Lesson Research Proposal for the Discovery of the Difference of Two Squares formula in a Transition Year class

Date of lesson:	12th March 2019
School name:	St Patrick's Lacken Cross
Teacher giving lesson:	Michael Walsh
Associate:	Michael Walsh
Lesson plan developed by:	Pauline Mc Gee, Sinead O'Malley and Michael Walsh

1. Title of the Lesson: Scrutinizing the Difference of two squares.

2. Brief description of the lesson

Students will use prior knowledge of factorising in algebra and apply it to factorising the difference of two squares. They will use algebra tiles to develop the idea and discover a pattern that will lead to a general algebraic method.

3. Research Theme

School aims:

We at St Patrick's College aim to improve the Literacy and Numeracy levels of our students. We hope our students will achieve at a very minimum, basic reading skills and basic skills in Arithmetic.

Math Department aims:

As teachers we want to be able to develop and implement consistent and dependable formative and summative assessment practice. We aim to support our goals by implementing the use of ICT in the classroom.

4. Background & Rationale

We are choosing Transition year students; the topic we are using is Algebra at a common level. We believe that in the future all students will be in the same room working on problems. Also, students will not be split up into different classes like they traditionally were in the old form of the senior cycle. In the area of Algebra students have common misconceptions.

With the new Junior Cycle, students will be entering senior cycle with a different mind-set to students we have taught in the past. We are concerned that our students will find the transition from the new Junior Cycle to the current Senior cycle challenging.

Students who are currently in senior cycle are having great difficulty with multiplying out binomials. They don't seem to have a good conceptual understanding of how to multiply expressions. We aim to address any misconceptions students have about this skill.

Questioning students in the classroom they are showing great difficulty with algebra especially in the area of multiplying out expressions.

We plan to research the students' misconceptions through a survey, we will use Microsoft forms and identify specific problems the students are having.

5. Relationship of the Unit to the Syllabus

Students should already know how to use the associative and distributive property to simplify expressions as:

$$a(bx + cy + d) + e(fx + gy + h)$$

$$\boldsymbol{a(bx + cy + d) + \ldots + e(fx + gy + h)}$$

$$a(bx^2 + cx + d)$$

$$ax(bx^2 + c)$$

$$where\ a, b, c, d, e, f, g, h \in Z$$

Also multiply expressions of the form:

$$(ax + b)(cx + d)$$

$$(\boldsymbol{x + y})(\boldsymbol{x + y});\ (\boldsymbol{x - y})(\boldsymbol{x - y})$$

$$(ax + b)(cx2 + dx + e) where\ a, b, c, d, e \in Z$$

$$a(bx \pm cy \pm d) + \ldots + e(fx \pm gy \pm h)$$

$$(ax^2 + bx + c) \pm \cdots \pm (dx^2 + ex + f)$$

$$where\ a, b, c, d, e, f \in Z$$

Students will learn how to perform the arithmetic operations of addition, subtraction, multiplication and division on polynomials and rational algebraic expressions paying attention to the use of brackets and surds. Apply the Binomial theorem.

The students will use this topic in their future learning while solving equations.

This topic will be important in their future learning at Senior Cycle when multiplying expressions.

6. Goals of the Unit

- At the end of this unit of study students will understand how to multiply out binomials and complete the square using an algebraic, graphical and a physical method using algebra tiles.
- The students will see the link between a physical, graphical and an algebraic approach to factorising, multiplying and completing the square.
- Students will understand the concept of Completing the square and the binomial theorem.
- Students will learn how to multiply out expressions by connecting the graphical and algebraic techniques discussed in class.
- Students should be able to multiply out any algebraic expression and be able to relate the solution visually.

7. Unit Plan

How the research lesson fits into the larger unit plan, helping to show the bigger picture of the whole unit and the progression of learning.

Lesson	Brief overview of lessons in unit
1	Revision of JC Algebraic expressions and extension to HL LC
2	Factorizing in algebra
3	Rearranging formula
4	Adding algebraic fractions
5	**Research lesson**

6	Solving linear and quadratic equations.

We plan to carry out the research lesson after the students learn skills from 'Algebra through the lens of Functions'.

Students will review Junior Cert Algebra before meeting the new learning outcomes at Senior Cycle.

We plan to assess the students learning by giving them a short quiz via Microsoft forms on the topics covered. Students will be able to demonstrate their learning by answering in a safe environment, with only the teacher seeing their response.

8. Goals of the Research Lesson:

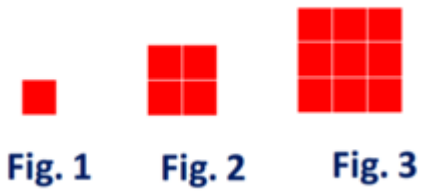
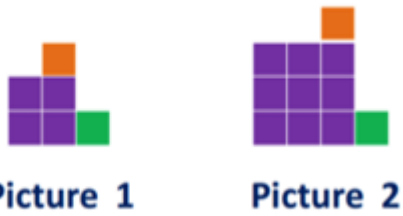
Looking at the goals of the research lesson itself:

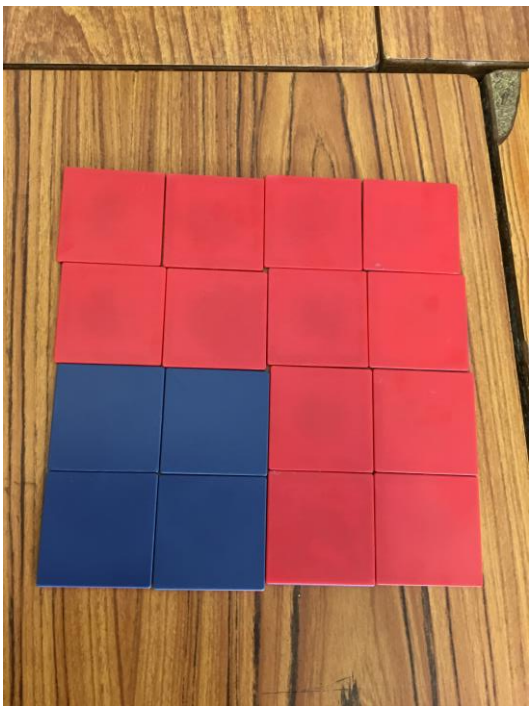
Mathematical goals:

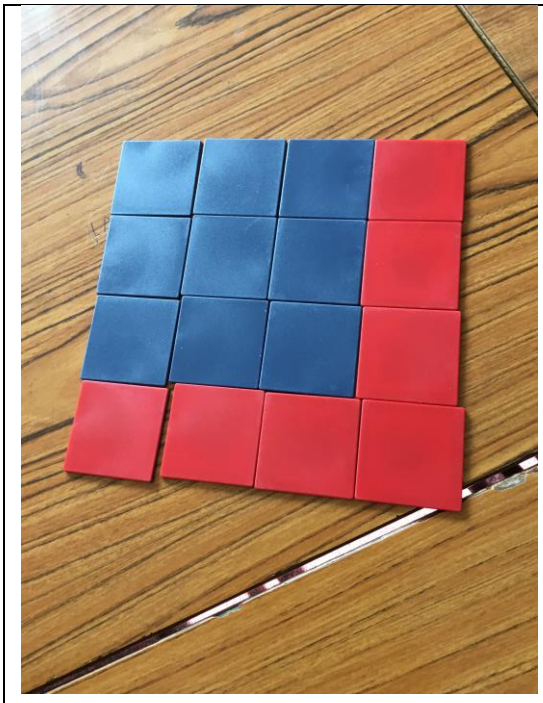
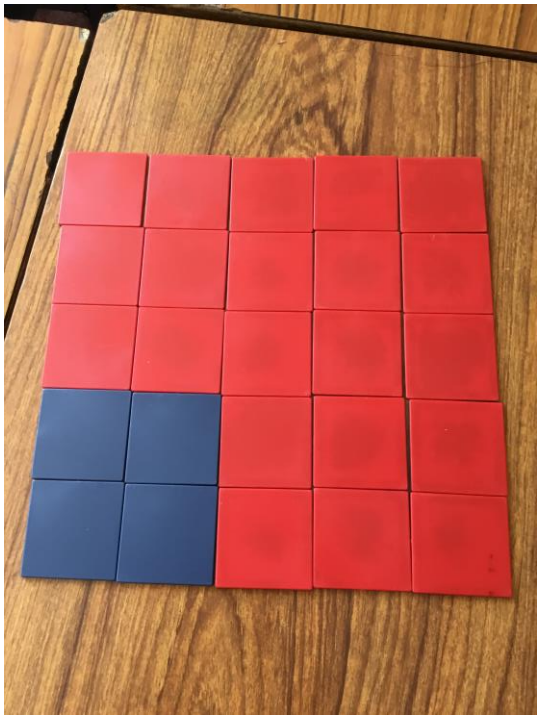
- Students will understand the application of the skills they have learned in Algebra by using both an algebraic and graphical technique.
- Students will use their learning from 'Algebra through the lens of Functions – Quadratics' to extend their understanding of algebraic expressions.
- Students will discover the formula of the difference of two squares by using algebra tiles.

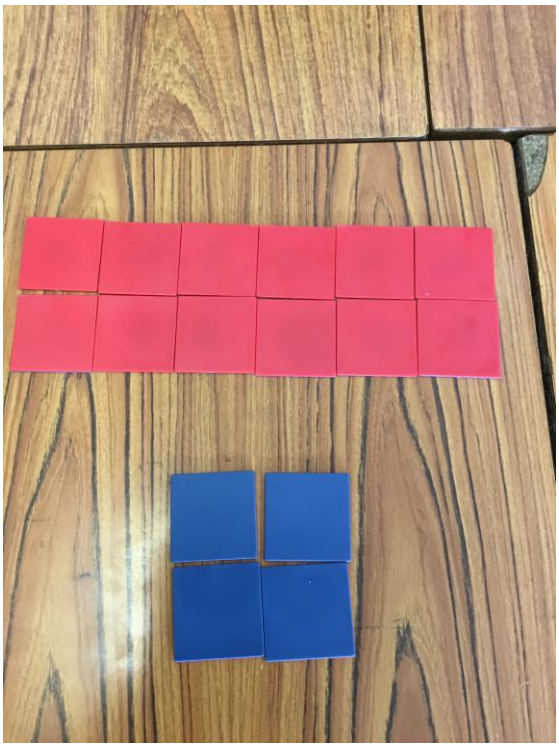
9. Flow of the Research Lesson:

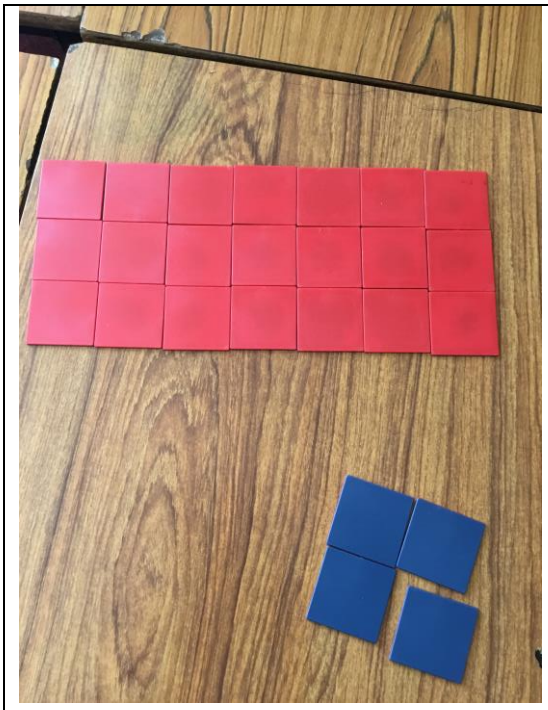
Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Re cap on previous lesson on quadratic patterns (10mins) Revise the formulae	Use one note of the growing visual quadratic patterns	Oral Feedback to assess prior knowledge

Pattern 5  *Pattern 5 = (n) ^2* Pattern 6  *Pattern 6 = (n + 1) ^2 = (n + 1) (n + 1)* Pattern 7  *Pattern 7 = (n + 1) ^2 + 1 = (n + 1) (n + 1) + 1* Pattern 8  *Pattern 8 = (n + 1) ^2 + 2 = (n + 1) (n + 1) + 2* Today we are going to use algebra tiles to help us with factorising.	Write up the formulae for patterns 5 – 8 Discussion on patterns	Students ability to generalize the formula Students will see a relationship between the diagrams in patterns 5 –8 and algebra
Posing the Task (2mins) Today we're going to use our mathematical knowledge to find the difference of two squares. We have learned that we can factorise	Group work tables with each table having algebra tiles showing the	Do students understand the question?

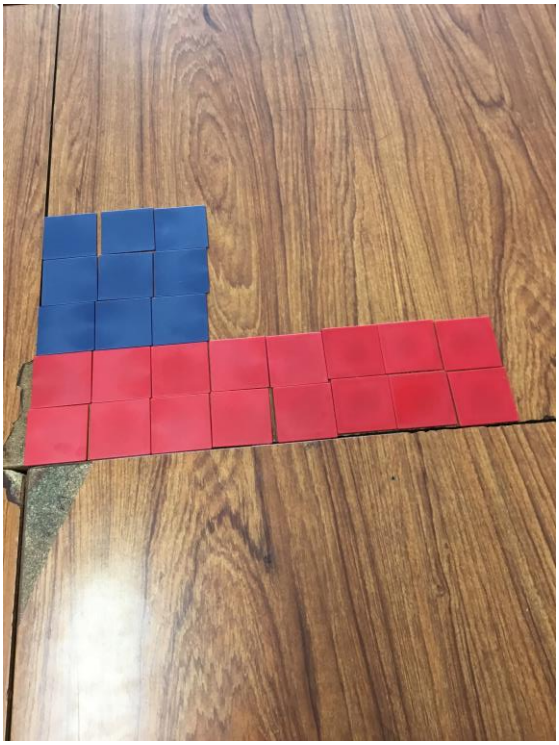
and multiply out expressions using the quadratics patterns resource. Today we are going to use algebra tiles to try and find a method of factorising the difference of two squares. "Shauna is trying to find the relationship between the red and blue tiles using subtraction. Can you help her?" There are algebra tiles left on each desk representing the difference of two squares: 	difference of two squares using different examples.	

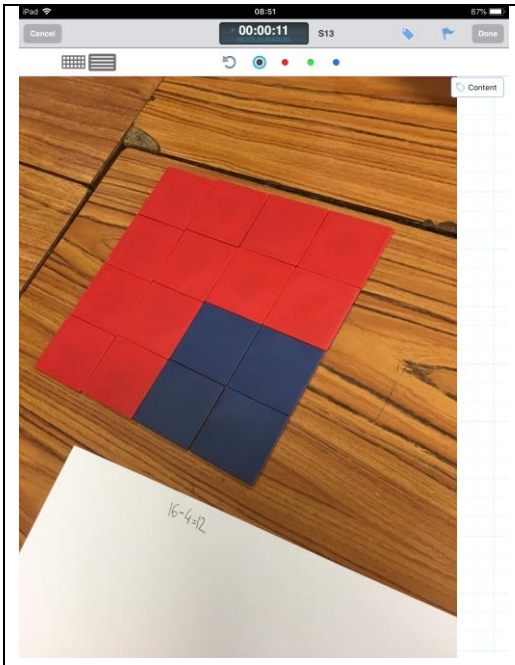
Student Individual Work (15mins) Students put into groups of 4 or 5. Time given to explore the visuals that have been left on their desks. Students are encouraged to move the tiles around (but not to turn them over). Students discuss possible relationships and jot down some ideas on a page. (Pics)  Area of rectangle = 6 x 2	Moving between groups and offering help if needed Sample questions: Did you take away the blue tiles? Why might you take away the blue tiles? Can you make another shape? What shape? Why that shape? Reminder to previous patterns! Is there a relationship to original diagram? Is there a relationship between the numbers? Reminder of factors Can you generalise this relationship?	Visually Verbally Looking at their generalisations



Area of Rectangle = 3 x 7



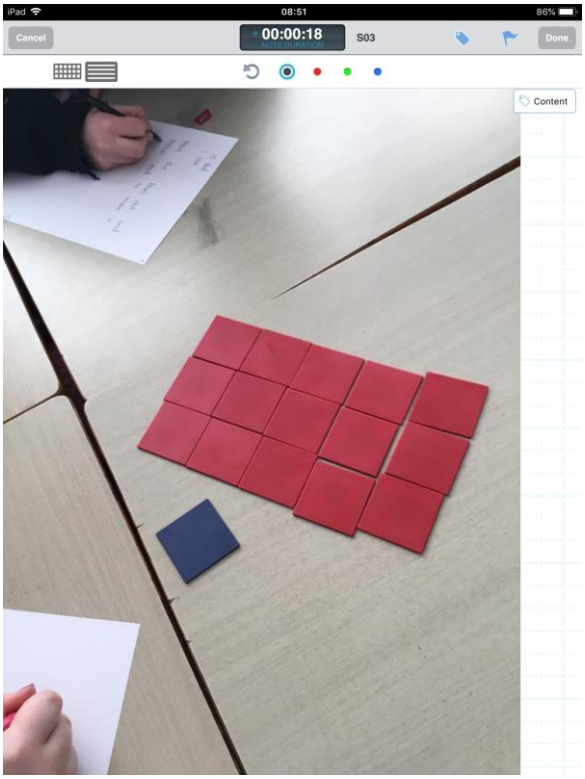
Area of Rectangle = 2 x 8



Area of squares:

16 – 4 = 12

(4) ^2 – (2) ^2 = 12



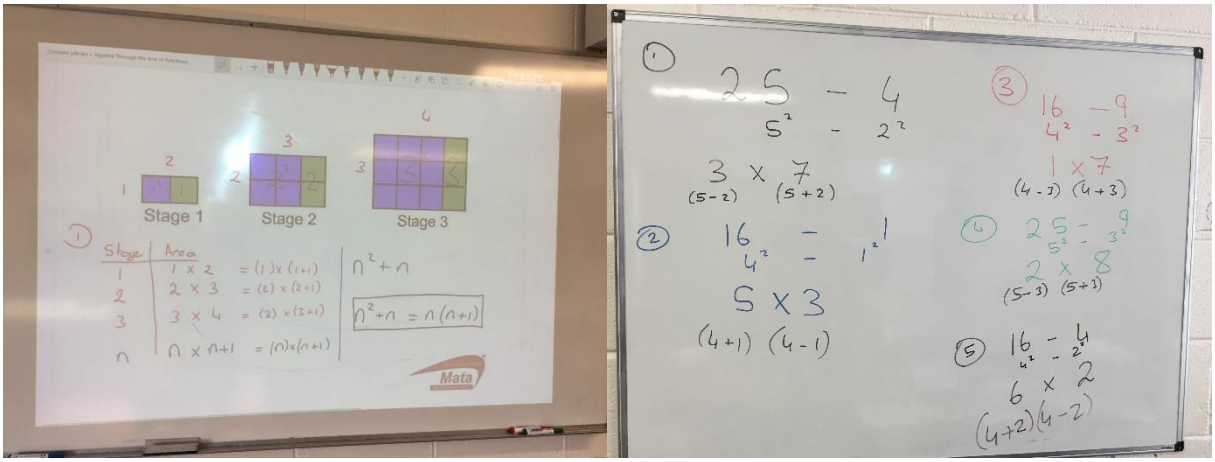
Area of Rectangle = 3 x 5

Ceardaíocht /Comparing and Discussing (10mins) Examples from tables will be shown on the board. Discussion based around the examples. Finding the relationship: **Example 1:** *25 – 4* *(5) ^2 – (2) ^2 = 3 x 7* *(5 – 2) (5 + 2)* **Example 2:** *16 – 1* *(4) ^2 – (1) ^2 = 5 x 3* *(4 + 1) (4 – 1)* **Example 3:** *16 – 9* *(4) ^2 – (3) ^2 = 7 x 1* *(4 + 3) (4 – 3)* **Example 4:** *25 – 9* *(5) ^2 – (3) ^2 = 8 x 2* *(5 + 3) (5 – 3)* **Example 5:** *16 – 4* *(4) ^2 - (2) ^2 = 6 x 2* *(4 + 2) (4 – 2)*	Students will be encouraged to write out their workings on a page. Each group shares a description of the original shape that was left on their desks. On the board: Re-write the squares as a number squared. Write the area of the resultant rectangle below this. Can you see a relationship between the numbers in the squares and the numbers in the rectangle? Now, can we generalize this relationship? *(x) ^2 – (y) ^2 =* *(x + y) (x – y)*	Each student has written a mathematical expression of the algebra tiles that were on their desks originally. They have changed these shapes to rectangles and written a mathematical expression to describe these rectangles. The students are challenged to see the connection in the numbers.

Summing up & Reflection (3mins) Did the students enjoy the lesson? Do the students understand where the formula for difference of two squares has come from? Would students be able to use this knowledge given the difference of two squares in algebraic format? – This will be the next lesson's extension		Students were in agreement that the difference of two squares formula has an important place in mathematics and that it makes sense.

10. Board Plan

Board Plan



The Lesson:

The students were not making the link in the beginning. They had to be reminded that we were subtracting. Maybe we could re-word the question. Students made one rectangle which turned out to

be the right one for each group. Had they made other rectangles consisting of other possible factors we would have guided them to the relevant factors for the given problem.

The students were solving the problem visually. They moved the tiles to form a rectangle. The student's presentation and discussion was teacher led by writing the workings of each group on the board and then comparing this work.

This led to a 'wow' moment where students discovered the formula for the difference of two squares. Students discussions and worksheet scribblings reflect that they discovered this. The time worked very effectively. Students were given ample time to work on and discover the relationship. To follow on from this lesson, students will need to see different examples of Algebraic problems to understand factorising the difference of two squares in varying contexts.

Teachers' Reflections on their Lesson Study

- What did you find useful about participating in a Lesson Study?

  "It was good for sharing ideas and collaborating inside the department. We are all on the same page in relation to practice in the classroom. We will always have a folder online that we share. If I have a problem in the future, I wouldn't hesitate to share it with my colleagues in the maths department. As a group will be able to work more cohesively together in future."

- Has participating in a Lesson Study caused you to think about your teaching in a different way?

  "Lesson study encourages you to be creative with your time and with specific topics. Lesson study also enables you to reflect on your practice and enjoy your teaching." Watching the demonstration by Professor Takahasi gives a great insight into how important appropriate teacher questioning can be.

- Having participated in one Lesson-Study cycle - can you see the difference between doing a Lesson Study for your professional development versus participating in a workshop? Consider the benefits of doing so, and the associated challenges and possible solutions.

  "Lesson study is more personal, you are more confident with your own students. Sometimes after a workshop the material has gone over your head or not being carried to your classroom practice. With lesson study you are working through the process over time, it matures and develops. It is absolutely necessary to have an outside influence and perspective so that we can have a fresh look at the problem/process ourselves."

- Does Lesson Study have a role in supporting other curriculum reforms e.g. junior cycle? If so, how?

  “Yes, in first year maths we could use the same problem. In French at junior cycle we could use the same process of lesson study to look at different ways of teaching material to students.”