Research Lesson

The Percentage Paradox

Coláiste Bríde, Enniscorthy

PDST Lesson Study 2018-2019

Topic: Proportionate Reasoning

Class: Higher level 2nd Year Class

School: Coláiste Bríde Enniscorthy

Teacher: Olive McGuinness

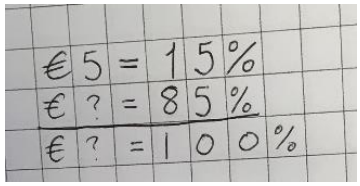
Lesson Study Team: Kim Nolan, Olive McGuiness and Aidan Roche

1. **Title of the Lesson:**
   The Percentage Paradox

2. **Brief description of the lesson**
   Structured problem solving lesson involving multiple approaches to a direct proportion percentages problem. Misconceptions around percentage equations are addressed leading to considering an algebraic approach.

3. **Research Theme**
   The school's SSE focus for 2018/2019 is to improve teaching and learning through engagement in Teacher Collaboration and Peer Observation.

4. **Background & Rationale**
   The mathematical area which our Lesson-Study group identified as causing problems for students were: Misconceptions and inefficient approaches in dealing with percentages. It was noted that when dealing with percentages a significant number of students used the equal sign even when values were not equal e.g. "5 = 15%".



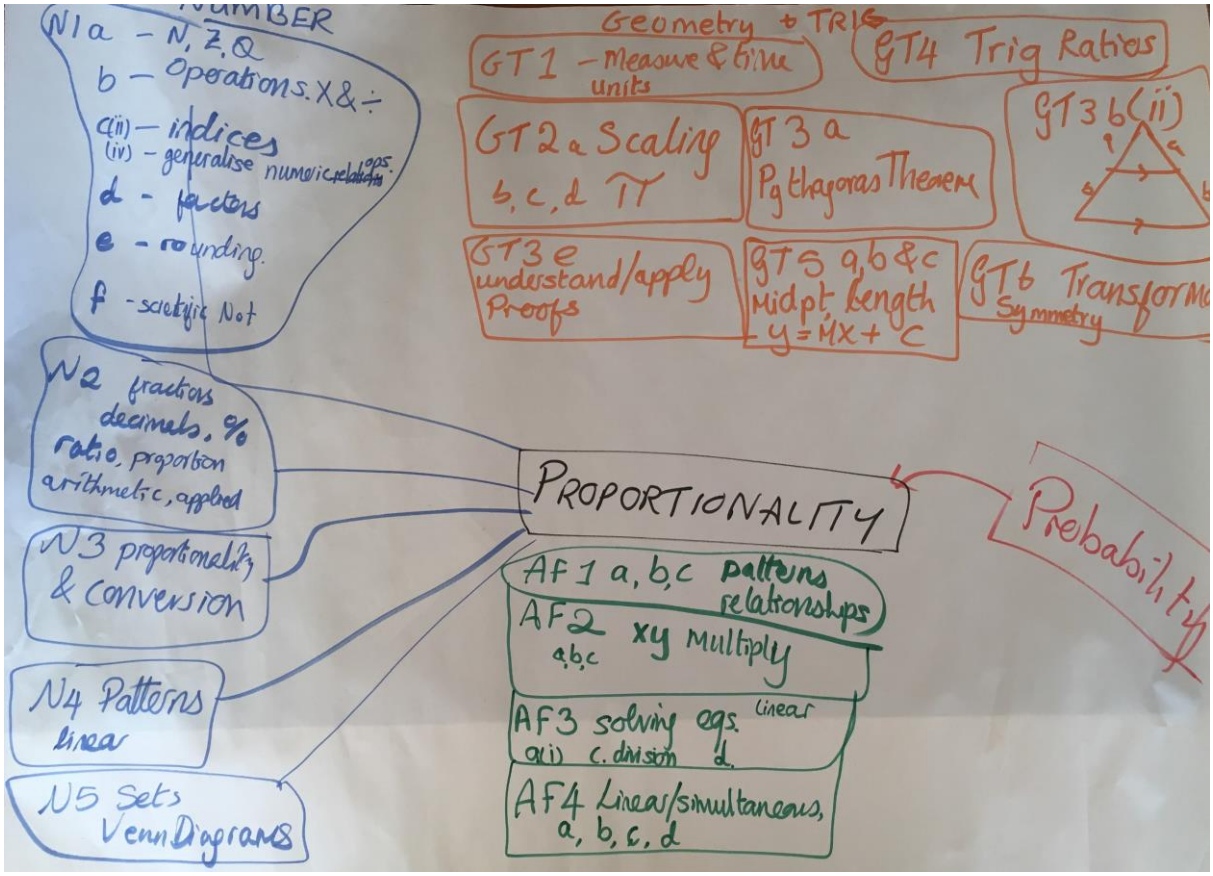
   Our solution… An algebraic approach when dealing with "percentage of" type problems is an efficient and mathematically rigorous method that addresses misconception.

5. **Goals of the research lesson**
   - Students use and consider the merits of multiple approaches to solving a problem
   - Students understand that €5 is not equal to 15%
   - Students can approach this problem using algebra
   - Students develop more efficient approaches to this problem
   - Students of all abilities are engaged and enjoy their learning
   - Higher order thinking is promoted during the lesson and achieved by students
   - Teachers collaborate in design and evaluation of teaching and learning

6. **Relationship to the syllabus**

   Though our current second year students are not following the revised curriculum, the entire mathematics department considered the topic of "proportion" in relation to the Junior Cycle Mathematics Specification and Learning Outcomes. The topic reaches into

every strand (see below). We also considered how Junior Cycle key skills would be met within the Research Lesson.

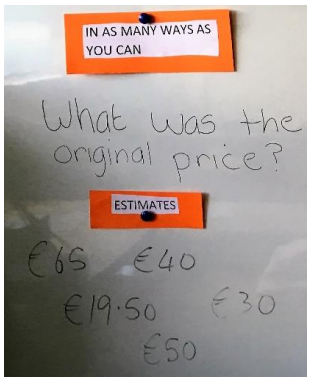
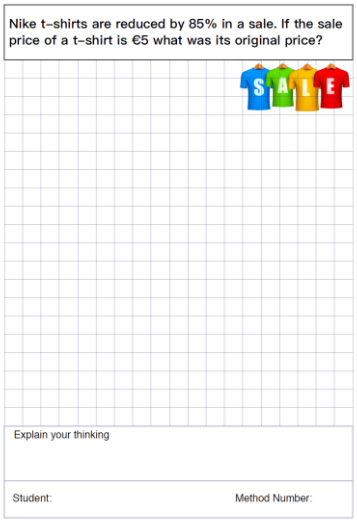
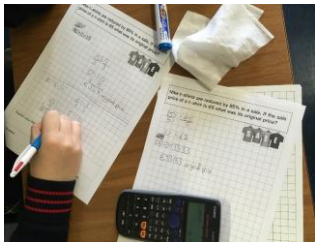


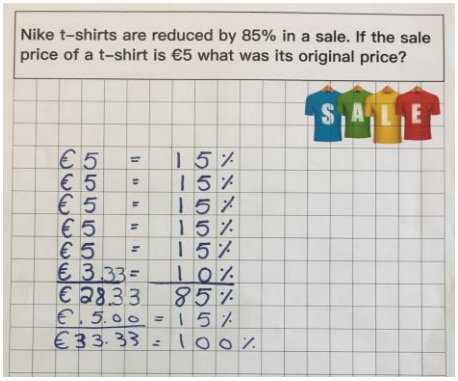
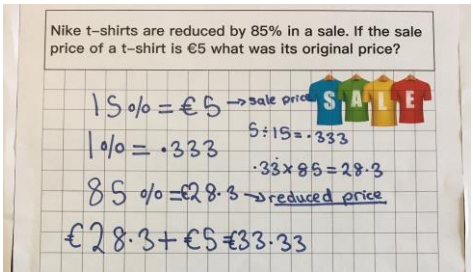
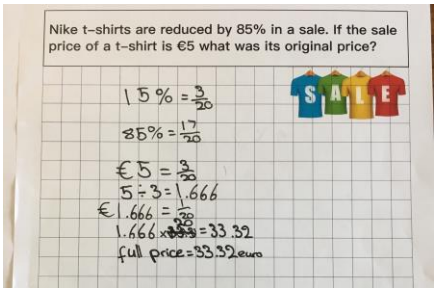
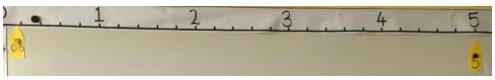
7. Flow of the Unit

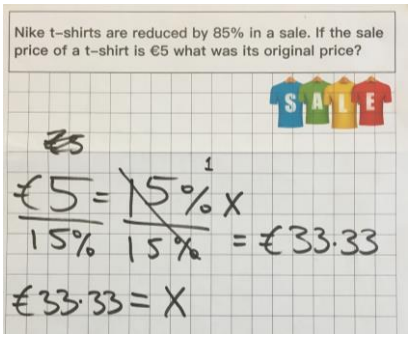
This is the first lesson in a series on direct proportion. The second lesson considers students homework responses and relates our work to a graphical understanding of direct proportion.

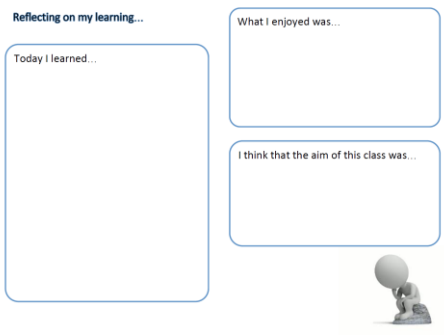
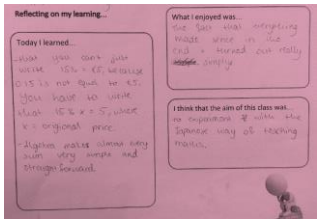
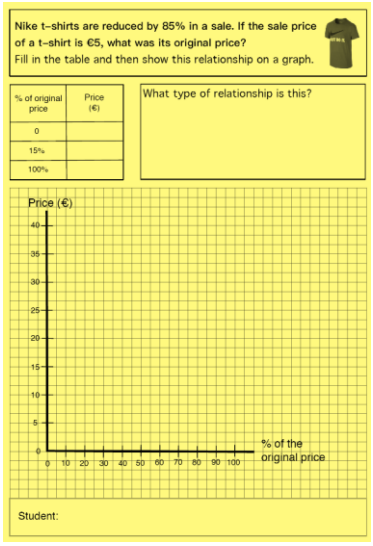
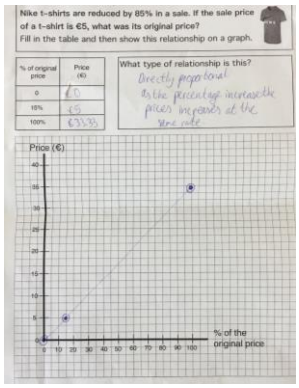
Lesson 1	Research Lesson
Lesson 2	Graphical approach to direct proportion
Lesson 3	VAT – Profit and Loss
Lesson 4	Household Bills
Lesson 5-6	Income Tax
Lesson 7	Currency Exchange
Lesson 8-9	Compound Interest
Lesson 10	Unit Test

9. Flow of the research lesson:

Steps	Teacher Support	Assessment
Posing the task (5 minutes)		
What question could we ask about this? **Clarifying the problem:** Estimating with show me boards	 What question do you think we could we ask? What does this mean? What percentage of the original price do we know? What percentage of the original price do we want to find?	Students ask: "What was the original price?"   Estimating with individual whiteboards
Student Individual Work (15 minutes)		
6 worksheets given to each student. Solve in as many ways as you can.		  Teacher notes engagement and student responses. Selecting student work to be presented.

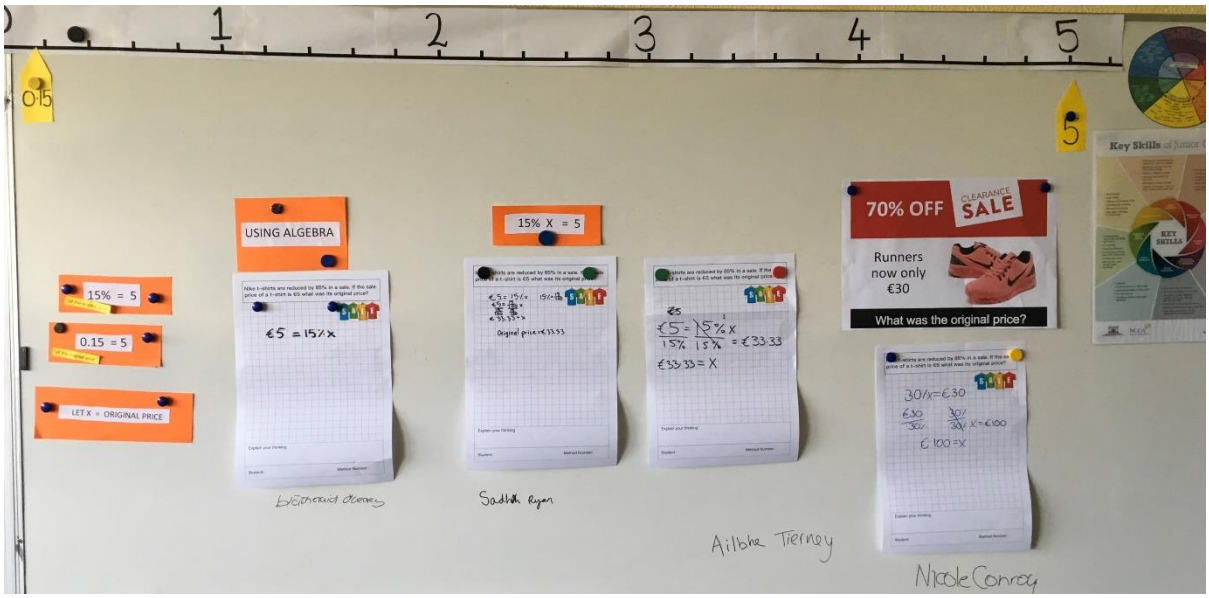
Presenting students' work at the board (30 minutes)		
Response 1 Using Addition		Can you explain your thinking? Do you agree? Did anyone else use a method like this?
Response 2 Using multiplication/division Finding 1% of the original price first.		Can you explain your thinking? Do you agree? Did anyone else use a method like this?
Response 3 Using multiplication/division Fractions or decimals		Can you explain your thinking? Do you agree? Did anyone else use a method like this?
Addressing a misconception: **Is €5 = 15%?**	Is 5 = 15%? How can we write/express 15% in another way? Is 5 = 0.15? Show me 5, 15% and 0.15 on a number-line  How can we correct this? Do we need to change anything on the board?	Can students understand that 5 is not equal to 15% "€5 is 15% of the original price. Can students stick the "of the original price" labels to correct on the board-work

Response 4 Using Algebra	How can we write a correct maths equation meaning "€5 is 15% of the original price? Can you try this problem again by solving "15%x = 5" (4 minutes) Student presents algebraic approach.  	Do students say: "let x = the original price"? "15%x = €5" Can students solve the equation. Do they use decimals? Fractions? Percentages?
Ceardaíocht (5 minutes)		
Second problem	Whose method do you prefer? Which method is the most efficient? Why? Do you agree? Solve this problem using any method you like on your whiteboard  Who did it using algebra? Why did you choose this method?	What do students say What methods do students use to solve this question? Do they write "€30=30%"?

Student Reflections (5 minutes)		
Each student given reflection sheet to complete. Learning intentions?	 What did you learn today? What did you enjoy? What do you think the aim of the class was?	What do students write, understand, enjoy, learn…? 
Homework Task		
Each student given homework extension work on the problem leading to a graphical understanding of direct proportion.		 Do students see the linaer pattern and can they describe the type of relationship?

9. Board-work:


Flash SALE
Now €5
85% off all Nike T-Shirts
Today Only
USING MULTIPLICATION
USING ADDITION
Find 1% of the amount first...
USING FRACTIONS
IN AS MANY WAYS AS YOU CAN
What was the original price?
ESTIMATES
€65 €40
€19·50 €30
€50
Asmaa Sallam
Sarah Walsh
Kayleigh Doyle


1 2 3 4 5
0·15
5
15% = 5
0.15 = 5
LET X = ORIGINAL PRICE
USING ALGEBRA
€5 = 15%x
15% X = 5
70% OFF CLEARANCE SALE
Runners now only €30
What was the original price?
Key Skills
Ailbhe Tierney
Nicole Conroy

10. Student Reflections

Today I learned…

I learned that you must say e.g. "15% of the original price"

That there is more than 1 way to do a question. 15%=5 is a wrong statement and for it to be right it must be 15% of the original price = 5 which can be written as 15%x=5

You don't have to stick to one method while doing maths. There is plenty of ways and if you really think about it you will find a much faster/easier way to do it.

That the most efficient way of completing the original price questions was through algebra and fractions (literally my least favourite units in maths, but somehow still fun).

To use "x" after a percentage when it is equated to a price.

What I enjoyed was…

Working with whiteboards, doing the maths for myself and seeing other peoples ways of doing things. I love actually doing maths and the practical application of it.

Getting to see how other people worked it out and comparing how maybe I could of (sic) done it differently and maybe better.

All the different methods until we found the most efficient method.

Seeing all the different ways to do maths and knowing there's a lot more than 1 way to do a maths problem.

I thought that it was interesting how many methods we came up with as a class group.

I think that the aim of the lesson was…

To help us to think about how the maths is written, and see that there's not only one method of doing things. Maths doesn't come in chapters, it's just maths, and sometimes you just have to use it all to figure out your answer.

To see there's many ways of doing maths and to find the most efficient way of doing it.

To get everyone thinking in a more efficient way of doing maths.

To make it easier to find the original price of something.

To get us to think different, outside the box.

11. Teacher observations and discussion

Five teachers participated in the observation and post-lesson discussion.

- Surprise at student engagement during the 16 minutes given to individual problem solving at the beginning of the lesson. Some students seemed to have finished early but when they noticed other students continuing to attempt multiple methods they re-engaged. Teachers thought that each student had multiple worksheets supported this.
- The large number line was effective in students being able to debate and decide conclude that “15% was not equal to 5”.
- Effective use of the “colourful” board allowed students to compare, contrast and evaluate different approaches from the most naïve to the most efficient.
- Observing teachers viewed this model of student collaboration as being radically different from what they previously understood and had experienced.
- The structured problem solving approach was seen as highly effective in terms of achieving higher order skills for all students, effectively meeting the goals of the lesson and formatively assessing the learning.
- Key Skills: communication, creativity, managing myself, managing information and thinking, being numerate and literate were all evident throughout the lesson.
- Consideration needed to be given in the follow-up lesson to address some misconceptions evident in the students approaches, check calculator skills, to build on the proportionate reasoning being developed in the homework task and embed effective strategies.

Nike t-shirts are reduced by 85% in a sale. If the sale price of a t-shirt is €5 what was its original price?

Explain your thinking

Student:

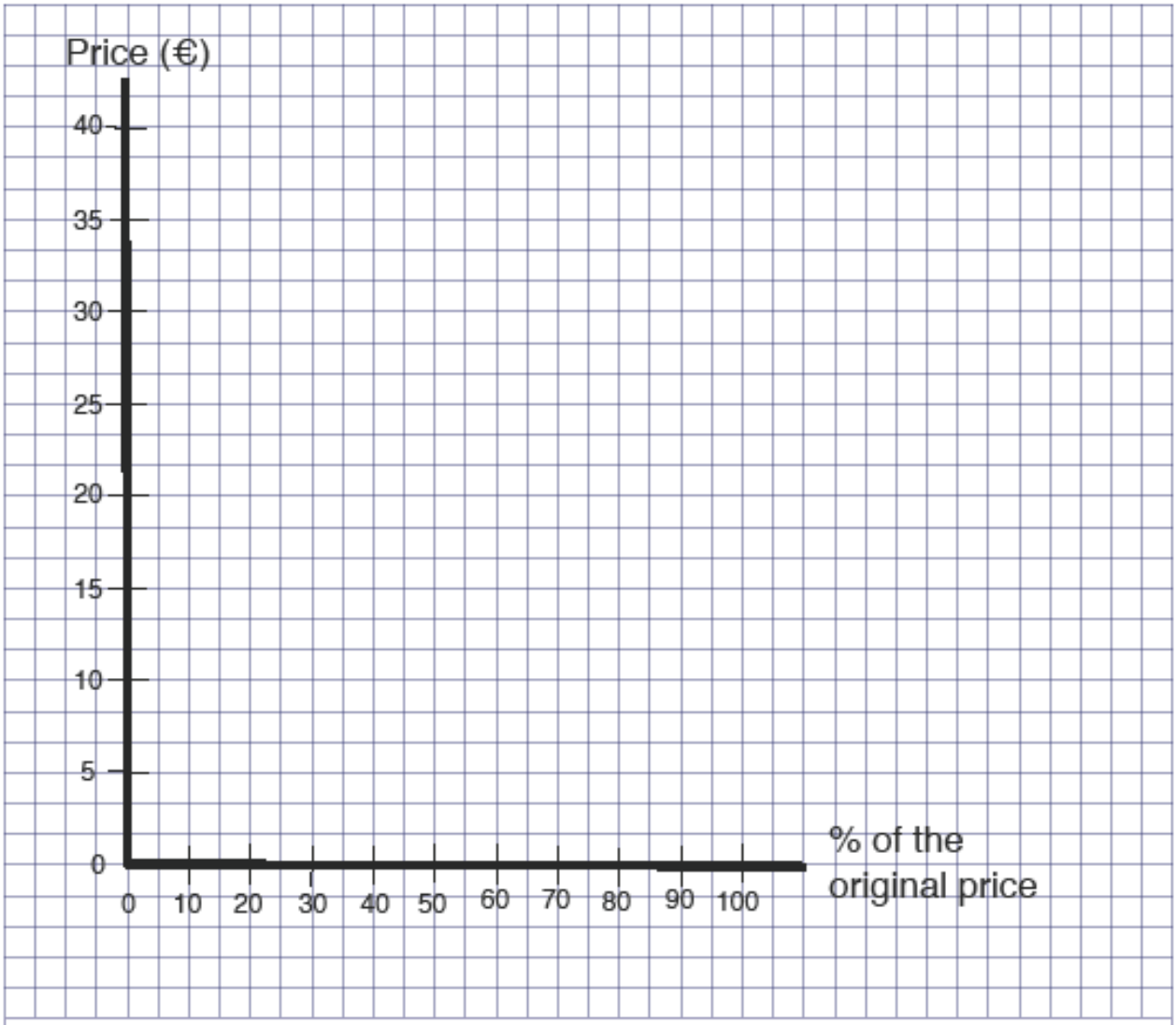
Method Number:

Nike t-shirts are reduced by 85% in a sale. If the sale price of a t-shirt is €5, what was its original price?

Fill in the table and then show this relationship on a graph.

% of original price	Price (€)
0	
15%	
100%	

What type of relationship is this?



Student:

Reflecting on my learning...

Today I learned...

What I enjoyed was...

I think that the aim of this class was...