Lesson Research Proposal for **GET UN-SN🎱🎱KERED!**

Date of lesson: 11th February 2019

School name: Holy Family Community School, Rathcoole, Co. Dublin

Teacher giving lesson: Seán Murphy

Associate: Derek Maher

Lesson developed by: Matthew Munds, Nikki Cowzer, Jim Hanley, Derek Maher, Seán Murphy and Ailbhe Keating

1. Title of the Lesson: GET UN-SN🎱🎱KERED!

2. Brief description of the lesson:

This lesson is based around students developing multiple methods of solution to a mathematics problem. It combines the approaches of trigonometry and algebra as a means of exposing students to the efficiency of theorems and trigonometric identities.

3. Research Theme

The schools' self-evaluation (SSE) priority is formed on the basis of numeracy, in particular stating that we aim to improve students' knowledge of the main mathematical skills and to 'the up-take of students at higher level. These targets are paramount for the Mathematics Department to achieve and nurture over the period of Lesson Study.

From examining the '*Looking at our Schools*' document we have selected two domains from the Teaching and Learning section; '*Learner Experiences'* and '*Teacher's collective/ collaborative Practice'.*

Learner Experiences:

In developing our research theme for Lesson Study, it was decided that a meaningful lesson be composed that encompassed multiple elements of the Leaving Certificate course. We want our students to build on their prior constructed knowledge from Junior Cycle and to ensure that there was conceptual fluency when it came to approaching this challenging problem. We also wanted to build on the approach of 'real-life' mathematics and to include a sense of competition that would excite and motivate students to finding a solution to the problem. This would help ensure that students were challenged to utilise mathematical literacy in the dissecting of a problem-solving question.

Teachers' Collective / Collaborative Practice:

In developing our research theme for Lesson Study, we wanted to ensure that the hours invested into this model of structured problem-solving carried longevity and impact and to ensure that it cited a catalyst of change in our collective and collaborative practice as Teachers within our larger Mathematics Department. In selecting this problem-solving question, it was paramount that different levels and abilities of students could start to initiate a logical response to the question based on their own prior knowledge. As a collection of Mathematics teachers it can be very upsetting and off putting for students who try to solve these types of questions without any suitable start point to be found within the question itself. Lastly, by engaging in this process it is hoped that we, as a department, will start to cohesively work together to develop more suitable lessons and activities for our students.

4. Background & Rationale

After much discussion and debate amongst the team, the teachers agreed that there was a repeated difficulty for students being able to derive and apply coordinate geometry and trigonometry formulae. Furthermore, students can have difficulties with the applications of such topics as trigonometry and co-ordinate geometry on rudimental questions. Throughout the course for Junior Cycle, these formulae are often prescriptive in nature and unfortunately rote learning can often prevail as a result.

As a department, we aim to nurture our students' understanding of the 'why' behind these questions and aim to develop a series of lessons that will assist our students to develop these skills. Due to the higher-level knowledge and material required, it was decided to focus on a higher level leaving certificate grouping. In addition, we hope that students would utilise their problem-solving skills or else use the knowledge of the trigonometric identities as a means of solution. From reviewing classroom-based assessments and end-of-term summative examinations, there tends to be issue with the applications of trigonometry and geometry. It is hoped that this process will assist with improving students' ability to use procedural skills developed in an applied context.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
2.2 Coordinate Geometry. 2.2 Solving problems involves slope of a line. 2.3 Calculate the area of a triangle using coordinates. 2.2 Angle between two lines.	2.3 Apply trigonometric formulae. 2.3 Use trigonometry to slope problems in 2D. 2.2/2.3 Relationship between coordinate geometry and trigonometry. 2.3 Use trigonometry to find the area of a triangle. 2.3 Solve problems using Sine and Cosine rules.	2.2 Relationship between 'angle between two lines formula' and trigonometric identities. 2.1/2.3 Using theorems to solve problems.

6. Goals of the Unit

- Students recognise that there are different methods of solving mathematical problems that evolve as a result of investigation of a problem.
- Students recognise the scope for creativity within a problem.
- Students develop their key skills within mathematics including working together and communication.
- Students develop a positive attitude towards problem solving and numeracy.
- Students recognise the links between the different strands of the curriculum.

7. Unit Plan

Lesson	**Brief overview of lessons in unit**
1	Revision of Trigonometric ratios from Junior Certificate
2	Trigonometric Functions
3	Sine Rule and Area of Triangle
4	Cosine Rule
5	Live Lesson: **GET UN-SN🎱🎱KERED!**
6	Recap of Angle between two lines in Coordinate Geometry

8. Goals of the Research Lesson:

Mathematical goals

- Students will revise the core topics of trigonometry, algebra and geometry.
- Students will utilise different mathematical areas to develop a cross-topic solution to a problem.
- Students will understand the relationship between trigonometric identities and the acute angle between two lines.

Key Skills and Statements of Learning

- **Communicating:** Students will be tasked with sharing their own mathematical reasoning and knowledge to other members. Furthermore, individual/group members will be tasked with presenting their solution to their peers.
- **Information Processing:** Students will be challenged to apply their prior mathematical knowledge to articulate a solution to a task.
- **Working with others:** Students will be working in smaller groups of three people as a means of finding as many solutions as possible to the task. Firstly, students will be tasked at having five minutes to find as many solutions as possible before communicating with their teams.
- **Critical and Creative Thinking:** Students will evaluate each other's solutions while debating their effectiveness and their elegance.
- **Being Personally Effective:** Students will apply their own independent knowledge autonomously to try and find effective solutions to this problem.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
<u>Introduction (5 minutes)</u> Welcome to the class and an introduction to today's event. Students to settle down and the facilitator to pose the task to the students.	Students will be placed in groups of three.	
<u>Posing the Task (3 minutes)</u> Students will be shown the task on the board and then be tasked with reading this. Copies will also be placed on the individual tables for students to read and refer to throughout. <u>Individual work (7 minutes)</u> Teacher will tell the students to work individually for a few minutes and come up with solutions to the problem. Students will work individually to try and articulate the question and create their own solutions to the problem	Teacher will read out the task and ensure that the students understand what is being asked of them Point out that the aim is for the solution *method* to be different and that solving it using the same method using different lengths is not the aim. If the students are struggling with the task the teacher will show a picture on the board of the question placed on the coordinate plane.	(a) Teacher should see that the students have applied their prior knowledge to the question and articulated a solution on paper

		before moving to group work. (b.) Observers should see that the students have applied their prior knowledge to the question and have made attempts at working solutions on paper
<u>Group work (15 minutes)</u> Teacher will call the students back and tell them that they are to now work in their groups of three and compare their solutions and try and create new/more solutions. Students will work in their groups of three and discuss their solutions and try and build on each other's work to create more solutions	Teacher may need to point out that there is table books and geometry sets at the top of the room in order to point students towards more solutions.	(a) Teacher should see that the students have worked together to create more solutions and that the solutions range in difficulty and not redoing the question using different lengths for the sides. (b.) Observers should see that students have a varied range of solutions and methods of answering.

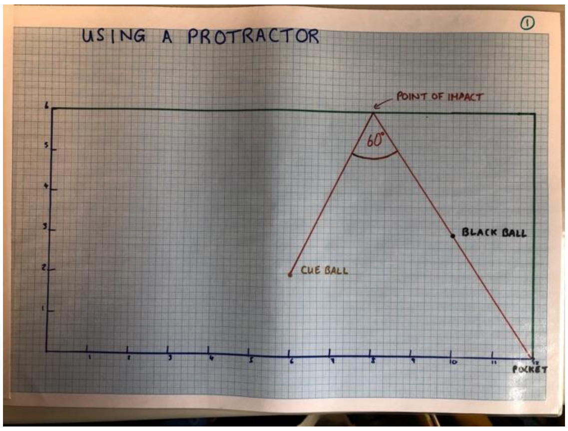
<u>Ceardaíocht /Comparing and Discussing (20 minutes)</u> Teacher will call on certain students (chosen during observation of group work) to come up and explain their method; these students will be brought up in order from the simplest to the most elegant solution.	Use of probing questions: • What concept is that from? • Is there a connection emerging here? • Can you explain why you chose that method? Teacher will have further solutions on hand that were done beforehand to stick on the board so that when students sit down the solution will remain in sight. 1. Using a Protractor 2. Tan of Interior Angles 3. Tan of Exterior Angles 4. Cosine Rule 5. Area of Triangle 6. Angle Formula 7. Tan (A+B) 8. Theorem 19	(a) Students are able to articulate their solution and others are able to ask questions or link it to other methods. (b.) Observers should see students discussing their work and questioning each other on the methods chosen and linking to other topics of the curriculum.

Summing up & Reflection (5 minutes)	Questions asked	
Teacher will recap the lesson and ask the students to reflect on their learning for the day and ask probing questions on what they learned during the lesson.	• What did you learn? • Is there a link between certain topics? • What do all the various solutions show you about maths?	(a) Students are able to reflect on the lesson and see the link between different topics on the curriculum (b) Students are able to see that there are multiple solutions to problems and that some are more elegant than others.

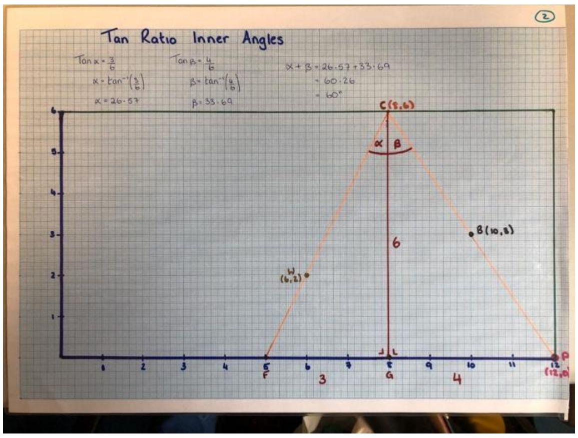
10. Board Plan

Board Plan

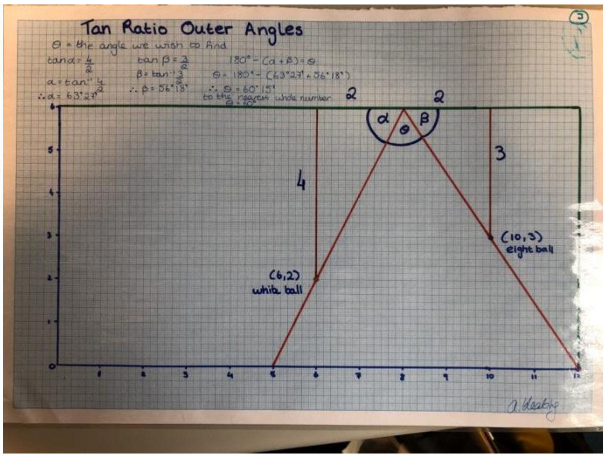
Solution 1: Using a Protractor



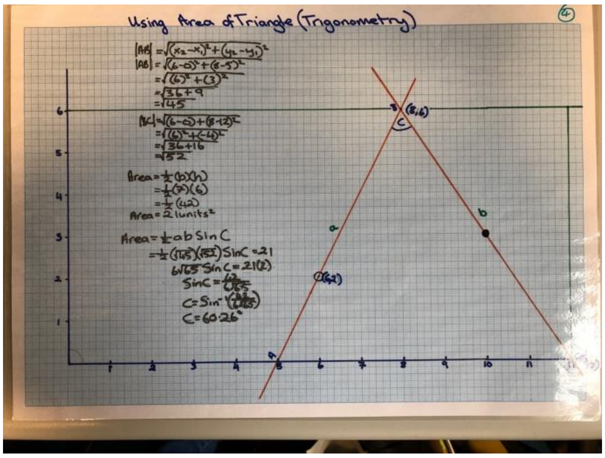
Solution 2: Using Tan Ratio Inner Angles



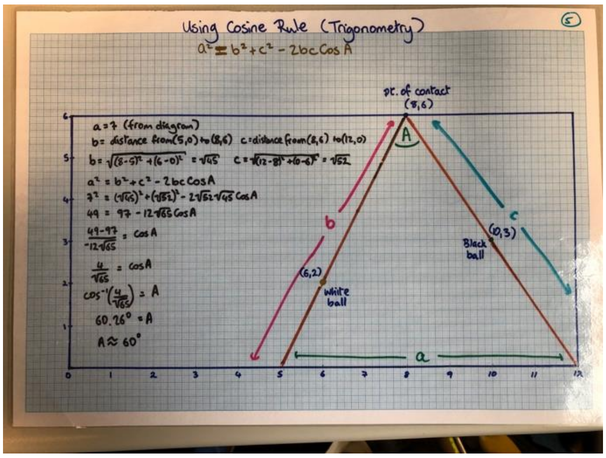
Solution 3: Using Tan Ratio Outer Angles



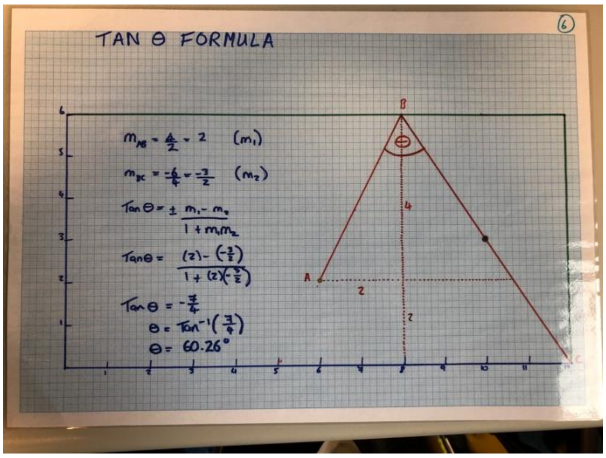
Solution 4: Using Area of Triangle (Trigonometry)



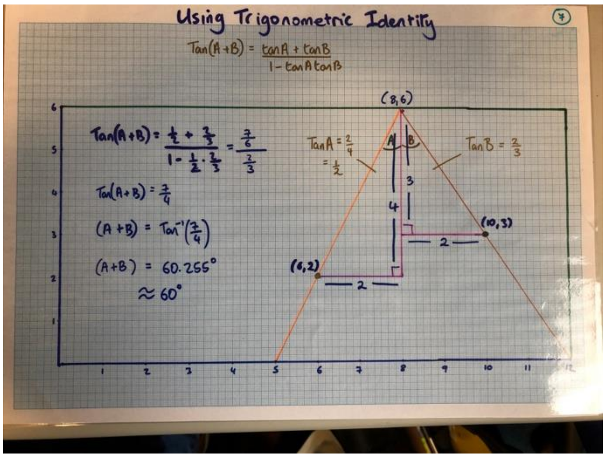
Solution 5: Using the Cosine Rule



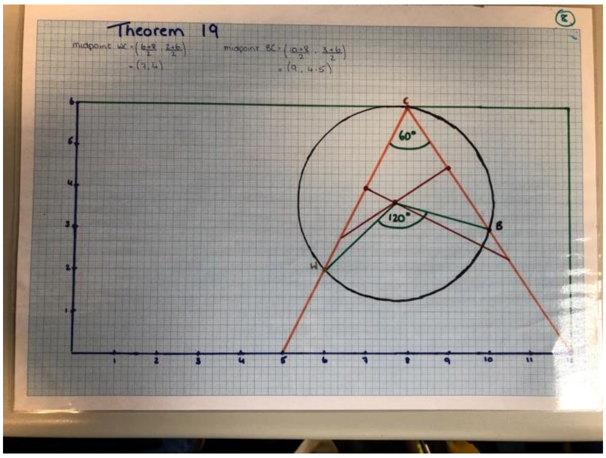
Solution 6: Tan Theta Formula



Solution 7: Using Trigonometry Identity



Solution 8: Theorem 19



11. Evaluation

On evaluation of the lesson, it was apparent that the lesson reached the desired research goals. Students utilised different aspects of algebra, trigonometry and coordinate geometry as a means for developing multiple solutions to this problem. It was imperative that students started to mentally develop the sophistication of theorems and an appreciation of the development of formulae.

This lesson successfully challenged students to initiate a word problem and to transfer this into mathematically literacy. Students over complicated the problem. There was a need for students to decode the problem from words into mathematics.

One member commented on the concept of 'plate spinning' that students were trying to try multiple ways at once, rather than focusing on one approach and developing it fully. As teachers we must construct one idea/concept well, prior to developing the next one and then interlinking as needed.

One part that we loved was that the teacher did not answer students' questions on whether their method/approach was 'right or wrong' instead he asked them to check with their team members. There was a need for affirmation from the classroom teacher that students constantly strived for. This over-reliance on the teacher is a difficult area to address. Students used statements like '*I am wrong'* and '*I am probably wrong'* automatically casting doubt on their mathematics.

In their selection of suitable methods; it was interesting to note that some students selected similar triangles as a means of solving. Furthermore, it was reassuring to see that instead of using the distance formula for calculating length, some students applied Pythagoras' Theorem. One student even referred to this as '*Pythag*' highlighting a confidence in their mathematics.

12. Reflection

On reflection, the team were very happy with the lesson as a whole. It was interesting to see the different outcomes and hear the discussions that occurred between the student groups about their mathematics. Initially, the association of which angles were the ones needed to solve this question was debated. This highlighted that real life mathematics can be somewhat abstract for students. Students were also provided with the mathematical formulae book but they failed to use this resource while working independently.

Once the individual time was over, the student voice emerged with students citing discussions on when to use the cosine rule and when to use the sine rule. This discourse caused students to critique their sheets and workings and compare with each other to see what elements of the questions that they had and which elements that their peers had.

As a school and in particular a mathematics department, we are moving to one-hour classes in September 2019. As a result of participating in this lesson-study project, we have now developed a sense of the activity learning methodologies that we could use for such classes.

During the observation, it was interesting to see that students initially struggled with the problem before starting to draw a diagram as a method of problem-solving. Once the individual time was over students were relieved and started to work well together. The teachers also noted the longevity of this lesson and its ability to cater to numerous lessons and to different year groups as an end of chapter / revision unit. Our knowledgeable other observed that there is a connection with the trigonometric identities that could be explored. This connection would be a nice way of introducing these concepts to leaving certificate higher level students, especially the $\tan(A+B)$ formula.

Our knowledgeable other also commented positively on the lesson and noted that the timing of the lesson worked well. It was discussed that students have to learn to trust their own abilities, but that there is an intrinsic level of self-doubt among many students.

Lastly, the concept that a triangle is composed of three lines was discussed and the utilisation of coordinate geometry is an additional tool for students to approach trigonometry problems. Furthermore, the need to reinforce the strong links between algebra and geometry is an area to develop over the course of the curriculum.

Problem Posed to the Students:

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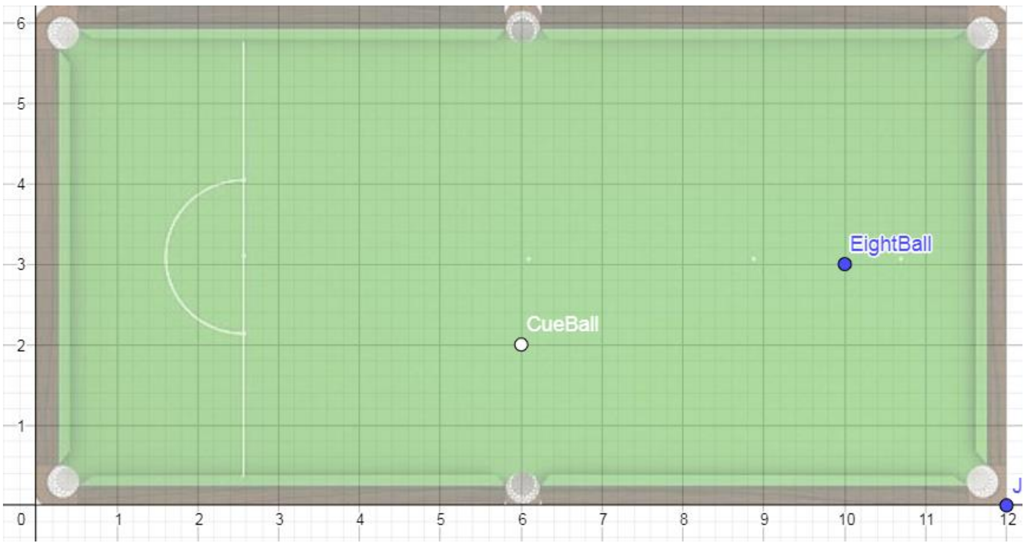
Matthew and his friends are playing pool. The pool table has the following dimensions 12 units by 6 units. There is no direct path to the Eight Ball, so Matthew must take a shot that rebounds off the cushion first before hitting the Eight Ball into the pocket. He knows that the white ball has the coordinates (6,2) and the Eight Ball has the coordinates (10,3). His selected pocket is positioned at (12,0).

He wants to calculate the required angle to complete this shot as necessary.

Find this angle in as many ways as possible.

Support Diagram if needed:



Result Diagram: