Lesson Research Proposal for St Mark's CS - Algebra

Date of lesson: February 12, 2018
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Lesson developed by: Rachel Byrne, Marion Coulon, Lynne Kelly, Abieyuwa Owie, Alison Ryan, Irene Stone

1. **Title of the Lesson:** The X Factor

2. **Brief description of the lesson**

Through exploring different ways to represent factorisation of a single algebraic term, students will discover how to factorise a two term linear expression.

3. **Research Theme**

From consulting with the document *Looking at our Schools 2016 A Quality Framework for Post-Primary Schools,* we have considered the school self-evaluation priorities for our school, St Mark's CS. With these priorities in mind, we aim as teachers of mathematics:

- to select and use teaching approaches appropriate to the learning intentions and to students' learning needs
- to use a range of questioning techniques effectively for a variety of purposes including stimulating substantial student responses and facilitating deeper engagement with lesson content
- to meaningfully differentiate content and activities in order to cater for the varying needs and abilities of students.

Through the Lesson-Study process, we as teachers aim to:

- develop tasks at the most appropriate level for our students
- consider multiple approaches to tasks and make connections across the syllabus
- develop a growth mindset in our students
- use formative assessment to inform learning and give our students diagnostic feedback on their work
- give students time to act upon that feedback.

4. Background & Rationale

The *Chief Examiner's Report (2015)* found that students often give up easily and candidates tend to struggle with questions involving Algebra. It was *"noticeable for most candidates that if they did not hit upon the correct answer immediately, they showed little purpose in their further attempts".* At higher level, some non-routine questions required candidates to engage accurately and effectively with Algebra and candidates tended to struggle with these questions. A survey of our 1st years in September 2018 reflected the difficulties documented in the report. The survey highlighted that many students have negative attitudes towards mathematics and have a fixed mindset.

> *"Students with a fixed mindset are those who are more likely to give up easily, whereas students with a growth mindset are those who keep going even when work is hard, and who are persistent."* (Jo Boaler[1]).

Some of our students believe that one is '*either good or not good at maths*' and that '*students who are good at maths will be able to solve a problem in less than 5 minutes*'. Other students believe that '*if I can't do a problem, I give up straight away'*. Many of our older students have demonstrated misconceptions in Algebra such as confusing x^2 and $2x$ and making errors multiplying out brackets for example:

- $(x + 8)(x + 7) = x^2 + 15x + 15$
- $(x + y)^2 = x^2 + y^2$
- $x + 3 = 3x$

We have also found inconsistencies in the way multiplying expressions is taught to students. It is important that our students are exposed to different methods. Some students appear to be relying on a procedural way of multiplying out brackets without having a deep understanding of what 'brackets' mean. We also found that our students struggle when factorising expressions and don't see the link between multiplying and factorising. It seems they are taught as distinct concepts. We are not confident that all our students have a true understanding of what a variable is.

We decided, therefore, to target our research lesson at 1st years. We hope to develop a growth mindset in them and we chose the topic of Algebra so as to avoid potential misconceptions developing. We wanted to encourage different ways to present their solutions, give them feedback through the lesson and allow them 'quiet' time to engage with the work.

[1] https://www.youcubed.org/resource/growth-mindset/

5. Relationship of the Unit to the Specification

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Primary curriculum The child should be enabled to ● identify factors and multiples from basic multiplication facts. ● identify common factors and multiples - explore and record factors and multiples to identify common factors and multiples ● write whole numbers in exponential form $1000 = 10\ x\ 10\ x\ 10 = 10^3$ ● multiply a two-digit or three-digit number by a one or two-digit number They should know that ● a product is the result of multiplying two numbers **Junior Cycle Specification:** Students have used the area model for multiplying numbers. AF2: They have investigated situations in which letters stand for quantities that are	To use the distributive rule in Algebra. AF3: apply the properties of arithmetic operations and factorisation to generate equivalent expressions so that they can develop and use appropriate strategies to add, subtract and simplify linear expressions in one or more variables with coefficients in $\mathbb{Q}$ multiply expressions of the form $a(bx + cy + d)$, $a(bx^2 + cx + d)$, $a(bx^2 + cx + d)$ $where\ a, b, c, d\ \in \mathbb{Z}$ $(ax + b)(cx + d)$ $where\ a, b, c, d \in \mathbb{Z}$ AF3 apply the properties of arithmetic operations and factorisation to generate equivalent expressions so that they can develop and use appropriate strategies to flexibly convert between the factorised and expanded forms of algebraic expressions of the form: $axy, where\ a\ \in\ \mathbb{Z}$ $axy + bzy\ where\ a, b\ \in\ \mathbb{Z}$	AF3: apply the properties of arithmetic operations and factorisation to generate equivalent expressions so that they can develop and use appropriate strategies to c. multiply expressions of the form $(ax + b)(cx + d)$ $(ax + b)(cx^2 + dx + e)$ $where\ a, b, c, d, e\ \in\ \mathbb{Z}$ d. flexibly convert between the factorised and expanded forms of algebraic expressions of the form: $dx^2 + bx$ $x^2 + bx + c$ $ax^2 + bx + c$ $where\ b, c, d\ \in\ \mathbb{Z}$ and a $\in \mathbb{N}$

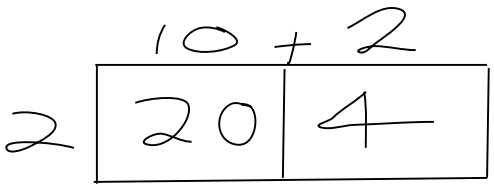
variable so that they can: a. generate and interpret expressions in which letters stand for numbers N.1: They have investigated the representation of numbers and arithmetic operations so that they can: b. perform the operations of addition, subtraction, multiplication, and division and understand the relationship between these operations and the properties: commutative, associative and distributive in $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ and in $\mathbb{R}\backslash\mathbb{Q}$, including operating on surds N1: d calculate and interpret factors (including the highest common factor)	$sx - ty + tx - sy,$ $where\ s, t\ \in\ \mathbb{Z}$ $dx^2 + bx$ $where\ b, c, d\ \in\ \mathbb{Z}$ and a $\in \mathbb{N}$ U3: recognise that equality is a relationship in which two mathematical expressions have the same value U4 represent a mathematical situation in a variety of different ways (numerically and using the area model) U13 communicate mathematics effectively: justify their reasoning, interpret their results, explain their conclusions, and use the language and notation of mathematics to express mathematical ideas precisely	

6. Goals of the Unit

Students will be able to:

- transfer their knowledge of the commutative and distributive properties from number to Algebra
- discover that there are different ways to factorise numbers and variables
- develop skills for using the area model
- develop problem-solving skills
- develop a better appreciation of the differences between sums and products in Algebra
- develop ownership of their learning through discovering how to factorise a linear expression
- be flexible in moving from expanding to factorising and back again
- be confident in multiplying and factorising algebraic expressions

7. Unit Plan

Lesson	Brief overview of lessons in unit
1	Algebraic Products; looking at different ways to show multiplication in Algebra, recap of what is a variable and constant. What do $3x, xy, (4)(a)(b), 3x^2$ mean?
2	Simplifying algebraic expressions $3x + 2y + 5x$, collecting like terms, identifying terms, variables, coefficients, expressions
3	
4	Multiplication of algebraic terms including using area models 
5	Recap of factorising numbers (as products) e.g.$24 = (2)(12)$ $24 = 2(10 + 2)$ with boxes 
6	Research Lesson
8	Factorising other expressions (e.g. with 4 terms)
9	Decomposition of numbers, area model with numbers
10	Multiplication of algebraic expressions (linear x linear) $(c + 5)(2e + 3)$ $(2z + 3)(x + 5)$

8. Goals of the Research Lesson:

Mathematical goals

- Students will understand how to factorise an algebraic term and know that they can represent it in different ways (e.g. using an area model and brackets)
- Students will discover how to factorise a linear two-term expression and represent it in different ways
- Students will recognise that equality is a relationship in which two mathematical expressions have the same value

Key Skills and Statements of Learning

- Being Numerate: By engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.
- Working with Others: Students will learn with and from each other by discussing different approaches to problem solving.
- Communicating: During Ceardaíocht, students will present and discuss their mathematical thinking.
- Managing myself: Students will have the opportunity to reflect on their own learning when the teacher asks them to write a reflection at the end of the lesson.
- Managing Information and Thinking: Students will be encouraged to think creatively and critically.
- Being Creative: Students will explore options as they actively participate in the construction of knowledge.
- Staying Well: By engaging in tasks that are appropriate to their abilities, students' confidence and positive disposition will be promoted.
- Being literate: Through Ceardaíocht, students will have the opportunity to express their ideas accurately and clearly with the correct use of mathematical language.

This lesson is designed to meet the following Junior Cycle Statements of Learning:

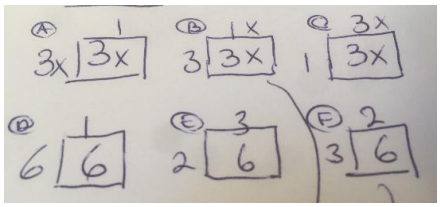
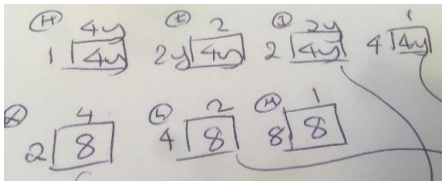
- SOL 1: The student communicates effectively using a variety of means in a range of contexts.
- SOL 15: The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
- SOL 17: The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

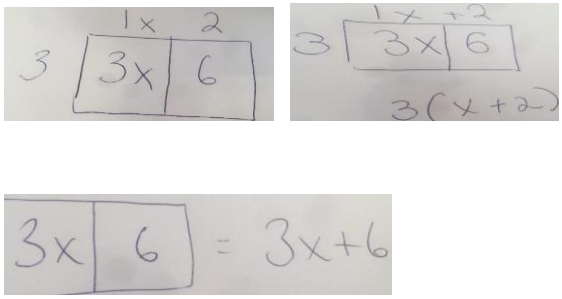
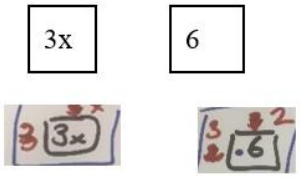
9. Flow of the Research Lesson:

Steps, Learning Activities **Teacher's Questions (Bold) and** **Expected Student Reactions (Italic)**	**Teacher Support**	**Assessment**
<u>Introduction - 5 minutes</u> Review of prior learning. **What are the factor pairs of 36?** **What does 5a mean?** **What does (5a) (4b) mean?** **What are we assuming here in our representation?** Recap key words Term Variable Expression **Can you remember what this means?** **What's the operation involved?** 5 | 15 - *5 + 15* - *I got 20 because I added 5 + 15*	Put up card with all the factor pairs of 36. **Can these be written a different way?** Put up card with factor pairs drawn as area models. Put up card on board with an expression. Identify the terms, variables, coefficients, expressions etc. **What are we doing with these boxes?** This box AND ... We are putting them together… For the student who says 20 **How did you get the 20?**	Are students able to answer the questions? Can students show different ways to multiply? Do they understand the 'boxes' are a shortcut to drawing a scaled diagram? Do students know what a variable is? Do students know how many terms there are? Do students know that this means 5 + 15? Do students recognise that is another way to represent 20?

Posing the Task **1 - 5 minutes** For the next 5 minutes, work on your own and factorise the terms in as many ways as possible 3x 4y 6 8	Students will be given A5 sized handouts with terms written on them. If students are finished early, they will be encouraged to draw more examples on the blank paper.	
Student Individual Work Response 1 3 1x 3x Response 2 1 3x 3x Response 3 1x 3 3x Response 4 3 2 6 Response 5 2 3 6 Response 6 6 1 6	As the teacher circulates the room, she looks for examples of the various methods. The order will be noted of the ways the solutions will be displayed on the board. Students names will be noted e.g. who to ask to present a board and who may have misconceptions (during Ceardaíocht.) For students who may be finished… …. they could be asked **Could you show that**	Are students writing down all the different ways on their handouts? Are there any misconceptions? E.g. look out for students who may write the factors of 6 as 2 and 4.

Response 7 4 1y 4y Response 8 2 2y 4y Response 9 2y 2 4y Response 10 2 4 8 Response 11 1 8 8 Response 12 8 2 8 Response 13 2 4 8	**another way?** **Could you make up your own example on the blank paper?**	

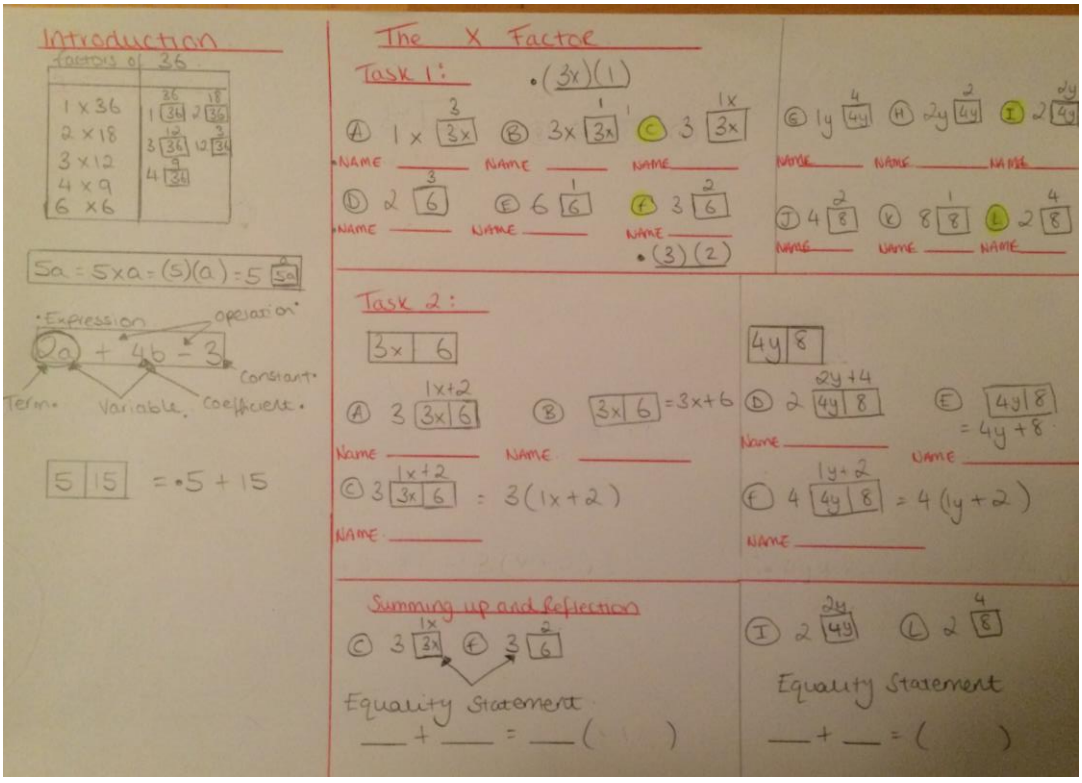
Ceardaíocht (10 mins) Some of the solutions that the students will be presented on the board as follows:   A discussion will take place around the different solutions. Suggested questions for discussion: • **What have you done here? Please explain to the class.** • **Do you understand?** • **Can you explain it back to me?** • **Did anyone else use these factors?** • **Can you explain it back to me?** • **(Teacher picks two examples that use the same factors but different order) Are these the same? Why/Why not? What do we call this?** • **Is there another way we can write the factors?**	Students are called to the board with all the different examples of how they could have factorised the terms. Student at the board will explain to the class what they have done. Teacher writes beside some of the 'boxes' (3x)(1), (2)(3)	Can a student explain back to the class what another student has done? (only for more challenging examples) Will students recognise and be able to explain the commutative property? Are students able to show the factors using brackets as well?

Task 2 (5 mins) Students are given handouts with two copies of $3x + 6$ and $4y + 8$  Students will be asked to write down ways to describe the diagram.	Suggested questions: **What operation is involved in here?** **Can you think of other ways to describe this diagram?** **You might need to look back at what we did before (e.g. 5 + 15) or what we just saw (in task 1).**	
Student individual Work 		
Ceardaíocht for Task 2 (5 mins) Students work is presented at the board. If it's felt that students do not come up with all the required solutions, then the teacher may prompt. **If I put the $3x$ and + 6 together like this, how would I say it?**  **Does anyone notice do they have**	Teacher may need to give a prompt if she feels students don't recognise to add the boxes. She can slide the two boxes together on the board. **What's another way to write what's inside the box?** $3x + 6$ After getting the solutions	Do students know that putting the two boxes together is the same as $3x + 6$? Through the Ceardaíocht, can students spot what the numbers have

anything in common? *They both have a factor of 3* *3 goes into both of them* *They both have a 3* **So we got all these 3 answers from this one mathematical statement so what does that mean?** *They are all the same* **Another way to say this?** *They are equal* **They are all equality statements** Repeat the process for 4y and 8	up on the board teacher writes on board $3x + 6 = 3(x + 2)$ *Can we fill in the blanks?* _____ + _____ = _____ (_____) "Equality Statements" Repeat for 4y and 8	in common?
Summing up & Reflection To consolidate the learning, the teacher will slide the $3x$ and the 6 together again. **What am I doing here?** *You are adding 3x and + 6* **What is the common factor?** **What goes into both of them?** *3* **What's on the top?** $x + 2$ **So I can write this as?** $3(x + 2)$ **These are all equal so we can say that** $3x + 6 = 3(x + 2)$ **For 4y and 8 what's another way of saying this?** $4y + 8$ **What's common factor?** 4 or 2 **So what goes on top?** $y + 2$ or $4y + 8$	Teacher will have equality statements written on cards ready to be stuck on the board.	

Task 3 (10 mins) & Reflection **On your handout try to factorise the boxes and fill in the blanks.** 4y | 8 ____ + _____ = ____ (_____) 5a | 20ab ____ + _____ = ____ (_____) 2xy | +4x | +8 ____ + _____ + _____ = ____ (_____) Students will fill out a reflection sheet.		Will students be able to fill in the sheet? Students work will be left behind and the work will be checked after the lesson.

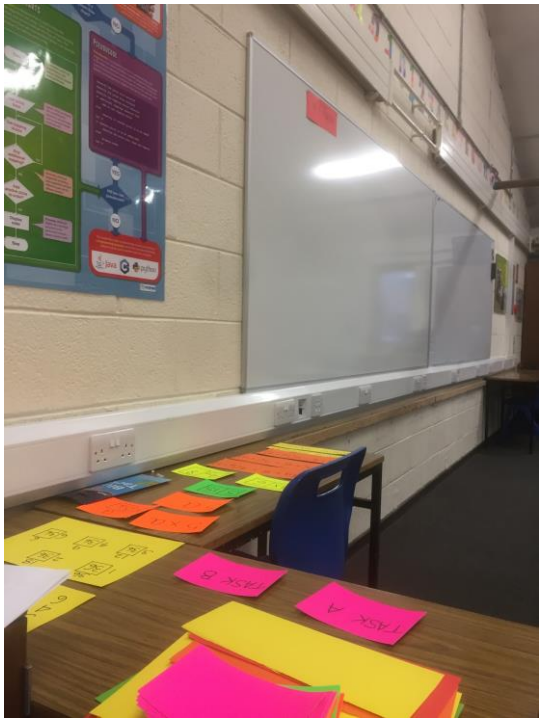
10. Board Plan



11. Evaluation

The classroom will be divided up in four sections, six students per observer. The fourth observer will use the “Lesson Note” app to observe the whole lesson. Observers will take note of student interactions, engagement etc., keeping in mind the goals of the lesson. Student worksheets will be collected and photographed. The completed board will be photographed. A post-lesson meeting will take place immediately after the lesson for reflection to take place. When observing the lesson, the following questions will be kept in mind:

- Were students engaged in the lesson? Were they on task at all times?
- Did students feel a sense of ownership of their learning?
- Were students giving constructive feedback to each other?
- Do students understand how to factorise an algebraic term?
- Do students know they can represent the factors in different ways? (e.g. using an area model and brackets)
- Do students recognise that equality is a relationship in which two mathematical expressions have the same value?
- Did anything “not work”?
- Were there any surprises?



Getting ready for the live lesson

12. Reflection

What had we hoped before the lesson?

- To develop tasks at the most appropriate level for our students
- To consider multiple approaches to tasks and make connections across the syllabus
- To develop a growth mindset in our students
- To use formative assessment to inform learning and give our students diagnostic feedback on their work
- To give students time to act upon that feedback.
- To see evidence of key skills embedded in the lesson
- To see if students could transfer knowledge from one lesson into the next and build on this prior knowledge.
- That students will understand how to factorise an algebraic term and know that they can represent it in different ways (e.g. using an area model and brackets)
- That students would discover how to factorise a linear two-term expression and represent it in different ways
- That students will recognise that equality is a relationship in which two mathematical expressions have the same value
- That the experience of lesson study would help inform our teaching and help us recognise the value of collaboration and reflection.

What was actually observed during the lesson?

During the lesson, students were fully engaged and were on task at all times. Students were allowed time to spend on both tasks. The teacher walked around encouraging students who were stuck asking questions like *"what happens if we put a 3 here?"*. Students who were finished were asked to *"try another way"*. She did not spend too much time with any one student or give direct answers to questions posed, rather, she provided some scaffolding.

Providing scaffolding to students

The teacher had a list of expected correct solutions and noted

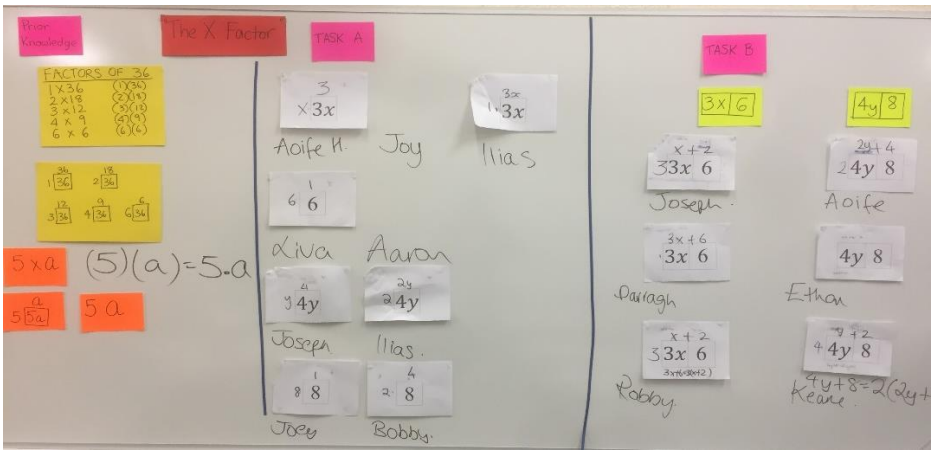
students who had come up with these solutions. These students were subsequently asked to present their solutions at the board. We felt the structured format of the lesson allowed for this to happen and prevents the focus of the lesson becoming about what students do *not* know. The teacher was checking for what the students had discovered instead of what they had not understood.

Checking solutions

Giving students the opportunity to present at the board was powerful as other students were very engaged when this was happening. One student was not able to answer a question that was posed to her; she said she didn't understand. The teacher made the student feel very comfortable and let her know it's ok if she can't answer a question, that it is ok to 'struggle'. This helps students with their growth mindset. The questions were rephrased and the teacher guided the student to come up with the answer.

Students work presented on board

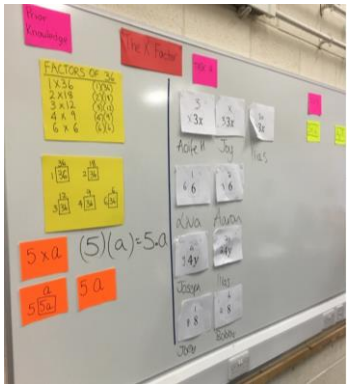
Board Work



Major points raised during the post-lesson discussion, and the team's own opinions:

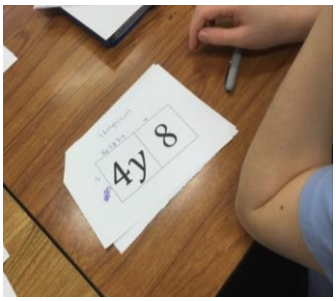
The teacher noted that some students who would normally 'shine' were quiet in class. We felt that this may have been because other teachers were in the room. However, some students who are normally quiet in class, were confident in showing off their knowledge and were keen to present their work at the board. Students were taking ownership of their learning as they presented their solutions at the board.

The team all felt that the lesson flowed smoothly. The observers noted that the teacher brought the students from their comfort zone at the start of the lesson (going over prior knowledge) to a point where most discovered something new.

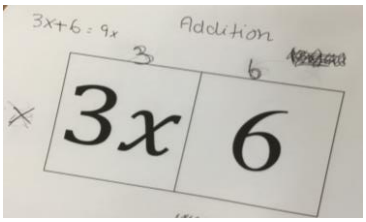


Going over prior knowledge and examples of the different ways students can factorise

Differentiated learning was evident in the lesson. The observers felt that all students learned something, even though they worked at different levels. Some students were unable to factorise the two term expression but they were able to represent single term factorising in different ways. Two-thirds of the class discovered how to factorise a two term expression. It was evident that students understood what an equality statement is.

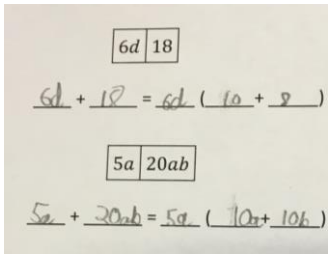


***Students were given time to work on the tasks; hence this student was able to factorise the 2 term expressio*n**



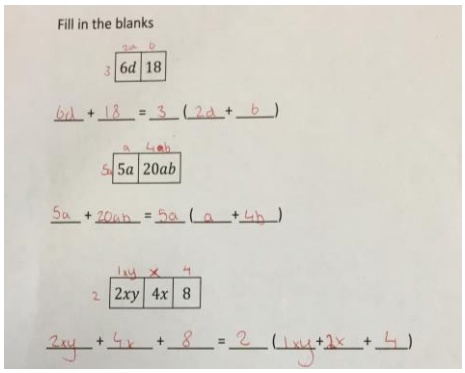
Misconception: This student factorised correctly but incorrectly wrote $3x + 6 = 9x$

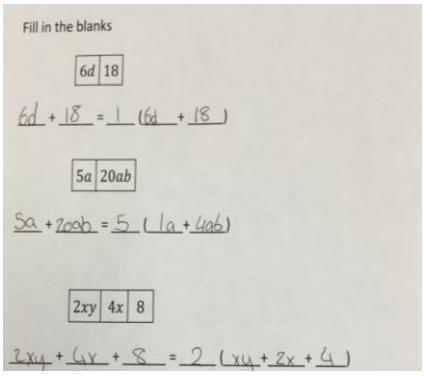
Misconception: This student doesn't understand what the brackets mean.



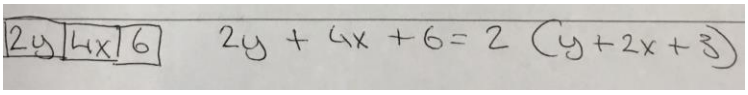
At the end of the lesson it was clear that the two out of three of the mathematical goals of the lesson had been met. This was evident when students were asked to fill out a worksheet. Approximately two-thirds of the class correctly filled out the worksheet.

Students

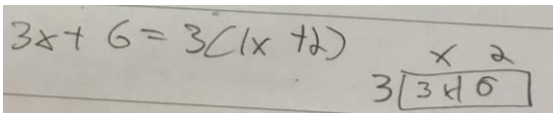


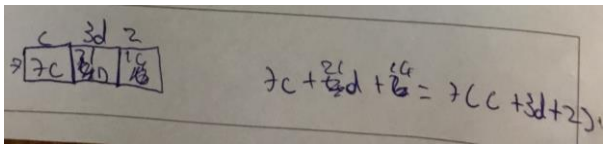


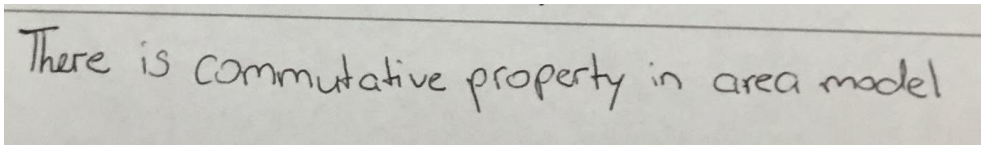
Students successfully factorising two term expressions

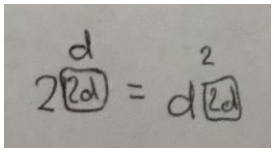


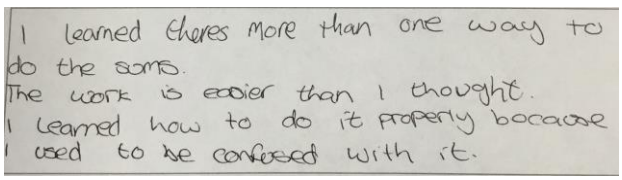
Students took ownership of their learning as they made up their own examples.







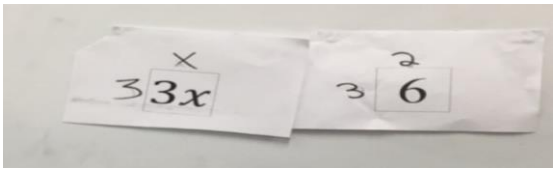




Ideas for future study

It would be worth exploring the following in a future or follow-on lesson.

- Students need more instruction when carrying out Task A, they kept writing the same thing. It is worth reiterating that they should show different approaches.
- We see the value in having quiet time for students, e.g. giving students the time to do individual work. An idea for future lessons would be to mix between group and individual work. Task 1 suits individual work but Task 2 might work better as a paired exercise. If students were working together in Task B, more of them may have successfully completed it.
- For a follow on lesson, it is suggested that the misconceptions are addressed, in particular $3x + 6 = 9x$.
- An emphasis on sliding the two individual terms together to show they are being added together may help deepen students' understanding of how to complete Task 2. A suggestion would be to highlight the common factor and to focus on the word common.
- Continue to facilitate students presenting their work at the board.



Idea for future lesson: 'slide' these two terms together to show the terms being 'added' together and emphasise the 'common' factor of 3

What did we learn from the process?

Lesson Study has helped us to become reflective practitioners. We see the value in allowing students the time to work quietly on a problem, encouraging them to struggle and persist while highlighting to them it is not about how fast they can solve a problem.

Lesson Study has allowed us to develop professionally through deepening our own understanding of subject knowledge.

> *"It was great to be able to discuss our common problems around Algebra. Think it worked really well in the end. I learned a lot from observing the teacher and through the process of talking to everyone - the 'unpacking' of the syllabus."*

> *"I would never have used the Area Model - it has completely changed my view of Algebra. Tried it with my TYs and 2nd years - it has changed my teaching."*

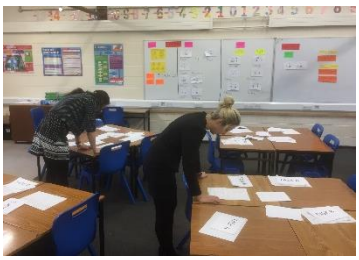
Lesson Study allowed us to work collaboratively with other teachers. We felt that we developed skills of listening to and valuing other people's opinions. Trust developed between us as we were nearing the end of the process and we felt able to constructively critique the lesson in the post-lesson reflection. The teacher who delivered the lesson noted that *"You were confidently able to be critical in a respectful way. I trust the group I'm with…"* She felt supported by the group *"I felt like I was being nudged by someone"*. The principal attended the lesson and commented afterwards about all the teachers involved:

> *"You are bringing back the skill of listening to your respective departments. … the ability to bring people around and getting people to listen. You have all gained a confidence from working it out with other people. The missing link in our profession is collaboration.*"

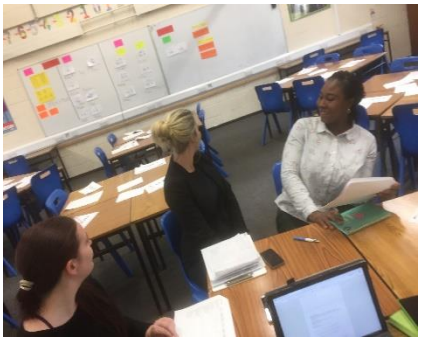
We all agreed that Lesson Study is a powerful form of professional development. One teacher felt that sometimes the material presented at other CPD events are not suited for the types of students that we may face daily. *"At a workshop you are talked to… Other CPD is 'not real' - not geared at students that are in front of us"*. Lesson Study allowed the teachers involved to tailor the material to suit our own students. *"Between us we came up with the merits ourselves. We took ownership of our own professional development"*.

The time we spent together over a number of weeks was critical to the Lesson-Study process as it allowed the relationships between us to develop. However, the time required for each meeting was challenging with other school commitments. We believe that if we were involved in a Lesson Study again that we may not need as much time especially during the earlier meetings; these were about getting to know each other and spending time learning about Lesson Study as a method of professional development. We therefore can see the value in doing it again with the same group.

> *"Time was a challenge. If we participated in Lesson Study again I think it would be much quicker. We spent time at beginning getting to know each other and learning about Lesson Study."*



Looking at students work after the lesson



Post Lesson Reflection

We believe that Lesson Study has a role in supporting other curriculum reforms; looking at the effectiveness of a digital tool, junior cycle reform, how to improve students' skills in discussion (dialogic teaching). To combat the challenge of time, a possible solution might be to use Professional Time or Croke Park time to facilitate the meetings between teachers. These meetings would still be structured, however, it may work if there were more of them but of smaller duration e.g. six hours of Croke Park and six hours of professional time. Schools need to be supported to allow for the final live lesson and post lesson reflection. *"I feel you could do it every year and get something out of it"*.

Summary

We felt that two out of three mathematical goals were achieved in the lesson. Students clearly understood how to factorise a single-term expression and represent it in different ways. While not all students could factorise a linear two-term expression, it will be revisited in subsequent lessons. Students were encouraged to develop a growth mindset in class through allowing time to work through problems and being made feel it's ok to struggle. Students certainly have a deeper understanding of what equality means; this is a key concept permeating the new Junior Cycle Specification. There were many key skills evident in the lesson; students were communicating as they presented at the board and managing their information and thinking through the reflections. We believe we developed tasks that were at an appropriate level for our students and gave them the time to work on the tasks. Students' growth mindsets were developed as they were encouraged to come up with different ways and allowed to 'struggle'.

Being involved in Lesson Study as a form of professional development has informed our teaching through deepening our own understanding of teaching Algebra. By working collaboratively, we have developed skills in listening and being critical constructively and respectfully. We have seen the value in reflecting on students' prior and future learning.

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