

# Lesson Research Proposal

Date of lesson: 13<sup>th</sup> February 2019

School name: St. Flannan's College, Ennis

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Lesson plan developed by: Teresa Considine, Philip Ryan, Martina Scully

## 1. Title of the Lesson:

Discovering the formula for the volume of a cylinder - "The volume of Money".

## 2. Brief description of the lesson

Through discovery, students derive the formula for the volume of a cylinder. This is achieved through examining the area of a circle (2D) and linking the relationship of the circle to a cylinder which has been created using a stack of coins.

## 3. Research Theme

At St. Flannan's College we want students to:

- Grow as learners while contributing their opinions and experiences to class discussion with confidence.
- Be able to report on present and explain the process and outcome of learning activities to a highly competent level.

As mathematics teachers we will actively support the achievement of our goals through:

- Assessing all relevant aspects of students learning using both Assessment of Learning and Assessment for Learning techniques and through engaging in collaborative practices
- Our intention to share teaching strategies and resources within the Mathematics department initially and subsequently other departments.

## 4. Background & Rationale

The research team chose to deliver this lesson to higher level second year students. The reason for choosing this topic and year group is that the all teachers within the team commented that students at this level generally learn a formula without truly understanding (or caring) as to how that formula was formed. According to the syllabus students are required to calculate the volume of a cylinder and apply to problem solving questions. We find that students are very competent at substituting into the formula to compute correct volume, however have difficulty manipulating the formula to less straightforward questions. A lack of understand of how the formula is derived is at the root of this difficulty. The research team feels that if the students are involved in deriving the formula, that this will lead to true understanding thus allowing for manipulation of the formula once more complex questions are encountered.

## 5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<p><i>In first year students have learned how to:</i></p> <p>Find the length of the perimeter and the area of the following: disc, triangle, rectangle, square and any combinations of these.</p> <p>Draw and interpret scaled diagrams. Investigate nets of rectangular solids.</p> <p>Find surface area of rectangular solids Solve problems involving the surface area of a cuboid.</p> <p><i>In second year students will:</i></p> <p>Investigate nets of cylinders.</p> <p>Solve problems involving the surface area of a cylinder.</p>	<p>In second year, students are expected to learn how to:</p> <p>Find the volume of rectangular solids and cylinders.</p> <p>Solve problems involving volume of cylinders.</p> <p>Solve problems involving volume of cuboids and cylinders.</p>	<p>In 3rd year, the students will: Investigate nets of other prisms (e.g. triangle based prisms), nets of cylinders &amp; cones.</p> <p>Solve problems involving the surface area of a cylinder. Solve problems involving the curved surface area of cylinders.</p> <p>Solve problems involving the surface area of triangular base prisms and cones.</p> <p>Solve problems involving cones and spheres.</p> <p>Perform calculations to solve problems involving the volume of cones, triangular base prisms, spheres and any combination of these.</p> <p>Having appreciation and understanding of volume for cylinder provides a building block for rates of change in senior cycle.</p>

## 6. Goals of the Unit

To help students discover that area relates to 2D shapes and volume relates to 3D shapes.

To help students develop problem-solving skills through examining a variety of 3D shapes.

To help students understand that the units used for 2D and 3D shapes stem from the number of variables used to calculate area and volume.

To allow students to transfer their knowledge of volume from the context of a cuboid to the context of all prisms.

To help students discover the formula to calculate the volume of a cylinder.

## 7. Unit Plan

Lesson	Brief overview of lessons in unit
1	<p>Students should know the difference between a 2D and 3D shape. This will consist of examples of each and connections to real-life objects. Introduce the idea of prisms. Identify the face/base of a prism. Know that a prism has a uniform cross-sectional area. Identify a cuboid as a rectangular based prism. Recall the area of a rectangle.</p> <p>Recall that a net diagram is a 2D representation of a 3D object. Revisit using net models of cuboids. Recall how to find the volume of a cuboid. Practice finding the volume of simple cuboids.</p>
2 Research Lesson	Discover how to calculate the volume of a rectangular solid using cross sectional area $\times$ height. Establish the formula for the volume of a cylinder and complete question on volume for homework.
3	Students convince themselves that this process can be applied to all prisms to find the volume through investigation. Solve problems involving the volume of a cylinder
4	Students will investigate surface area and volume of a sphere and solve problems from textbook.
5	Students will investigate the surface area and volume of a cone and solve problems from textbook.

## 8. Goals of the Research Lesson:

Looking at the goals of the research lesson itself from two perspectives:

- (i) Mathematical goals:
  - We want students to develop the formula for the volume of a cylinder through exploration of applying surface area and height.
  - Understand how to calculate the volume of a cylinder
  - Students will apply the formula in order to problem solve
- (ii) Key Skills and Statements of Learning

### Key Skills

- (a) Being Numerate: By engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.
- (b) Working with Others: Students will learn with and from each other by discussing different approaches to problem solving.
- (c) Communicating: students will present and discuss their mathematical thinking.
- (d) Managing myself: Students will have the opportunity to reflect on their own learning when the teacher asks them to write a reflection at the end of the lesson.
- (e) Managing information and Thinking: Students will be encouraged to think creatively and critically.
- (f) Being Creative: Students will explore options as they actively participate in the construction of knowledge.

### Statements of Learning

SoL 1. The student communicates effectively using a variety of means in a range of contexts.

SoL15. The student recognizes the potential uses of mathematical knowledge, skills and

understanding in all areas of learning.

SoL16. The students describe, illustrates, interprets, predicts and explains patterns and relationships.

SoL17. The student's devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

**(iii) Flow of the Research Lesson:**

<b>Steps, Learning Activities Teacher's Questions and Expected Student Reactions</b>	<b>Teacher Support</b>	<b>Assessment</b>
<p><b>Introduction (5 mins)</b></p> <p>Review of previous lesson (Prism)</p> <p>What can we measure on a prism?</p> <ul style="list-style-type: none"> <li>● Length</li> <li>● Width</li> <li>● Height</li> <li>● Area of a face</li> </ul>	<p>Remind students what a prism is.</p> <p>Put up card on board that describes Area = length x width</p>	<p>The teacher uses effective questioning to ensure that the students illustrate understanding of a prism, and what can be measured on a prism. This also allows the teacher the opportunity to assess if the students clearly understand the language used.</p>
<p><b>Task 1</b></p> <p><b>What is the formula for the volume of a cuboid? (5mins)</b></p> <p>Put up card on board that describes Volume = length x width x height</p> <p><b>Introducing a disc- what can you measure and calculate? (5mins)</b></p> <p>Place a card with the word cylinder on board. Definition of prism.</p> <p>Students illustrate learning by placing the labelled measurements that they found, onto the board.</p> <p><b>What shape is this 3D object? Is a prism? Why? (2mins)</b></p> <p>Students will need to put forward</p>	<p>Ask students to see a connection between Area formula and volume.</p> <p>Is there another way of shortening the volume formula to two pieces of information?</p> <p>Call students to the board to place up information re diameter, radius, and area.</p> <p>The teacher looks out for potential misconceptions e.g. Calling it a Sphere, not recognizing it as a Prism</p>	<p>Students put forward the suggestion of the formula for the volume of the cuboid from previous learning.</p> <p>Extension learning looks for students to derive the formula Area*Height from the formula Length*Width*Height</p> <p>The teachers observe the students as they work in pairs to figure out what measurements they can derive from the disc.</p> <p>Observe student's reactions and discussions, being aware of potential misconceptions.</p>

suggestions for the name of the 3D shape and if it is a Prism

**Task 2**

**Can you calculate the volume of this shape? (10mins)**

Can you come up with a formula for the volume of the stack of coins?

Student Pair Work

Teacher circulates to see solutions as stacks of coins are distributed.



Can students link prior learning of the cuboid and the disc to the cylinder?

Can they derive a formula for the volume of the cylinder linking the area of the disc and height?

Students hopefully derive:

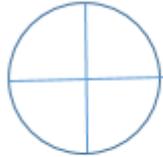
Area\*Height

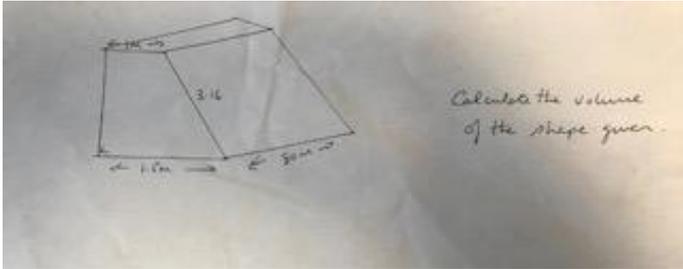
OR

$\pi r^2 h$

## **STUDENT WORKSHEET**

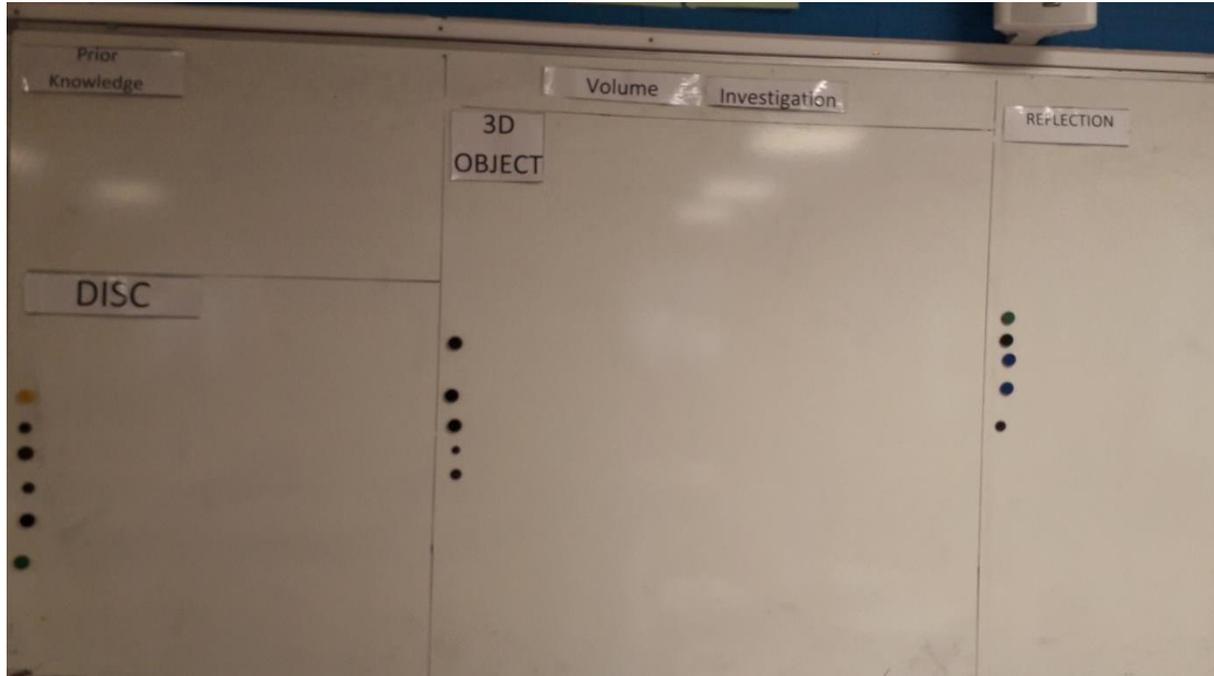
**What can you measure and calculate?**



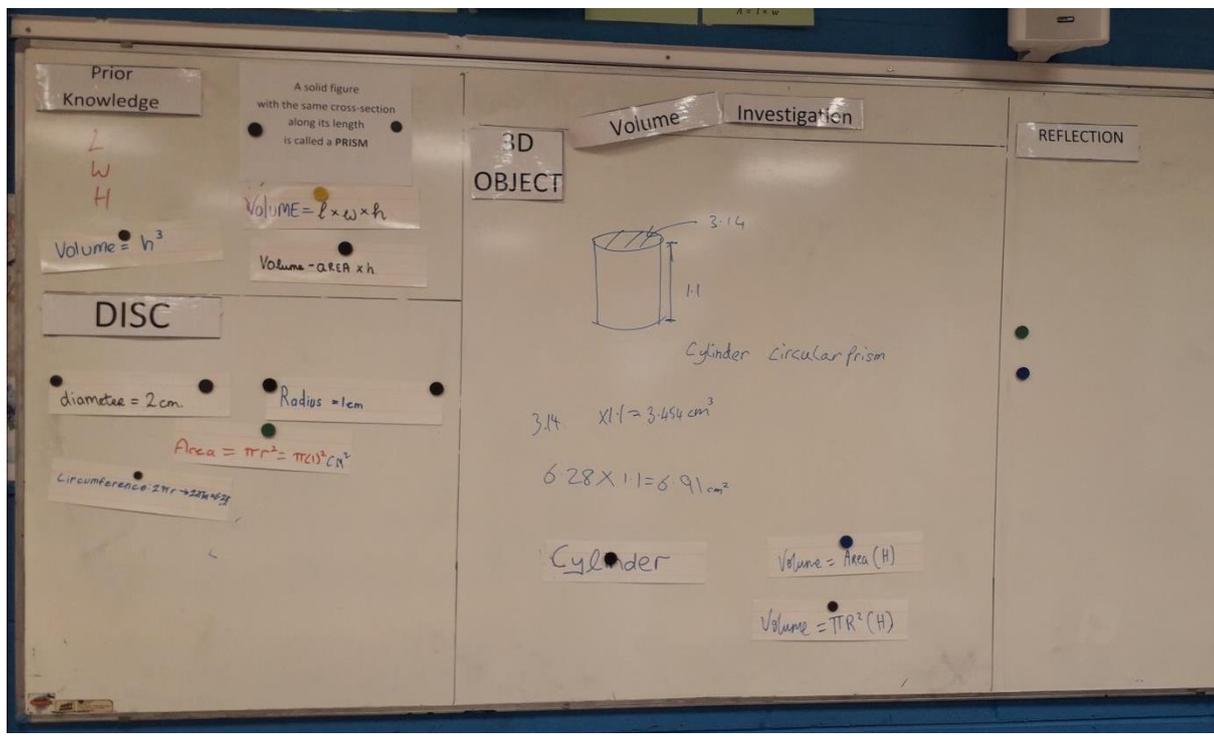
<p><b>Ceardaíocht /Comparing and Discussing (10 minutes)</b></p>	<p>Through clever questioning the teacher will get the students to link prior learning of cuboids and prisms to discs and cylinders.</p> <p>The goal is to get students to identify:</p> <ul style="list-style-type: none"> <li>• That a cylinder is a prism</li> <li>• That there is a link between the 2D (Disc) and the 3D (Prism)</li> <li>• That the relationship between the area of the face and the height a formula for volume can be derived for all prisms</li> <li>• That the formula for a cylinder is <math>\pi r^2 h</math></li> </ul> <p>All of this should be done without the teacher explicitly feeding ideas or terms to the students.</p>	<p>Observe student engagement and self-correction.</p> <p>Observe student interaction.</p> <p>Observe and record students board work.</p> <p>Were the students able to identify the link between area of the face and height in order to find the volume?</p> <p>Were students able to link the prior work with a cuboid to that of the cylinder and apply that learning</p> <p>Did students generate this learning independently of the teacher.</p> <p>Was their co-creation of learning amongst the pairs and the class as a whole</p>
<p><b>Summing up &amp; Reflection (3 mins)</b></p> <p>Give homework task</p>	<p>Students will be asked to fill in the reflection sheet and leave it behind.</p> <p>Collect reflection sheets and classwork.</p> <p>Homework given.</p>	<p>As above</p>
<div style="display: flex; align-items: center; justify-content: center;">  <div style="margin-left: 20px;"> <p>The homework task.</p> </div> </div>		

## Board Plan

The board before the lesson.



The board towards the end of the lesson.



#### (iv) Evaluation

##### **Did the students learn from the lesson?**

Based on the work in class along with student's feedback and classroom interaction, student learning was seen throughout the class.

##### **Were the research goals achieved?**

Students interacted with their colleagues initially and then in class to such an extent that a student corrected another student's board work. The class engagement was evident here as a lot of students said "yes" or "yeah" at that moment. Students explained different methods of calculating the volume with one another which created "oh yeah" moments for some students. One example of this was when a student spotted that the area of a cuboid was length by width by height and his colleague then spotted that this was the same as area  $\times$  height.

##### **Flow of the lesson.**

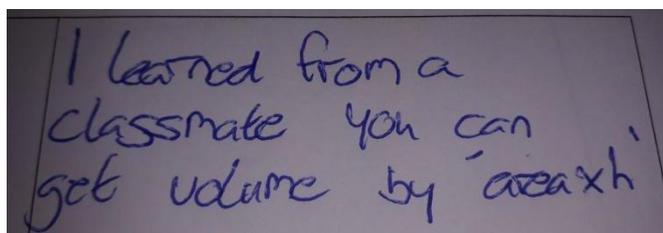
It worked. Students had an opportunity to tackle the problems which they did on their own and in pairs. A number of students were observed completing their tasks but after a brief pause they considered other tasks which hadn't been explicitly asked. This included looking for the surface area of the cylinder and exploring whether the curved surface area was a rectangle with the perimeter of the circle and the height of the curved surface area being the dimensions. This would be worth exploring in subsequent lessons or if time allowed could be explored in a further iteration of the research lesson.

##### **Misconception**

One student stated that the volume of the cylinder was area squared  $\times$  height. This was corrected by colleagues without prompting. The same student stated in his feedback that you don't square area to find volume. Another student stated that he had the same misconception in feedback.

##### **Feedback**

Student feedback was extremely positive and all excluding one student from 30 stating that they learnt / understood the cylinder formula. Students also enjoyed learning from their colleagues and in general they enjoyed and were learning in the class. They enjoyed working and learning with their colleagues and improving their social skills.



# STUDENT WORKSHEET

What can you measure and calculate?



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