Research Lesson Proposal on:

Factorising quadratic expressions in algebra – with emphasis on 2nd year unit plan for expansion and factorisation of algebraic expressions using

The Array Model

Date of lesson: 12/02/2019
School name: CBS Carrick-on-Suir
Teacher giving lesson: Martin Molloy
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Lesson plan developed by:
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Year group	2nd year
Topic	Algebra - Factorising
Level	Higher

1. Title of the Lesson: Using array models to factorise quadratic expressions

2. Brief description of the lesson

Student led lesson on factorising quadratic expressions in algebra using an array model – with emphasis on a 2nd year unit plan for expansion and factorisation in algebra.

3. Research Theme

At CBS Carrick-on-Suir we want our students to:

- grow as learners through respectful interactions and experiences that are challenging and supportive

We want our students to experience an interactive class, where individual and group participation is valued, shared and reflected upon. Evolve basic ideas of maths.
We want our students to feel safe, grow in confidence and get more actively engaged in class.

We want our teachers to:

- value and engage in professional development and professional collaboration
- work together to devise learning opportunities for students across and beyond the curriculum

This means as members of our staff department, we work more collaboratively with regards to lesson planning & sharing of our resources & expertise. Our learning outcomes through structure problem solving can be incorporated into our subject plans.

4. Background & Rationale

We have chosen a second year class as our target group for this proposal. The topic is Algebra with a focus on factorising. It is a higher level class but the lesson may be used for both higher & ordinary.

We have chosen this topic - factorising, because we feel it's a concept that students struggle with and students had to be retaught these skills again at Senior Cycle as they had very poor retention of it from Junior Cycle.

"They struggled noticeably with questions that involved substantial amounts of algebra"
Chief Examiners Report, Junior Certificate Mathematics, 2015

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
6th class students should be able to: • Identify factors and multiples	Multiply expressions of the form: • $(ax + b)(cx + d)$ • $(ax + b)(cx^2 + dx + e)$ where $a, b, c, d, e \in Z$	Solve a Quadratic of the form $ax^2 + bx + c$ Divide expressions of the form:

• Identify common factors and multiples • Multiply a whole number • Calculate Area of 2D shapes 1st year students should be able to: • Investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers • Understand N: the set of natural numbers, *N = {1,2,3,4….} Z*: the set of integers, including 0 • Evaluate expressions of the form: *ax + by*, where *a, b, c, d, x, y* $\in Z$ Simplify expressions • *a(x + y)* • *ax^2 + bx + c* • *axy*	Factorise expressions such as: • *ax, axy,* where *a* $\in Z$ • *abxy + ay*, where *a, b* $\in Z$ • *sx - ty + tx - sy*, where *s, t, x, y* are variables • *ax^2 + bx*, where *a, b, c* $\in Z$ • *x^2 + bx + c*, where *b, c* $\in Z$ • *x^2 - a^2* • *ax^2+ bx + c*, *a* $\in N$ *b, c* $\in Z$	• *ax^2 + bx + c ÷ dx + e*, where *a, b, c, d, e* $\in Z$ • *ax^3+ bx^2+ cx + d ÷ ex + f*, where *a, b, c, d, e* $\in Z$ Solve one linear equation and one equation of order 2 with two unknowns Factorise expressions of order 2 Add and subtract expressions of the form: • *(ax+by+c)+...+(dx+ey+f)* • *(ax^2+bx+c)+...+(dx^2+ex+f),* where *a,b,c,d,e,f* $\in Z$ Use associative and distributive properties to simplify expressions such as: • *a(bx+cy+d)+...+e(fx+gy+h),* where *a,b,c,d,e,f,g,h* $\in Z$ • *(x+y)(w+z)* Rearrange formulae

• $ax^2+ bx + c$ • $ax^3 + bx^2 + cx + d$ where $a, b, c, d, x, y \in Q$		

6. Goals of the Unit

- Students will experience an interactive class where they can discuss mathematics and participate at many levels
- Students will discover and understand the relationship/connection/link between expansion and factorising.
- Students will have a stronger foundation to retain the skills of expanding and factorising for future learning.
- Students will have a good platform in which to handle later learning outcomes.
- Students will have a deep understanding of the symbols and appropriate language used in algebra, and will practice using these symbols and language.
- Students will appreciate the importance of the operations used in algebra, and that the same operations have been practiced when studying number systems.

7. Unit Plan – a bank of 8 proposed lessons to help with student understanding in this area

Lesson	**Brief overview of lessons in unit**
1	Note: Teacher input is in **bold** Revise the area of a rectangle  Area of a rectangle = length x width Students solve some simple numerical examples

<table>
<tr>
<td></td>
<td>
Introduce the array model to multiply number expressions
For example:
(i) Multiply out, using an array model:
3 X 27 = (3)(20 + 7)

Solution:
<table>
<tr><td></td><td>20</td><td>+7</td></tr>
<tr><td>3</td><td>60</td><td>+21</td></tr>
</table>
Ans: 60 + 21 = 81

(ii) Multiply out, using array model:
47 X 13 = (40+7)(10 +3)

Solution:
<table>
<tr><td></td><td>10</td><td>+3</td></tr>
<tr><td>40</td><td>400</td><td>+120</td></tr>
<tr><td>+7</td><td>+70</td><td>+21</td></tr>
</table>
Ans: 400 + 120 +70 + 21 = 611
</td>
</tr>
<tr>
<td>2</td>
<td>
Replace the lengths of the rectangle with algebraic expressions

For example;
Multiply out, using an array model:
(3)(x+2)

Solution:
<table>
<tr><td></td><td>x</td><td>+2</td></tr>
<tr><td>3</td><td>3x</td><td>+6</td></tr>
</table>
Ans: 3x + 6
</td>
</tr>
</table>

3

Expansion of more (and longer) expressions in algebra

For example:

Multiply out, using an array model:

(x + 2)(x + 3)

Solution:

	x	***+2***
x	*x^2*	*+2x*
+3	*+3x*	*+6*

Ans: *x^2 + 5x + 6*

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Using the array method to factorise linear expressions

For example:

Factorise, using an array model:

(2a + 4)

	a	*+2*
+ 2	***2a***	***+4***

Ans: *(a + 2)(2)*

Division of a quadratic expression

For example:

using an array model, divide the following expression

(a^2 + 3a + 2) / (a + 2)

	a	*+2*
a	***a^(2)***	***+ 2a***
+1	***+1a***	***+ 2***

<table>
<tr><td></td><td>Ans: (a + 1)</td></tr>
<tr><td>5
Research
Lesson</td><td>Using the array model to factorise quadratic expressions</td></tr>
<tr><td>6</td><td>Using the array model to factorise quadratics of the form ax^2 + bx + c, where a≠1

Factorise:
4x^2 + 2x - 6

Student Solution:
<table>
<tr><td></td><td>2x</td><td>+3</td></tr>
<tr><td>2x</td><td>4x^2</td><td>+6x</td></tr>
<tr><td>-2</td><td>-4x</td><td>-6</td></tr>
</table>

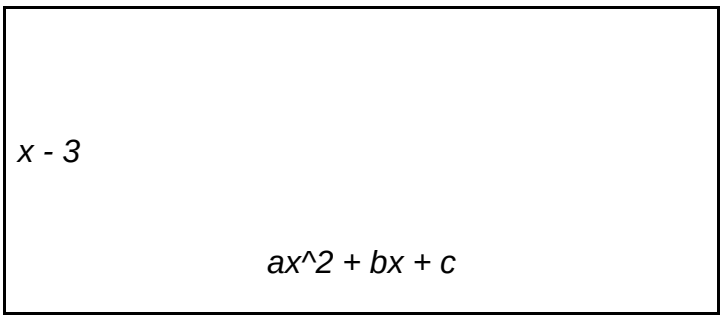
Ans: (2x + 3)(2x - 2)</td></tr>
<tr><td>7</td><td>Using the array model with other expressions
○ Difference of two squares
Factorise:
x^2 - 9

<table>
<tr><td></td><td>x</td><td>-3</td></tr>
<tr><td>x</td><td>x^2</td><td>-3x</td></tr>
<tr><td>+3</td><td>+3x</td><td>-9</td></tr>
</table>
Encourage students to identify that this is the difference of two squares, as -3x + 3x = 0</td></tr>
</table>

	○ Other expressions **Expand:** *(a + b)(c + d)* Ans: *ac + bc + ad + bd* **Factorise (2016 JCOL Q8):** *3ax + ay + 3cx + cy* Ans: *(3x + y)(a + c)*
8	Problem solving in algebra **(2017 JCHL Q12)** Q1. Use factorisation to simplify *(8e^2 - 18) / (2e^2 + 3e - 9)* Q2. A rectangle has sides of length *x - 3* units and *ax^2 + bx + c* units, where a, b, c ∈ Z. The **area** of the rectangle is: *2x^3 - 13x^2 +25x - 12* square units. Find the value of *a*, the value of *b*, and the value of *c*.

	a	***+b***
c	*ac*	*+bc*
+d	*+ad*	*+bd*

	3x	*+y*
a	***3ax***	***+ay***
+c	***+3cx***	***+cy***



Area: *2x^3 - 13x^2 +25x - 12*

Solution:

	2x^2	*-7x*	*+4*
x	***2x^3***	*-7x^2*	*+4x*
-3	*-6x^2*	*+21x*	***-12***

Ans: *a = 2, b = -7, c = +4*

8. Goals of the Research Lesson:

Looking at the goals of the research lesson itself from two perspectives:

A. Mathematical Goals:
   a. Students will discover how to factorise a quadratic expression.
   b. Students will discover and understand the relationship/connection/link between expansion and factorising.
   c. Students will have a good platform in which to handle later learning outcomes.

B. Emotional goals:
   a. Students will discover factorising in a new and more enjoyable way, helping to improve their motivation for learning algebra.
   b. Students will be given the freedom to be creative with their mathematics.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	**Teacher Support**	**Assessment**
Prior Learning: - Expanding two expressions, factorising linear expressions and dividing a quadratic. Learning Intentions: - Understand and be able to factorise a quadratic expression. - Discover the connection between expansion and factorisation. - Identify a quadratic expression. Success Criteria: - Draw the box for using the array model - Inserting into the appropriate sections (the x^2 and the constant) - Factorise the x^2 portion and the constant - Finding the correct factors		
Introduction (5 min) Recap previous lesson using following question: **Question on board:**	Teacher will have a question written on the board as students enter the classroom.	

<table>
<tr>
<td>Find the second factor in the following algebraic expression:

<table>
<tr><th>x</th><th>-4</th></tr>
<tr><td>x^2</td><td>-4x</td></tr>
<tr><td>+2x</td><td>-8</td></tr>
</table>

- Students will simplify the given quadratic: x^2 - 2x - 8
- use of colour to identify the correct locate for each term may be helpful to students.

Solution: (x + 2)</td>
<td>Students will be put in pairs and given mini whiteboards to write their solutions.</td>
<td>Students will write their solutions on the mini whiteboards and hold up the whiteboards so that the teacher can assess the solutions.

Students will give oral feedback on how they reached their solutions and that they understand question.</td>
</tr>
<tr>
<td>Posing the Task
Section 1: Quadratics - perfect squares

Factorise:
x^2 + 6x + 9

Student Solution:
<table>
<tr><th></th><th>x</th><th>+3</th></tr>
<tr><td>x</td><td>x^2</td><td>+3x</td></tr>
<tr><td>+3</td><td>+3x</td><td>+9</td></tr>
</table>

Discussion:
- Remembering to place x^2 in</td>
<td>Teacher will write the question on the board. Teacher will check with students of their understanding of question posed. Teacher will monitor students' progress during task completion.</td>
<td>Students will write their solutions on the mini whiteboards.

A pair of students will be invited to the board to demonstrate & explain their solution.
The class will then</td>
</tr>
</table>

top left corner
- Remembering to place constant (9) in bottom right corner
- Use of trial and error to fill in *x* boxes
- Using factors of the constant (9)

Factorise:

x^2 - 6x +9

Solution:

	x	***-3***
x	*x^2*	*-3x*
-3	*-3x*	*+9*

Discussion:

Note the difference in signs

The teacher will write this question on the board.

After students complete question, teacher will ask students to hold up their answers and then will write solution on board ensuring understanding of the whole class.

take down the solution into their notes copy.

Students will write their solutions on the mini whiteboards.

Students will show their answers to their peers.

Section 2: Quadratics - Constant as a prime number

Factorise:

x^2 + 6x +5

Solutions:

	x	***+5***
x	*x^2*	*+5x*
+1	*+1x*	*+5*

	x	***+1***
x	*x^2*	*+1x*
+5	*+5x*	*+5*

Discussion:

- Use of trial and error to fill in *x* boxes
- Students discuss the fact that they are using a prime number therefore only have two options to choose from

Factorise:

x^2 + 6x - 7

Solutions:

Teacher will write the question on the board.

Teacher will check with students of their understanding of question posed.

Teacher will monitor students' progress during task completion.

The teacher will write this question on the board.

Students will write their solutions on the mini whiteboards.

A pair of students will be invited to the board to demonstrate & explain their solution.

The class will then discuss the solution(s) and take down the solution into their notes copy.

	x	***+7***
x	*x^2*	*+7x*
-1	*-1x*	*-7*

	x	***-1***
x	*x^2*	*-1x*
+7	*+7x*	*-7*

After students' complete question, teacher will ask students to hold up their answers and then will write solution on board ensuring understanding of the whole class.

Students will write their solutions on the mini whiteboards.

Students will show their answers to their peers.

Section 3:

Quadratic - Constant Not prime

Factorise:

x^2 - 2x -8

Solutions:

	x	***-4***
x	*x^2*	*-4x*
+2	*+2x*	*-8*

	x	***+2***
x	*x^2*	*+2x*
-4	*-4x*	*-8*

Teacher will write the question on the board. Teacher will check with students of their understanding of question posed. Teacher will monitor students' progress during task completion.

Students will write their solutions on the mini whiteboards.

A pair of students will be invited to the board to demonstrate & explain their solution.

The class will then discuss and take down the solution(s) into their notes copy.

Discussion:

- Use of trial and error to find two factors
- Use of trial and error to fill in *x* boxes
- Need for prior knowledge of the factors of constant (8)
- Ability to choose these factors depending on the coefficient of *x*

Factorise:

x^2 - 8x - 9

Solution:

	x	***-9***
x	*x^2*	*-9x*
+1	*+1x*	*-9*

	x	***+1***
x	*x^2*	*+1x*
-9	*-9x*	*-9*

The teacher will write this question on the board.

After the students complete question, teacher will ask students to hold up their answers and then will write solution on board ensuring understanding of the whole class.

Students will write their solutions on the mini whiteboards.

Students will show their answers to their peers.

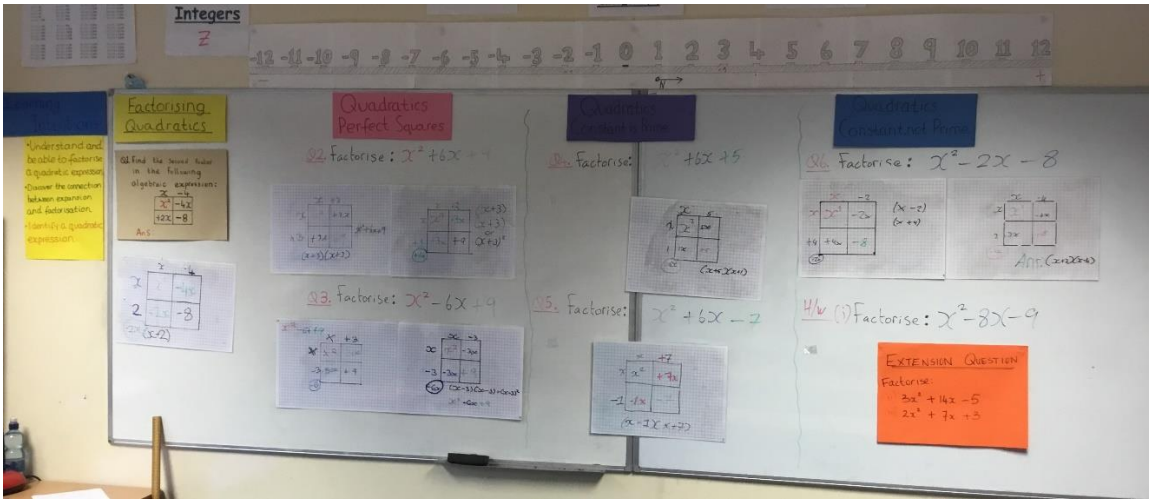
Extension / Homework Questions:

Factorise a quadratic expression with a prime coefficient on x^2:

Factorise: *3x^2 + 14x - 5* **Factorise:** *2x^2 + 7x + 3*		
Summing up & Reflection - Discussion on how expanding, factorising and dividing quadratic expression are linked. - Reflection on the skills needed to complete these tasks. - Reflection sheet handed out to students to use.	Recapping and reinforcing ideas from the lesson.	Students will write up a reflection of what they learned from today's lesson. Students will also reflect upon any area of the lesson that they had difficulty with & / need more clarification on.

10. Board Plan

Board Plan



11. Evaluation

Students discovered how to factorise quadratic expressions themselves by using the array model. By the end of the lesson students were beginning to see the connection between expansion and factorisation, however more time will need to be spent in the next lesson to make this connection more tangible for the students.

We believe that this lesson successfully promoted student-to-student discussion. Students explained the problems to each other, worked well in groups and kept all students involved in the lesson. We witnessed students motivating each other and correcting other students' mistakes.

Some of the approaches that the students used when tackling these factorisation questions included making meaningful connections with their previous learning. Students understood the vocabulary needed and practiced using this vocabulary within the class, in particular when they were presenting their solutions. Students also linked the perfect square to a lesson that had been done the previous week. There was also evidence of students writing their factors in brackets beside each other, without a prompt from the teacher.

One of the main issues that arose in this lesson was students' confusion over when to use positive and negative signs. This highlighted a common issue in the teaching of mathematics and will need to be addressed again during this unit.

During this lesson, observers saw students using trial and improvement to get to their answers, they gained confidence during this lesson and discussed their methods in groups. This resulted in more confidence when answering the final question, and almost every group could factorise this quadratic without asking for help. This was observed as a successful lesson that reached the goals set at the beginning, including evidence of the students' enjoyment and engagement during this lesson.

Emotional goals – some examples of student reflections:

- I learned that when doing on array method make sure to have the signs be right because a different + or – makes it wrong.
- I thought today's lesson was very informative on the topics I wasn't sure on.
- I learned how to factorise quadratics.
- I learned how to identify a quadratic expression.
- I thought today's lesson helped me improve my maths ability.
- The first thing I learned in today's lesson was proper words for the numbers.
- Today I learned how to correctly factorise quadratics with perfect squares, when constant is prime and when constant isn't prime.
- I thought today's lesson was fun because we got to use whiteboards.
- I would like to try do other ones which are harder.
- It helped me understand quadratic expressions.
- I would like to know more about when the constant is not a prime.
- If the constant is prime it will always be multiplied by 1 and itself.
- It was a very good learning experience.
- I think today's lesson was well explained.

12. Reflection

During this lesson the team hoped that the students would engage in the tasks and attempt them in a meaningful way. It was also hoped that they would present their solutions in a neat and legible way. What was observed in this 40-minute lesson was

that everyone was engaged and that they worked together and discussed the process. Students seem to find the use of mini whiteboards and group work enjoyable.

The teachers observed the benefit of using the array model when exploring factorisation and agreed that this method worked successfully for students of all abilities and should work as effectively at other levels also. Teachers agreed that what was covered in one lesson using the array model, would normally take two to three lessons using their old methods, however they did question if more time could be given to this topic in the next lesson. In this lesson, there were fewer questions attempted, but in a more effective way, leading observers to discuss the merits of having quality over quantity in their lessons. There should also be more time given to allow students to make a deep connection between expansion and factorisation.

The success of this lesson was the mathematical and emotional goals being reached. It was evident that a huge amount of collaboration went into the unit plan, which was designed to make factorising quadratic expressions accessible to students. Students were interested and engaged in the lesson. All students showed a greater understanding and knowledge of the topic by the end of the lesson.

Ideas for future study:

- Extension – higher level leaving cert students, long division in algebra.
- Thinking about changing own practice and spreading this method to other maths teachers.

Reflecting on the Lesson Study process:

Lesson Study was a worthwhile process, we needed that amount of time for planning as there was a lot of planning and thinking involved. Teachers were very happy with the amount of collaboration and felt it was an important use of time. They can now use other proposals from Lesson Study. Use of Lesson Study would be a worthwhile process in the planning of whole department. Teachers involved felt they had their own learning experiences during the meetings. They felt that if this concept had been introduced to them in a one-off workshop they would not have been as willing to change their practice. It was only through the series of meetings and discussions

involved in the Lesson Study process that they felt the confidence to change their practice to using the array model in algebra.

Involvement in the Lesson Study process as a whole department would be more worthwhile. It creates an emphasis on:

- collaborative planning
- improvement in practice
- should form part of development of strategies
- could be used with any topic
- is a good approach to planning.

This process would encourage a department to agree on common approaches, use of vocabulary in maths and consistent language resulting in more consistency in teaching and learning for their students.