d=2/lb=1</math> and works it out like a sum</p> <p>Response 9: <math>3 + 16 + 2 = 21</math> <math>B + W \times 4 + D = 21</math></p>	<p>10 minutes given for individual work.</p> <p>Teacher uses a seating chart to record each student's solutions and to prepare for the whole class discussion.</p> <p>Students who finish early or who have difficulty with writing an expression, will be encouraged to try and work through the problem.</p> <p>If students have difficulty writing an expression, they will be encouraged to focus on the letters and the points each letter is worth.</p>	<p>Each student comes up with at least one mathematical expression to express how Munster's points are calculated.</p>																					
<p><b>Ceardaíocht /Comparing and Discussing (15 mins)</b></p> <p>Invite a student who came up with an expression to come to the board and show his/her solution.</p> <p>Let other students interpret that expression before the selected student explains it to the class.</p>	<p>When student presents work at the board make sure to attach their name to it.</p> <p>Ask students to raise their hands if they used this method.</p> <p>Did anybody use a different</p>	<p>Can students explain their approach?</p> <p>Do students recognize similarities/differences between their approach and that presented on the board?</p>																					

<p>Repeat these steps with every mathematical expression.</p> <p>Did anyone else solve it the same way? Can you explain this method?</p>	<p>approach?</p> <p>Explain to students that there are several ways to write an expression.</p>	<p>Do students offer alternative approaches to solving the problem?</p>																																			
<p><b>Posing the Task (2 minutes)</b>  <b>Task 2</b>          Show the problem and table on the board and also distribute out the problem and table to the students.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Pick two of your preferred methods from task 1 to write down how the points were calculated for Glenstal</p> </div> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>National League</th> <th>P</th> <th>W</th> <th>L</th> <th>D</th> <th>B</th> <th>Pts</th> </tr> </thead> <tbody> <tr> <td>Glenstal</td> <td>8</td> <td>6</td> <td>1</td> <td>1</td> <td>2</td> <td>21</td> </tr> <tr> <td>CBC</td> <td>8</td> <td>5</td> <td>1</td> <td>2</td> <td>3</td> <td>21</td> </tr> <tr> <td>PBC</td> <td>8</td> <td>5</td> <td>2</td> <td>1</td> <td>1</td> <td>16</td> </tr> <tr> <td>Crescent</td> <td>8</td> <td>2</td> <td>4</td> <td>2</td> <td>3</td> <td>9</td> </tr> </tbody> </table>	National League	P	W	L	D	B	Pts	Glenstal	8	6	1	1	2	21	CBC	8	5	1	2	3	21	PBC	8	5	2	1	1	16	Crescent	8	2	4	2	3	9		
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<p><b>Student Paired Work (5 Minutes)</b>  <b>(Task 2)</b></p> <p><i>Anticipated responses</i></p> <p>Response 1:  <math>W = 6 \times 3 = 18</math>  <math>L = 0 \times 1 = 0</math>  <math>D = 1 \times 1 = 1</math>  <math>B = 2 \times 1 = 2</math>  <math>P = 21</math></p> <p>Response 2:  <math>6(w)+1(d)+2(b)=21</math></p> <p>Response 3:  <math>6w + 1d + 2b = 21</math></p>	<p>5 minutes given to students to come up with expressions in pairs.</p> <p>Having used their favourite expressions from task 1, students will be familiar with process but will be tasked with a different value for a win.</p>	<p>Each student comes up with at least one mathematical expression to express how Glenstal's points are calculated.</p>																																			
<p><b>Discussing (5 Minutes)</b></p>		<p>Do you notice anything about the expressions?</p>																																			

<p>Let's see how you can use what you have learned from Task 1 to help you write an expression in Task 2.</p> <p>What did you do that helped you write an expression?</p> <p>Help students realise that the value for a win has changed in task 2.</p> <p>By discussing the two tasks, students should realise that a value/letter can change and impact a result.</p> <p>Do you notice any pattern in the numbers in these expressions?</p> <p>Which number in the expression were different? Which numbers are staying the same? (constant)</p>	<p>Are students able to select the most relevant pieces from Task 1 in order to solve Task 2?</p> <p>Students show that they can use the expression efficiently.</p>	<p>Is there a pattern occurring here?</p> <p>What do you call a number that stays the same/ doesn't change? (constant)</p> <p>Do you know a term for the number that keeps changing? (variable)</p> <p>What can we use to represent the number that keeps changing? (x)</p>
<p><b>Summing up and Reflection (8 Minutes)</b></p> <p>We learned:</p> <ul style="list-style-type: none"> <li>• How to formulate a mathematical expression by studying rugby tables.</li> <li>• Meaning of the terms 'Constant' and 'Variable'</li> <li>• Ask students to write a reflection.</li> </ul>	<p>The teacher recaps briefly the different approaches that can be taken, and reasserts that they are all valid.</p>	<p>Do the students' reflections represent the teacher's view of the lesson?</p>

## 10. Board Plan

Board Plan
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## 11. Evaluation

The general consensus was that the lesson was successful, with the goals of the lesson achieved. All of the students were positive about the whole lesson, in particular the weaker students who grasped the concept and felt comfortable and creative with it. Students became more comfortable with the process of the lesson after addressing misconceptions, and in particular in task 2. Students felt they had been challenged and learnt a lot on their own and also from their peers and their responses that were demonstrated on the board. Strong students who were frustrated by the process found it hard to be creative and not just taught the concept.

It was found that all students came up with at least 1 way of writing an expression. Many of the students were comfortable with using letters to write the expression but unsure how to assign points. With gentle encouragement students were able to branch out and come up with a few more ideas. When presented with the second task students were happy to work in pairs to see what had changed and if the expression would be the same. By the end of the lesson they noticed that the same letters can have different values and came up with the term variable and constant, even considered using other letters if needed. Students had a good foundation of the concept of writing expressions after task 2.

## 12. Reflection

It was agreed by all that the task was engaging for all due to relevance to the students and their keen interest in rugby. Most students attempted at least one method to write an expression, although we had hoped that they would write more. Many had used their knowledge of the points system for rugby to write an expression using words and letters. We feel that the lesson really fostered a positive mindset regarding the concept of algebra and the introduction to a variable and writing expression. Even the shy, more relaxed students were the more creative and engaged by with the process; they made notes of concepts raised and addressed those that they did not themselves discover.

There were some misconceptions regarding the points rather than how to describe the building of the points. We feel the difficulty with this in the lesson stemmed from the wording in the problem '*how many ways can you tell me how Munster built their points' total*'. We noticed that students went to calculating points and moving towards substitution of variables too early in task, and in our post-lesson discussion, we discussed removing the words 'points' and 'calculate' from the problem. We also expected focus on the expression  $1st$  - but students went straight to solving for the variables.

## Recommendations and Future Study

The original difficulty with lesson stemmed from the wording in the problem, so perhaps changing the wording and removing the words 'points' and 'calculate' from the problem would be effective. The teacher revisited the problem again after the live lesson and changed the wording to see if it would be effective. He asked them '*Task 1 - describe the way Munster have progressed in the league; only tell me how their results have gone*'. There was a higher frequency of students 'telling the story' – i.e. Munster won 6 drew 2 etc. Some still started figuring out the value of the win or, already knowing its value, started saying what the points total was, but the discussion moved toward expression and use of a suitable variable much sooner than the live lesson.

In task 2 the teacher removed the points total from the Task 2 league table and asked '*state using a chosen method from task 1, how each team has fared in the league*'. Students now began to build the expression and chose the same variables. An interesting discussion took place again, about use of  $x$  or  $y$  in favour of  $w$  or  $d$  etc. Students were more connected with the role of variable - which it stood for something. Student's homework was to try and figure out the value for  $w$ ,  $d$ ,  $l$   $b$  in the Task 2 league table. They had 6 teams and the teacher replaced the point's total that he had previously removed and 11 students out of 15 got the correct points for each variable.

An extension on from the task was to ask students to create a problem using two variables. The best example was '*If it costs €10 for an adult and €5 for a child, write and expression to show the gate receipts at today's Munster Rugby match*'. The teacher took that example and explored the gate revenue for different numbers of adults and children. They tried changing the price of admission for each and both - did it affect the variable, so we distinguished between coefficient and variable. All students displayed a positive disposition, and were really engaged in the activity, and tried to come up with a couple of different solutions. They had to really think hard to come up with new expressions and they

also had to focus when their classmates presented a solution to see if they could understand somebody else's work. One student wasn't sure about his own solutions, but his classmate helped him see his own answer was correct. All students made an effort to complete both tasks and were respectful and listened when their classmates were presenting.