Lesson Research Proposal for Ballinamore Community School

1. **Title of the Lesson:** Point of Pick – up!

2. **Brief description of the lesson**

Students will be presented with a coordinate grid where an aerial map of Ballinamore will be superimposed onto it. Using their knowledge of algebra, coordinate geometry and synthetic geometry/constructions, the students are asked to find where would be the fairest/best position for a bus collection for pupils coming from three different villages.

3. **Research Theme**

At Ballinamore Community School, we want our students to:
- Enjoy their learning and are motivated to learn.
- Engage in their learning experience and develop as independent learners.
- One of our SSE targets is homework and that students can identify the success criteria for quality homework.

As mathematics teachers, we will support the achievement of these goals by:
- Adapting to the individual learning needs of our students by using differentiated approaches in our mathematics classrooms and through employing effective questioning as a means of assessing our students in our teaching
- Providing an environment where students can see the purpose to their learning by regularly making connections to build on our students prior learning from Junior Cycle so as to consolidate and improve upon for Leaving Certificate
- Create problems that will stimulate their mathematical reasoning and problem solving skills
- Using resources that support and allow opportunities for our students to discuss their thinking and share their ideas with their peers, thus allowing the student voice to be heard in the class
- Providing opportunities in class for students to reflect on their work and homework and engage in self-assessment
- Developing opportunities for our teachers to assess students understanding of material content and inform us on our setting of appropriate homework tasks

At the end of a lesson students will complete a reflection of learning as part of their homework, to give feedback to the teacher
- aspects of the lesson that went well for them
- one thing they would like to learn more about
- **One mistake the made and how they corrected it.**

4. **Background & Rationale**

The lesson is aimed at fifth year ordinary level students.

Why we chose this topic– students fail to recognise the relationship between Mathematics and everyday life and see that Mathematics is of the utmost importance when it comes to solving everyday problems. We noted through our SSE that our students were failing to study additional information and

were failing to make connections in their Maths. For this problem students need to recognise that the problem can be solved in many ways – through use of algebra, constructions, geometry and, it highlights that Mathematics is linked by many topics.

This topic reinforces information learned at Junior cert level as well as building on new information learned at leaving cert level.

The stated objective of the Leaving Certificate Mathematics syllabus is that learners develop mathematical proficiency, which it breaks down into five areas:

1. Conceptual understanding
2. Procedural fluency
3. Strategic competence
4. Adaptive reasoning
5. Productive disposition

Our lesson touches on many of these areas which will help our learners achieve mathematical proficiency and also promotes many of the recommendations from the Chief Examiners Report:

- *"In terms of skills, the new syllabus has an increased emphasis on problem solving, as well as on the skills of explanation, justification, and communication"*

- *At Ordinary level, students are not prepared to make a number of attempts in many questions and to persevere in solving problems "Candidates at this level generally abandoned the work as soon as difficulty was encountered, rather than trying different ideas"*

- *"They had more difficulty with questions which required them to draw on multiple strands of the syllabus at once, so there is clearly still a sense in which their knowledge and skills are compartmentalized"*

- *"Teachers should provide opportunities for students to apply the skills and knowledge from one strand to material from another strand"*

- *Teachers should provide students with opportunities to practice solving problems involving real-life applications of mathematics, and to get used to dealing with "messy data" in such problems. Students should also be encouraged to construct algebraic expressions or equations to model these situations, and / or to draw diagrams to represent them.*

Our research findings

From looking at our own students and discussions within the department, we find that students struggle to see the application of simultaneous equations and tend to compartmentalize its use to algebra only, instead of linking it with other areas like geometry.

Because of these deficits we have decided to commence teaching of simultaneous equations using a problem-solving situation which naturally gives rise to this concept.

We also find that students who struggle with algebra can often think that there is only one way to solve a problem and fail to see connections with other topics. If they recognized these other topics, it would help them realise that there are other alternative ways to approaching a problem.

We notice that some students draw on information from other subjects, for example, Geography and DCG, when solving problems and we feel it is important to address these methods to show the relevance of everyday Maths and its cross-curricular links.

5. Relationship of the Unit to the Syllabus

Describe how this unit relates to the syllabus/learning outcomes from prior years, for this year and for future learning.

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<u>Overview third to sixth classes</u>	Leaving Certificate Construction 16.	Students should be able to apply their learning to novel

Applying and Problem solving	Co-ordinate geometry: use slopes to show that two lines are	situations and recognize linear and geometric relationships. Students' will develop greater confidence and increased ability to recall facts in relation to linear equations, midpoints and slopes. Students' should be able to apply mathematical processes such as solving problems involving intersection of a line and a circle.
-apply concepts and processes in a variety of contexts	• parallel	
-analyse problems and plan an approach to solving them	• perpendicular select and use suitable strategies (graphic, numeric, mental) for finding solutions to real-life problems involving	
-select and apply a variety of strategies to complete tasks and projects or solve problems	up to two linear relationships	
-reflect upon and evaluate solutions to problems		
-discuss and explain the processes used and the results of mathematical activities and problems and projects in an organized way	Revisit constructions - Perpendicular bisector of a segment, using only compass and straight edge. Circumcentre and circumcircle of a given triangle, using only straightedge and compass.	
-listen to and discuss other children's mathematical descriptions and explanations		
-discuss problems and carry out analysis	• Students will be able to solve simultaneous linear equations with two unknowns and interpret the results	
2.2 Coordinate geometry - use slopes to show that two lines are • parallel • perpendicular – recognise the fact that the relationship $ax + by + c = 0$ is linear – solve problems involving slopes of lines 2.1 Synthetic geometry - Performing constructions - junior cert 1-15, construction 2 - perpendicular bisector of line segment Leaving cert 16. Circumcentre and circumcircle of a given triangle, using only straightedge and compass. 4.2 Solving equations - simultaneous linear equations with two unknowns and interpret the results		

2.2 Junior Cert - Co-ordinate geometry - Co-ordinating the plane. Properties of lines and line segments including midpoint, slope, **distance** and the equation of a line in the form.		

6. Goals of the Unit

- Students will understand that some type of problems do not have a single approach rather there are many approaches to solve the problem however accuracy is paramount in determining which method is best.
- Students will have the opportunity to interpret information presented in graphical and pictorial form and recognise the link between a graph and real life - Find the point of intersection from a graph visually/ algebraically
- Students will apply a variety of approaches to locate the best position of the bus
- Students will recall basic facts related to geometry
- Students will understand the concept of midpoint, distance between two points and intersection of lines
- Students will understand that situations involving distance, midpoint, slope, etc., may be expressed both algebraically and graphically
- Students will be familiar with Synthetic geometry, performing constructions, build on their prior knowledge from Junior Cert
- Students will be confident in their abilities to select the correct formula/approach to solve problems
- Students will be able Interpret a graph and recognise the link between a graph and real life - Find the point of intersection from a graph visually/ algebraically

7. Unit Plan

How the research lesson fits into the larger unit plan, helping to show the bigger picture of the whole unit and the progression of learning. Clarify where the research lesson will be taught.

Lesson	Brief overview of lessons in unit
1	Construction: bisector of an angle, perpendicular bisector of a line, line perpendicular to a given line.
2	Construction: line perpendicular to a given line through a point on the line, line parallel to a given line, division of a line segment into equal parts
3	Construction: construct an angle of 60 degrees, tangent to a circle at a given point on the circle.
4	Construction of triangles
5	Construction of triangles and quadrilaterals
6	Construction of circumcenter and circumcircle of a triangle
7	Construction of incentre and incircle of a triangle
8	Construction of centroid

9	Research lesson
10	Practice abstract exam questions relating to the goal of the research lesson.

8. Goals of the Research Lesson:
   a. Mathematical goals

   Students will be able to;
   - Interpret a graph
   - Recognise the link between a graph and real life
   - Find the point of intersection from a graph visually, algebraically and through use of constructions
   - Students will have a conceptual understanding of the meaning of solutions to simultaneous equations
   - See the use of constructions and how they are used to solve problems
   - Form the equation of a line and recognize that coordinate geometry and algebra can work together to find a solution.

   b. Key Skills

There are five key skills identified as central to teaching and learning across the curriculum at senior cycle. These are information processing, being personally effective, communicating, critical and creative thinking and working with others
   - **Information processing**

"Information processing relates to the ways in which learners make sense of, or interpret, the information to which they are exposed" – our students will have to make sense of the problem given to them They will need to interpret the graph and arrive at various at a way to solve the problem.

   - **Critical and creative thinking**

   "Learners are encouraged to solve problems in a variety of ways and are required to evaluate methods and arguments and to justify their claims and results" – our students will see that by the end of the lesson there is more than way and in fact many ways to solve mathematical problems. Students will be able to justify and back up their solution and show results which equal that of their peers who have used alternative methods
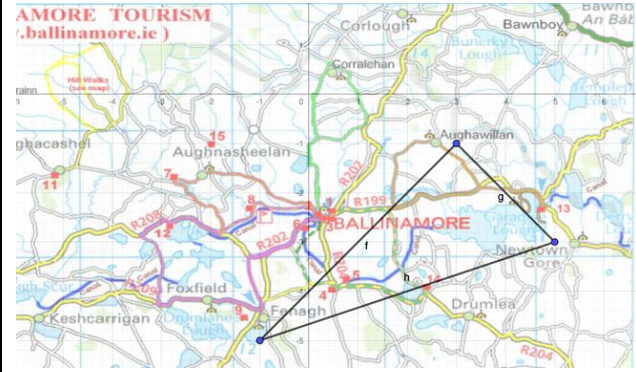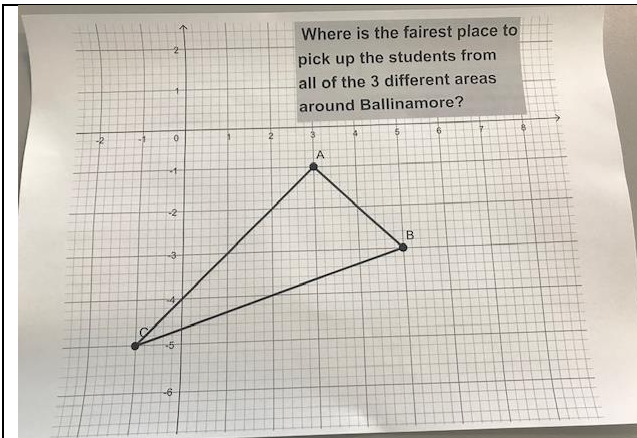
   - **Communicating**

"In mathematics learners are encouraged to discuss approaches and solutions to problems and are expected to consider and listen to other viewpoints. Since mathematics emphasizes investigation an important aspect of this is communicating findings to a variety of audiences in different ways." – the students will have completed their own investigation and some students will be chosen to communicate their findings to the class, all students will have to listen to other students findings, this should promote confidence and build communication skills as students will presenting to their peers and teachers.

   - **Being personally effective**

"Studying mathematics empowers learners to gain knowledge and skills that will benefit them directly in other aspects of their everyday lives" – our students will see how Maths can be used to solve practical real life problems. Some students will benefit from cross curricular links with geography and DCG.

9. Flow of the Research Lesson:

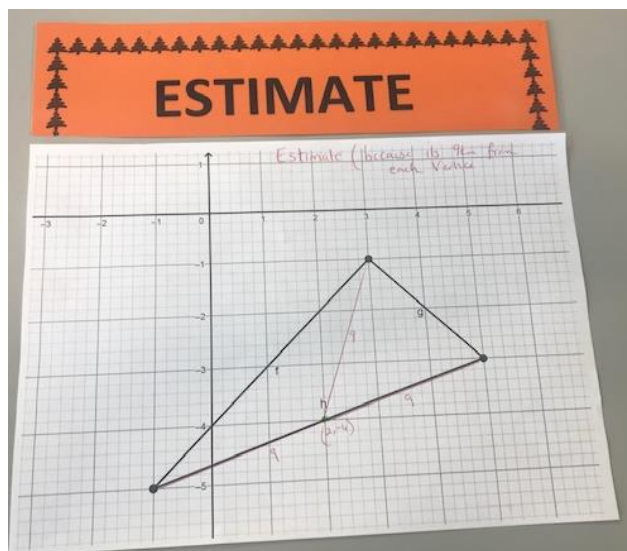| Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction (5 Minutes) Welcome students. Give a brief outline of lesson study and the process of the live lesson and observation in today's class. Provide students with the necessary equipment and paper.	**Resources:** Geometry sets Written problem (3 copies) Graph paper Extra plain Paper Mathematical tables Calculators Scissors Individual whiteboards with grid Board Magnets White board and data projector	
Posing the Task (3-5 Minutes) 3 students from 3 different areas are trying to decide where the fairest location for the bus to pick them up. Where is the fairest place for the bus to pick up the students from all of the 3 different areas around Ballinamore? 	-A visual image of the problem will be projected onto the board using the data projector. -The students will also be provided their own copy of the question and the resource materials. The teacher may have to explain what is meant by the word fairest – as in some cases students main say the best road is the road to Ballinamore.	The teacher will read the question and clarify what the students are expected to find, i.e. the coordinates of the fairest bus pick-up location. The students were told there were 4 different ways to finding the solution.

Where is the fairest place to pick up the students from all of the 3 different areas around Ballinamore?

Student Individual Work

Students will work individually for 25 minutes to try to find the coordinates of the fairest bus pick-up point.

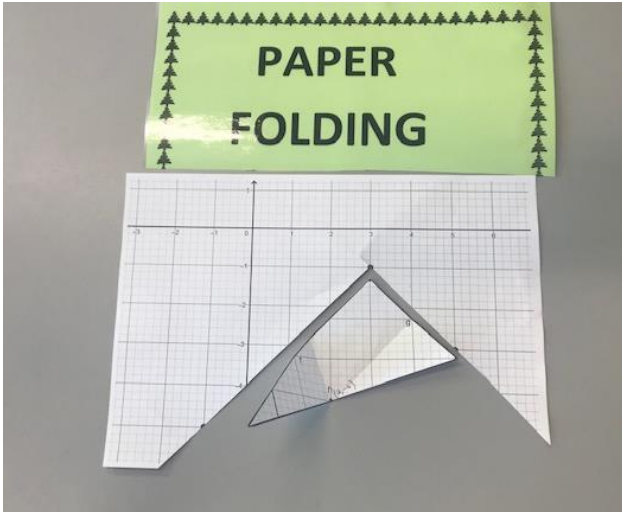The anticipated responses to the students working on solving this problem are:

The interactive classroom timer will be displayed for 25 minutes on the data projector. The timer can be found at (think about using this)https://www.online-stopwatch.com/classroom-timers/

1. **Estimation**

Students will look at the diagram and estimate by guessing the co-ordinates or by measuring with a ruler the fairest location for the pick-up point.

The teacher will move around and check students work. The teacher will encourage students to attempt solving the problem. Teacher will question students on the accuracy of estimation and ask them to validate their method. Teacher will encourage students to attempt other solutions that are more accurate.

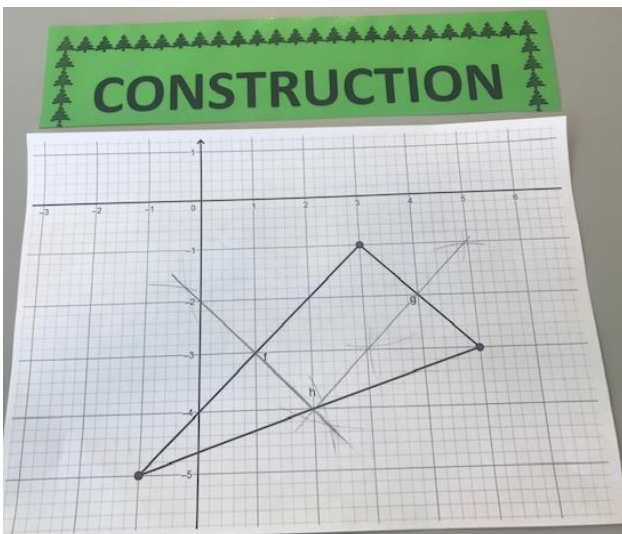Effective questioning will be used by the teacher to check students thought process and strategy.

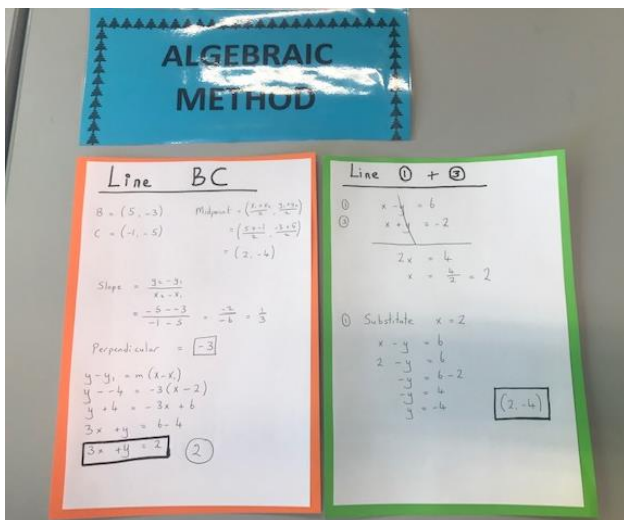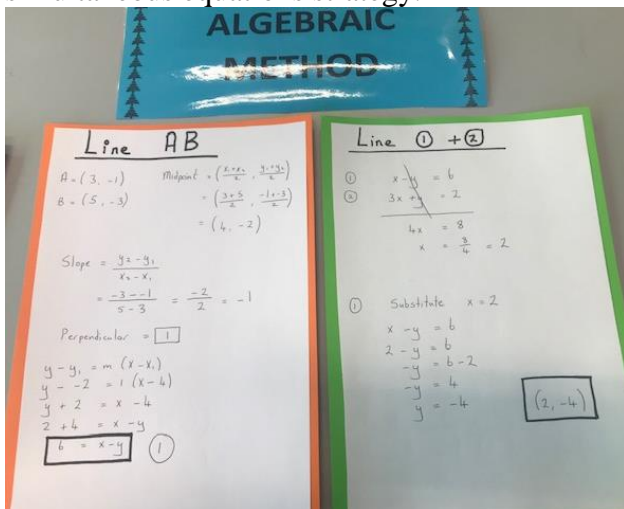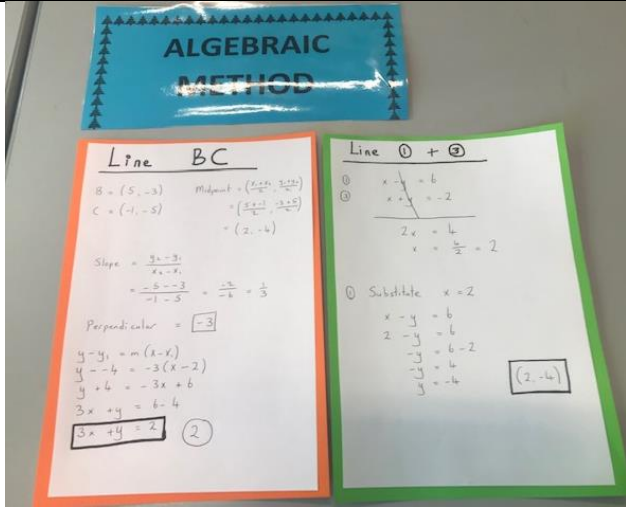|		
2. Paper Folding Students will cut out the triangle off their question sheet and by using paper folding techniques try to establish the common point. 	The teacher will check students approach and question this technique. Students will give the reason behind using this approach.	Questioning strategies will be used here by the teacher to gain an understanding of the students thought process and why they used this strategy. Teacher should question method of paper folding and check for any misconceptions.
3. Construction Students will attempt this question by constructing the perpendicular bisectors of the sides of the triangle and locating the point of intersection. 	Commend and affirm the exactness of this approach at locating the intersection of the sides. Acknowledge students and approach.	Check for understanding of the distance from the points to the co-ordinates found. Is the student able to validate and justify their approach? Observers should check on the accuracy of this method and student's ability to do the construction.

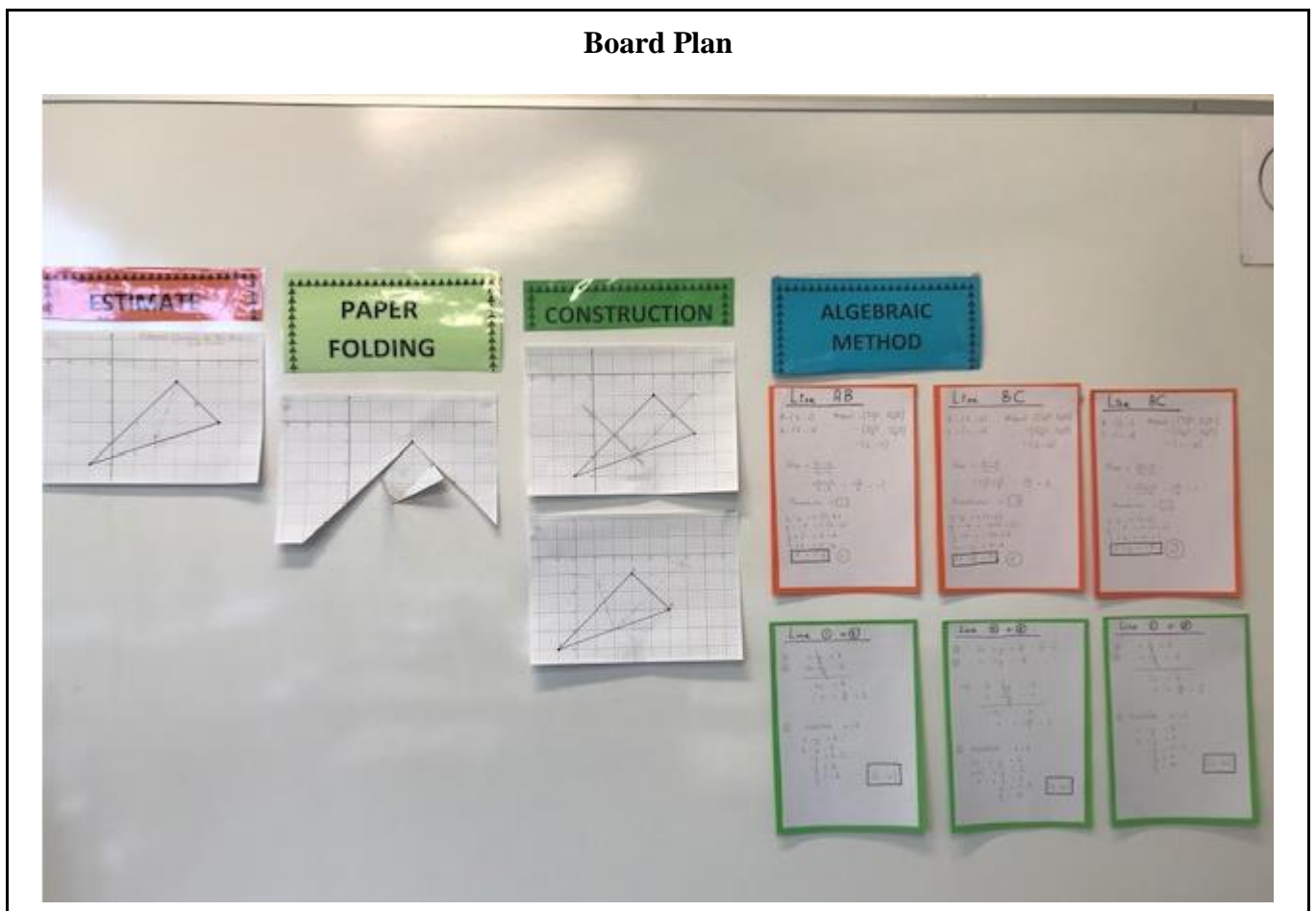|		
4. Algebraic Method Students will use two coordinates to form the slope of the line. The will locate the midpoint using the same coordinates. They will create an equation of a line perpendicular to the side of the triangle use perpendicular slope and the midpoint. They will repeat this process with another two coordinates. Having created the equations of both perpendicular bisectors they will find the intersection of these lines using a simultaneous equations strategy. 	Teacher will view students approach to coordinate geometry here using slopes and understanding equations of lines. Teacher will acknowledge and affirm students in their approach.	Questioning for students understanding of slope and perpendicular lines will be assessed by observation of algebraic method and effective questioning strategies for coordinate geometry. The students must be able to validate and justify this approach?

| **Ceardaíocht /Comparing and Discussing** (20 Minutes)
The aim of this section is to discuss with the students their approaches to problem solving and drawing on most accurate methods. Ceardaíocht will allow students to verbalise and emphasise where one approach is more reliable and definite than other approaches. Constructions and graphing can be limited to finding exact coordinates and approximation is called upon. The connection to geometry and students reasoning here should be affirmed and commended. Discuss on the coordinate and algebraic method should highlight the concept of accuracy. | Students will be called upon to present their solutions in order at the board. Teacher will lead the students through their solutions by asking questions and allowing the students to explain their approaches. The teacher should facilitate a whole class discussion and verification of the approaches used.

This type of question can then lead to further problem solving tasks based on written questions or solving equations or solving geometrical or graphical problems. | The teacher will assess the learning through questioning and also the student's justification and understanding of their methods. Whole class directed questioning will take place to assess the students who used similar approaches. |
| **Summing up & Reflection** (5-10 Minutes) | The teachers will draw student's attention to the board plan and the approaches used for solving the problem. Affirmation will be given with respect to all the approaches. Questions will be asked in relation to accuracy and making connections between the approaches, showing how one concept can lead to a | Student's answers to the questions posed by the teacher will assist in the assessment of learning and understanding. |

	stronger more justified method.	
Homework	Students will be handed out a reflection sheet on the lesson to fill in for homework. This will be fixed into the students notebooks.	The teacher will be able to assess the learning that took place from the feedback the students give in this homework task.

10. Board Plan
Carefully plan the board work before the lesson takes place to decide on the order of the solutions and the links that will be made at the board. Put an image or a diagram of the pre-prepared board work here.

Board Plan

11. Evaluation

We feel as teachers we achieved a lot of our goals.

- ***Were the students motivated and engaged in their learning?***
  During the introduction of the lesson, students built on each other's understanding of the term 'fairest, until they all agreed what the term meant in relation to mathematics, showing their engagement from the beginning. When the time was up for the students to stop working on the problem, they continued writing and some asked for additional minutes so they could finish the algebraic method. Even though the students started their own methods first, they were determined to find other ways to solve the question towards the end even when they weren't sure. This was evident when 2 students started doing methods that weren't one of the solutions.

- ***Are students able to recognize the relationship between Maths and everyday life?***
  The students were able to make the link between Maths and real-life problems by being able to fully explain what the problem was asking and what it meant in terms of Maths i.e. they came up with the reasoning that 'fairest' implied equal distance to all three villages. By making this relationship, all students were able to achieve at least one of the methods.

- ***Were students able to make connections in their Maths?***
  The students recognized that there are many ways to solve a problem, however, during the summary of the lesson, the students concluded that accuracy is paramount when deciding on the best method. They also came to the conclusion that the algebraic method was the most accurate because the other methods may not suit if the point of pick-up involved fractions or decimals. Other methods can help verify and guide students in their use of algebra. Many students were observed checking their paper folding along with the algebraic method to see if they were getting the same answer for both techniques.

12. Reflection

We hoped we would see all 4 methods attempted during the class, and that students would be motivated to find other ways to the solution. Both of these were achieved as the students demonstrated at least one of the methods, with a few requesting more time at the end to finish finding the solution. Presenting a problem that was specifically relevant to the students and where they lived also contributed to the engagement of the class.

The majority of students started with paper folding and constructions, no student started with estimation and eventually one student used estimation. Many students found the mid-point and stopped. 3 students used simultaneous equations with one of the students completing it correctly. Students checked/ verified their work by comparing their paper folding and constructions with the algebra. Some students drew the circumcircle to verify their answer. No student used the white boards to graph work out solutions. Other students looked at the Maths tables in the geometry section at triangles as if for inspiration, as they failed to make the link to co- ordinate geometry.

The majority of the students still cannot make the connection with making the equation of a line using the formula from the log tables, even though they have made it twice with their construction and paper folding, and even when they have their midpoint worked out. Estimation doesn't register with students as a method for finding the point of pick up but in the summary of the lesson they realized that it can help to get a general idea of what the solution might be. Students mathematical language was very good, and their explanations of methods used were clear and accurate. The teachers use of effective questioning allowed for the teacher to assess the learning of the students and their understanding.

Prior knowledge was drawn on for simultaneous equations and co - ordinate geometry from other

students. When the student that completed the algebraic method was brought to the board, other students were still questioned and encouraged to develop and explain the technique to promote involvement. Once they realised that the problem could be solved using coordinate geometry and simultaneous equations, the were able to contribute and their previous knowledge on these topics became evident.

Making connections was an important goal for our students and in the future we would consider looking at making links between algebra, graphs and patterns. Again, this would relevant and inclusive to all our students and our Maths Department.