# Lesson Research Proposal for Deele College Raphoe

Date of lesson: March 6th 2019, 11.15am-12.35pm
School: Deele College, Raphoe, Co. Donegal
Class: 1B [1st Year Mixed Ability]
Teacher giving lesson: Shauna Kelly
Associate: Patrick Flood
Lesson plan developed by: Patrick Flood, Marilyn O'Riordan, Stephen Gibson, Shauna Kelly

**1. Title of the Lesson:** What about parallelograms?

**2. Brief description of the lesson**

Exploring properties of parallelograms using various approaches, and looking for connections between approaches.

**3. Research Theme**

At Deele College, we aim to develop students who engage purposefully in meaningful learning activities.
Also, we want our teachers to select and use teaching approaches appropriate to the learning intention and students' learning needs.
As a Mathematics Department we will actively support the achievement of these standards in the following ways:

(a) by delivering creative and effective instruction which is directed at encouraging deep student engagement through various methodologies.
(b) by enabling teachers to use a range of questioning techniques effectively for a variety of purposes including stimulating substantial student responses and extending learning beyond the lesson.
(c) by meaningfully differentiating content and activities to challenge all students.
(d) by encouraging teachers to view collaboration as a means to improve student learning and to enhance their own professional development.

**4. Background and Rationale**

According to the subject specification students should be making connections and use problem solving skills to fully understand each part of Strand 2. In common with most schools, and as is clear from SEC examiner reports, teachers in Deele College have identified students having problems in geometry at all levels as a major difficulty. Our students seem to lack confidence in approaching geometric tasks and applying basic concepts. Our aim in this lesson, as part of the overall unit, is to give students time to explore straightforward geometric properties in a variety of ways. We hope the discussion and consideration of these concepts in 1st Year will form a basis of more successful handling of geometry questions at future stages. In this lesson students should be able to explore geometric properties and patterns, looking to move from particular observations to general concepts. Students need be familiar with the basics of geometry, angle, triangles, quadrilaterals and their properties, as well as the area of triangles and quadrilaterals.

Students must develop confidence and certainty in basic geometric operations applied to simple situations. They must learn to apply variety in approaches and methodologies. Furthermore, students need an understanding of the basis of these procedures, particularly how various elements of Strand 2 connect with each other. If students cannot refer back to these links between elements that may have been introduced to them separately, then they will struggle to apply geometric rules consistently correctly. They will also miss an essential purpose of geometry.

## 5. Relationship of the Unit to the Syllabus

| Related prior learning outcomes (Primary School 5th and 6th classes) | Learning outcomes for this unit (Refer to Junior Cert Specification 2018) | Related later learning outcomes (Refer to Junior Cert Specification 2018) [in addition to learning outcomes for this unit] |
|---|---|---|
| Estimate and measure length using appropriate metric units<br><br>Select and use appropriate instruments of measurement<br><br>Estimate and measure the perimeter of regular and irregular shapes<br><br>Discover that the area of a rectangle is length by breadth<br><br>Calculate area using square centimetres<br><br>Make informal deductions about 2-D shapes and their properties<br><br>Use angle and line properties to classify and describe triangles and quadrilaterals<br><br>Construct triangles from given sides or angles | GT.2 investigate 2D shapes and 3D solids so that they can:<br><br>a. draw and interpret scaled diagrams for students<br><br>c. find the perimeter and area of plane figures made from combinations of discs, triangles, and rectangles, including relevant operations involving pi<br><br>GT.3 investigate the concept of proof through their engagement with geometry so that they can:<br><br>b. recall and use the concepts, axioms, theorems, corollaries and converses, specified in<br><br>"Geometry for Post-Primary School Mathematics" (section 9 for OL and section 10 for HL)<br><br>U.1 recall and demonstrate understanding of the | GT.2 investigate 2D shapes and 3D solids so that they can:<br><br>b. draw and interpret nets of rectangular solids, prisms (polygonal bases), cylinders<br><br>d. find the volume of rectangular solids, cylinders, triangular-based prisms, spheres, and combinations of these, including relevant operations involving π<br><br>e. find the surface area and curved surface area (as appropriate) of rectangular solids, cylinders, triangular-based prisms, spheres, and combinations of these<br><br>GT.3 investigate the concept of proof through their engagement with geometry so that they can:<br><br>a. perform constructions in "Geometry for Post-Primary School Mathematics" |

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| Use 2-D shapes and properties to solve problems<br><br>Classify 2-D shapes according to their lines of symmetry<br><br>Tessellate combinations of 2-D shapes<br><br>Recognise, classify and describe angles and relate angles to shape and the environment<br><br>Recognise angles in terms of rotation<br><br>Estimate, measure and construct angles in degrees<br><br>Explore the sum of the angles in a triangle and in a quadrilateral | fundamental concepts and procedures that underpin each strand<br><br>U.2 apply the procedures associated with each strand accurately, effectively, and appropriately<br><br>U.8 apply their knowledge and skills to solve a problem, including decomposing it into manageable parts and/or simplifying it using appropriate assumptions<br><br>U.10 evaluate different possible solutions to a problem, including evaluating the reasonableness of the solutions, and exploring possible improvements and/or limitations of the solutions (if any)<br><br>U.11 generate general mathematical statements or conjectures based on specific instances<br><br>U.13 communicate mathematics effectively: justify their reasoning, interpret their results, explain their conclusions, and use the language and notation of mathematics to express mathematical ideas precisely | b. recall and use the concepts, axioms, theorems, corollaries and converses, specified in "Geometry for Post-Primary School Mathematics" (section 9 for OL and section 10 for HL)<br><br>d. create and evaluate proofs of geometrical propositions<br><br>e. display understanding of the proofs of theorems and corollaries<br><br>GT.5 investigate properties of points, lines and line segments in the coordinate plane so that they can:<br><br>a. find and interpret distance, midpoint, slope, point of intersection, and slopes of parallel and perpendicular lines<br><br>GT.6 investigate transformations of simple objects so that they can:<br><br>a. recognise and draw the image of points and objects under translation, central symmetry, axial symmetry, and rotation<br><br>b. draw the axes of symmetry in shapes<br><br>U.4 represent a mathematical situation in a variety of different ways, including: numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations<br><br>U.6 make connections |

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| | | between mathematics and the real world<br>U.7 make sense of a given problem, and if necessary<br>mathematise a situation<br>U.11 generate general mathematical statements or conjectures based on specific instances<br>U.12 generate and evaluate mathematical arguments and proofs |

## 6. Goals of the Unit

- Students will understand the basic properties of geometric shapes.
- Students will apply problem solving strategies to explore the problem using measurement, construction, synthetic geometry and other approaches.
- Students will make use of resources and concrete materials such as geostrips, and possibly explore properties using suitable IT.
- Students will apply prior knowledge of key elements of geometry such as properties of triangles and quadrilaterals in different contexts.
- Students will be encouraged to extend these ideas to further properties of polygons.

## 7. Unit Plan

| Lesson | Brief overview of lessons in unit |
|---|---|
| 1, 2 | Reintroducing basic concepts of geometry - points, line, line segments, rays, angles; measure and constructions using rulers, compasses, set squares and protractors; types of angles; real life examples (Maths Eyes) |
| 3, 4 | Establishing general angle properties (straight, full turn). Using synthetic geometry to calculate angles. Introducing concrete resources such as geostrips. |
| 5, 6 | Parallel and perpendicular lines. Constructions and measure. Observe and deduce alternate and corresponding angles. Real life examples. |
| 7, 8 | Revising and extending types and properties of triangles. Constructing and generalising. Exploring patterns. Introducing appropriate IT such as GeoGebra to illustrate properties and examine general properties of triangles and parallel and perpendicular lines. |

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| **9**<br>**Research Lesson** | Introductory exploration of quadrilaterals. Use of a parallelogram with one diagonal to reinforce and link earlier ideas and to open up a consideration of the characteristics of quadrilaterals such as lengths, angles, area and congruence. Investigate how these properties apply to particular quadrilaterals. Identify general attributes through analysis and problem solving. Assess students at all stages through appropriate questioning and requiring them to demonstrate learning in a variety of ways. |

## 8. Goals of the Research Lesson:

The design of this lesson includes mathematical goals and is underpinned by Junior Cycle Key Skills.

a) Mathematical Goals

- Students will consider and explore fundamental geometric concepts associated with parallel and perpendicular lines, triangles and quadrilaterals.
- Students will use a variety of approaches.
- Students will recognise key elements and distinguishing features of different types of geometric shapes and polygons.
- Students will deduce the significance of lengths and angle properties and the connections between them.
- Students will communicate their learning effectively.
- Students will build on this knowledge to explore ideas around other polygons.

b) Key Skills and Statements of Learning

In the planning and design of this lesson Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will promote Key Skills in the following ways:

- Being numerate: seeing patterns trends, relationships and the significance of generalisation.
- Being literate: expressing ideas clearly and accurately.
- Managing myself: students will have the opportunity to reflect on their own learning.
- Communicating: students will present, explain, justify and discuss their mathematical thinking and exploration of shape, space and geometry.
- Being creative: students will be encouraged to explore options and alternatives and will actively participate in creative learning.
- Working with others: students will cooperate and will learn with and from each other.
- Managing information and thinking: students will be encouraged to be curious, to think creatively and critically, and to reflect on and evaluate their learning.

This lesson is also designed to meet the following Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.

15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.

16. The student describes, illustrates, interprets, predicts and explains patterns and relationships.

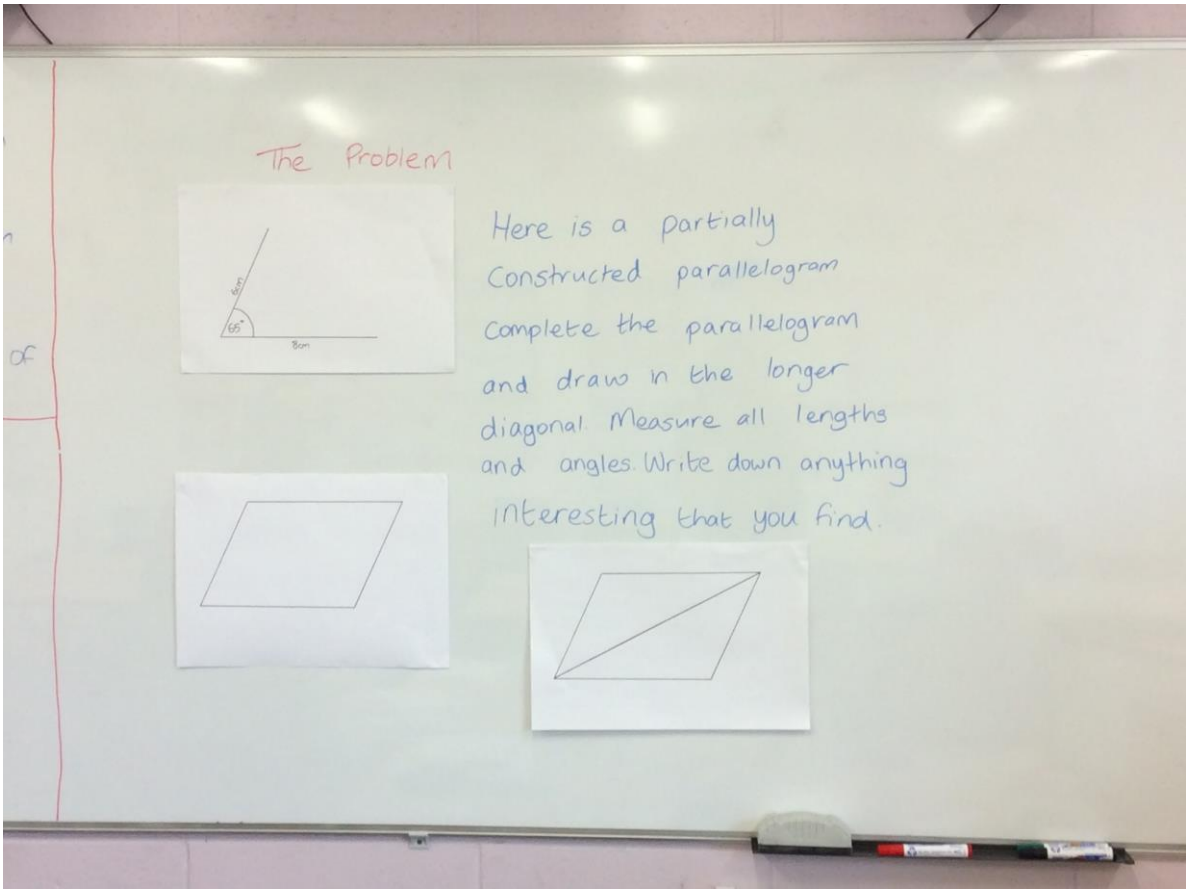
17. The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge reasoning and skills.

18.The student observes and evaluates empirical events and processes and draws valid deductions and conclusions.

24. The student uses technology and digital media tools to learn, communicate, work and think collaboratively and creatively in a responsible and ethical manner.

## 9. Flow of the Research Lesson

**THE PROBLEM**




| Steps, Learning Activities; Teacher's Questions and Expected Student Reactions | Teacher Support | Assessment |
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| **Introduction [5 minutes]**<br>To start proceedings the teacher will introduce any guests and observers and will emphasise to students that class will be conducted as normal. The teacher will briefly explain the role of the observer (observing work, gathering information, taking photos as a record of work, etc.).<br><br>The teacher begins by presenting the learning objectives of the lesson: | At the start of the class the teacher will divide the class into groups of four (two pairs) at tables, ensuring differentiated levels of student abilities within each group.<br><br>Teacher will examine homework as a basis for today's work. | Can students express to a reasonable level what they know about parallelograms?<br><br>Are constructions from homework satisfactory? |

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| • to investigate angles in a parallelogram<br>• to investigate lengths in a parallelogram<br>• to explore other properties of parallelograms, possibly related to diagonals or extending sides<br><br>Previous night's homework consisted of constructing a parallelogram any way they could using geometry equipment.<br>"Before we start today's work I want to have a quick look yesterday's homework."<br>"Are you happy that you have constructed a good parallelogram? Why?"<br>"We will look at this parallelogram later in class" | We want to be certain that students understand basic ideas and terms associated with parallelograms and the use of geometric instruments.<br><br>We need to use effective questioning to ascertain students' prior knowledge. In particular, we want to encourage the thought of using different approaches to deal with geometric problems. Possible approaches include construction and measure, cutouts, synthetic geometry, along with geostrips or IT packages at a later stage in the class. | |
| **Posing the Task**<br>Students will be presented with a partially constructed parallelogram, with adjacent lengths of 8cm and 6cm and an angle of 65º between them. This will be presented on blank A4 paper.<br><br>Students will then be asked to complete the parallelogram and to draw in the longer diagonal.<br><br>Finally, students will be asked to examine their constructions, measure all lengths and angles and to communicate what they observe.<br><br>***"Here is a partially constructed parallelogram with joining lengths of 8cm and 6cm and an angle of 65º between them. Complete the parallelogram and draw in the longer diagonal. Measure all lengths and angles. Write down anything interesting you see. We will then get some students to come up to the board to explain and discuss their solutions. I would also like you to use tracing paper and*** | Present an image of the problem on the board using the data projector and/or cut outs.<br><br>Ensure that each student has a calculator, drawing equipment and markers or colouring pencils. Also students will be asked to bring their school diaries, which include "traffic lights" to indicate levels of understanding.<br><br>Give each student an A4 copy of the problem. Also leave spare copies of the problem on each table (to facilitate cutouts or other approaches).<br><br>Give each pair an A3 template of the problem. | Do the students understand the task clearly?<br><br>Do the students understand that they are fundamentally dealing with geometric patterns?<br><br>Do the students understand that they are required to look at the problem in different ways, at least by constructing and measuring, as well as cutouts?<br><br>Are the students engaged in the work?<br><br>Are weaker students developing solutions? |

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| ***scissors to look at the parallelogram in that way."***<br>[SEE DIAGRAM ABOVE] | | |
| **Student Individual Work [10 minutes] and Collaborative Pair Work [10 minutes]**<br><br>*Student Response 1*<br>The diagonal creates two triangles.<br><br>*Student Response 2*<br>Opposite sides are equal in length, by measuring.<br><br>*Student Response 3*<br>Opposite sides are equal in length, by cutouts.<br><br>*Student Response 4*<br>Opposite angles are equal in size, by measuring.<br><br>*Student Response 5*<br>Opposite angles are equal in size, by cutouts.<br><br>*Student Response 6*<br>Three angles in a triangle add up to 180º, by measuring.<br><br>*Student Response 7*<br>Three angles in a triangle add up to 180º, by cutouts.<br><br>*Student Response 8*<br>The angles in a parallelogram add up to 360º.<br><br>*Student Response 9*<br>A diagonal divides a parallelogram into two equal triangles (area or congruence), by measuring.<br><br>*Student Response 10*<br>A diagonal divides a parallelogram into two equal triangles (area or congruence), by cutouts. | Give students extra copies of the completed construction along with tracing paper and scissors to enable them to approach the problem using cutouts. Place a set of Geostrips on each table. If they have enough time they may explore properties using these.<br><br>Use teacher's seating chart (or equivalent on Lesson Note app on iPad) to record the approach used by each student and each pair. Note the order in which you will call a representative from each pair to the board during Ceardaíocht, based on increasing sophistication of each pair's work. Observers will use the *Observation Template* designed by the group of teachers (see Appendix 1) to gather and record relevant data.<br><br>If students are having difficulties, particularly with constructions, guide them by asking appropriate questions.<br><br>Students or groups may struggle to see different possible approaches. The teacher may gently guide them towards approaches that are not occurring to | Are students able to meaningfully engage with the problem?<br><br>Can students demonstrate an ability to look at the problem in different ways?<br><br>Can students develop multiple solutions?<br><br>Are students identifying key features of parallelograms?<br><br>What limitations may apply to different approaches to solving the problem?<br><br>What strengths and weaknesses does each proposed solution have? Do any approaches have so many advantages that we may wish to focus on them in future?<br><br>Why are some approaches more useful than others?<br><br>By identifying key features in each approach, could solutions have wider applications? |

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| *Student Response 11*<br>Alternate angles are equal in measure, by measuring.<br><br>*Student Response 12*<br>Alternate angles are equal in measure, by cutouts.<br><br>*Student Response 13*<br>In a triangle, the longest side is opposite the biggest angle. | them after a period of time.<br><br>During "Ceardaíocht" phase discussion, students will be asked to comment, reflect and elaborate on what other students say. The teacher will use this and other strategies to ensure that all students remain engaged.<br><br>The teacher will look out for opportunities to use student errors or misconceptions to consider and analyse common difficulties among students.<br><br>After 10 minutes individual work, the teacher will ask students to work in pairs. The aim of the pair work is to debate the various approaches offered and to enable effective communication. This pair work almost serves as a pre-Ceardaíocht. | |
| **Ceardaíocht /Comparing and Discussing [20-25 minutes]**<br>From the anticipated student responses, the teacher will select at least one solution to be presented at the board based on each of the following (where available), and in this order:<br>● opposite sides equal<br>● opposite angles equal<br>● sum of angles in triangles or parallelograms<br>● diagonals dividing parallelograms in to triangles of equal area, or identical (congruent) triangles<br>● alternate angles equal.<br>The teacher will ensure that measuring and cutout approaches will both feature in solutions presented at the board.<br><br>As students or groups are developing solutions, if it becomes apparent that some of the above | It is a central component of the whole Lesson Study process that, when students are presenting solutions at the board, they must communicate verbally with the teacher and the class. The teacher must ensure this. Appropriate comments or questions may be helpful.<br><br>Write any student answers and predictions on the board.<br><br>If there is enough time, the teacher could employ GeoGebra to explore the | Can each student clearly explain his/her approach to solving the problem when up at the board?<br><br>Can each student justify elements of his/her solution and communicate this effectively?<br><br>Can students express an understanding of solutions developed by others?<br><br>Do students recognise |

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| approaches are not emanating from any student or group, the teacher may hint at or give some basic guidance towards considering them.<br><br>The teacher must ask questions of the presenter and of other students, to try to ascertain levels of understanding of key elements of solutions.<br><br>It is important that the whole class is engaged and that students understand they may be called on at any stage to reflect on what another student said. The teacher must keep in mind that clear student communication and understanding are goals of the lesson.<br><br>In keeping with the goals of the lesson, teacher questioning will focus on the most important properties of parallelograms arising from each solution. For example, "How did you find this?"; "Could anyone suggest another approach in this case?"; "Did anyone else get a similar result?"; "Do you think this happens all the time?".<br><br>The teacher should highlight comparable features in different solutions. For example, "Can anyone see any similarities between what's happening when you are measuring or cutting shapes out?"<br><br>Teacher questioning should lead debate on interesting features, merits or misconceptions of solutions.<br><br>When all solutions have been presented, the teacher may pose the question "Was any solution particularly good or interesting or surprising? Why?". The teacher could also ask students to consider if the comparisons between solutions may apply generally to parallelograms (although this may be work for another day).<br><br>To finish the Ceardaíocht stage, the teacher will ask students to move around classroom on a "one stay rest stray" basis, i.e. one student from each group will remain at his/her table to answer any questions, while the other students move around to look briefly at the work of others. | general nature of the properties of parallelograms.<br><br>At the Ceardaíocht stage it is important to keep in mind the goals of the lesson. The focus of the teacher's questioning and direction of the discussion should be guided by these goals.<br><br>Students may need to be reminded that a major goal of the lesson is to examine the problem in a variety of ways, not just to find a solution. This also allows for differentiated work, allowing stronger students to be challenged while weaker students can maintain confidence in their work.<br><br>For each anticipated student response the teacher (in conjunction with the full Lesson Study group) will have prepared a basic initial position (typically an appropriate diagram) as a starting point for the student to build on and develop his/her solution on the board.<br><br>When a student presents work on a board make sure to attach his/her name (or a group's name) to it.<br><br>Ask other students to raise their hands if they used the same method.<br><br>When each anticipated solution has been presented | similarities or differences between their own approach and that presented on the board?<br><br>Do students offer alternative approaches to solving the problem?<br><br>What major misconceptions or errors are arising as discussion of solutions is proceeding?<br><br>If a solution does not work out, can students see exactly where the problem arose? |

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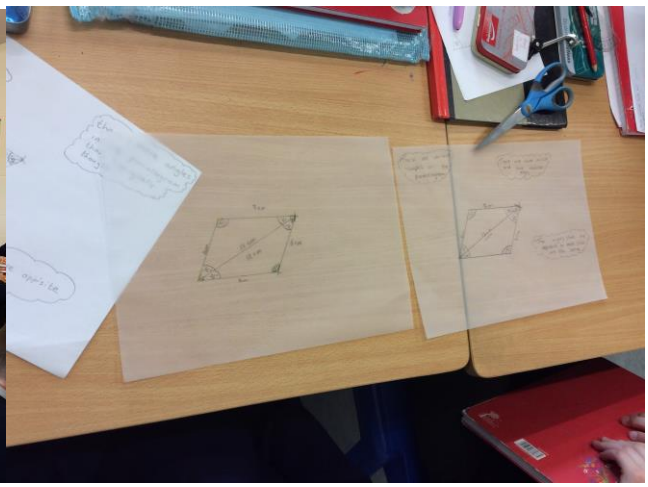
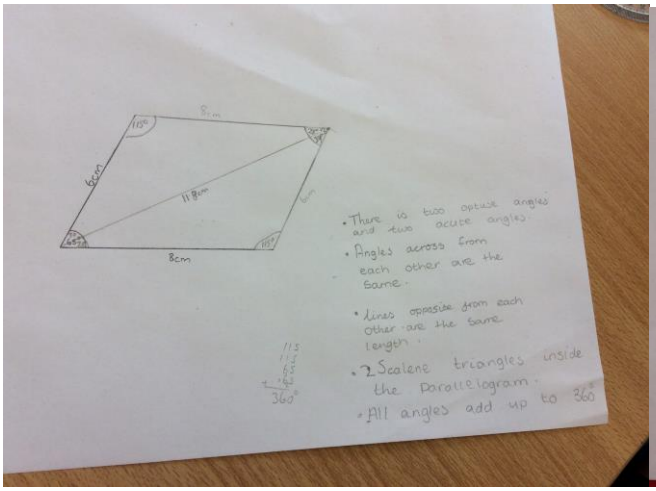
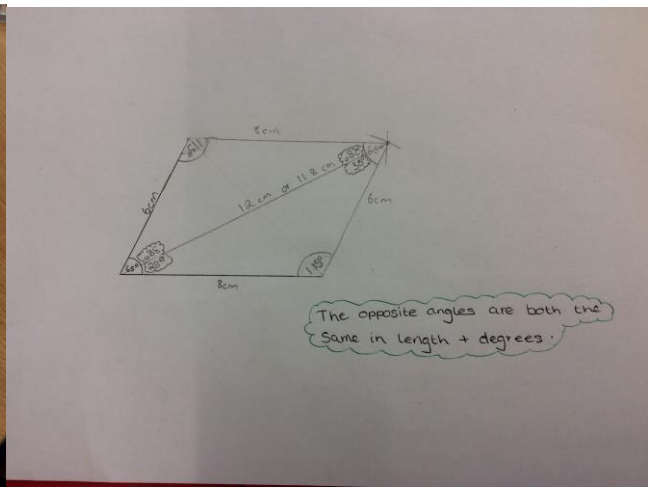
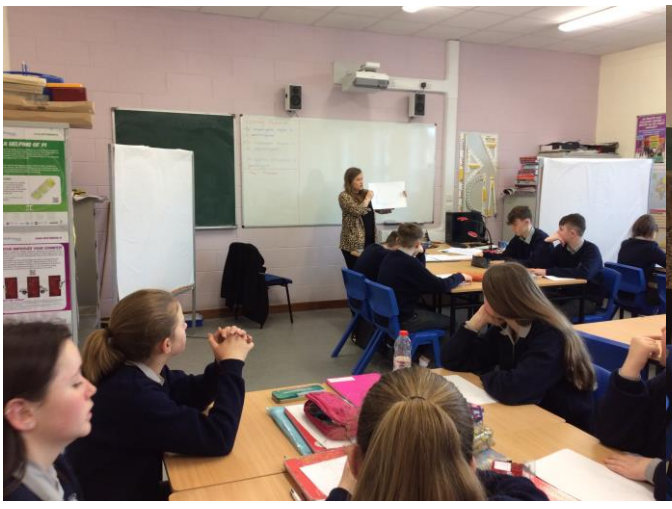
| | and discussed, ask did anybody use a different approach.<br><br>The teacher should use any opportunity in the discussion to introduce terminology, e.g. triangle, quadrilateral, parallelogram, diagonal, vertex, opposite sides, opposite angles, alternate angles, congruent triangles. | |
| **Summing up & Reflection**<br>Teacher will use Post-Its to ask students to write down any major thing they learned in class. [Padlet a possibility here too, but might require whiteboard space already being used]. | | |

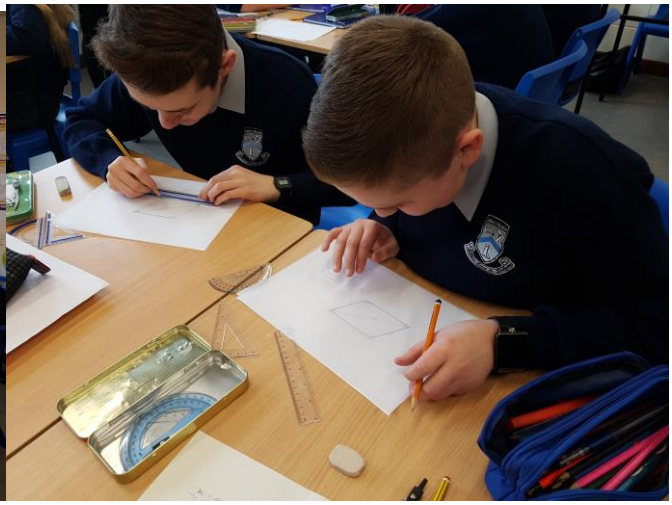


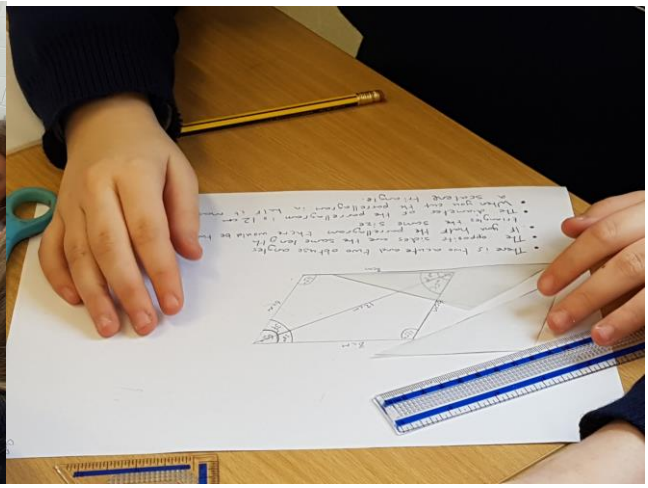

8cm
6cm
11.8cm
8cm
• There is two obtuse angles and two acute angles.
• Angles across from each other are the same.
• Lines opposite from each other are the same length.
• 2 Scalene triangles inside the Parallelogram.
• All angles add up to 360°

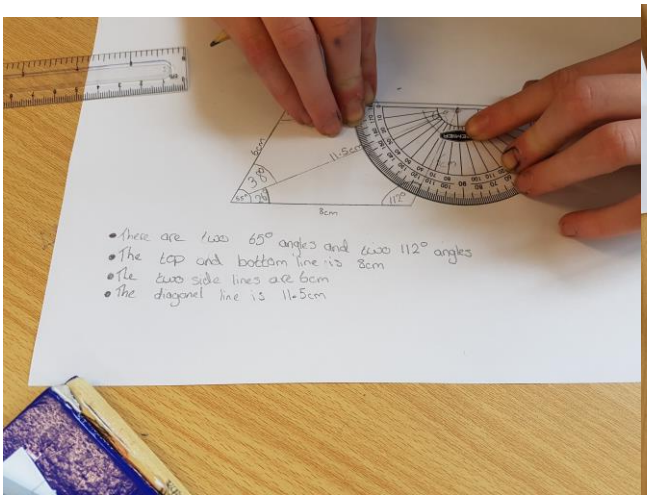

8cm
6cm
12cm or 11.8cm
6cm
8cm
The opposite angles are both the same in length + degrees.

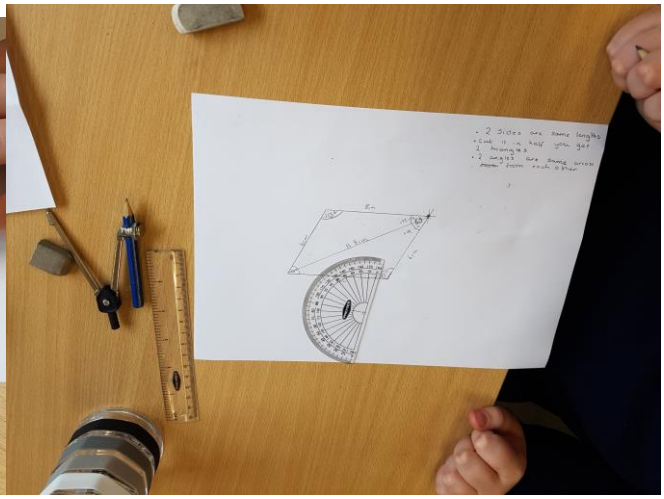


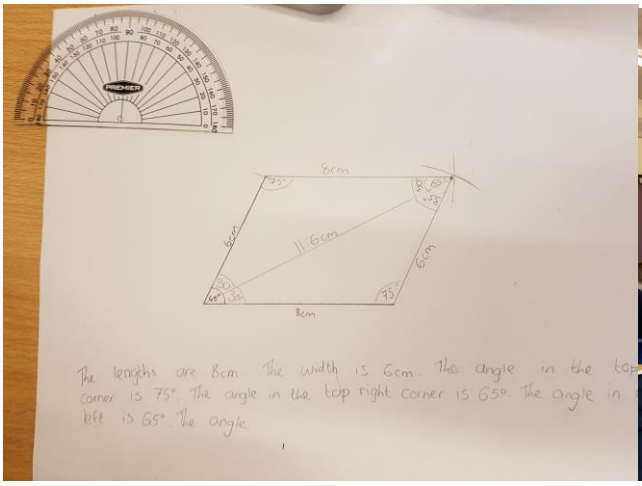


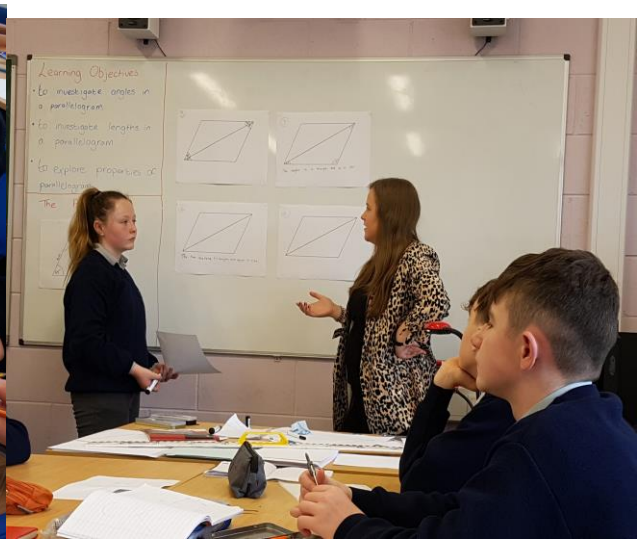





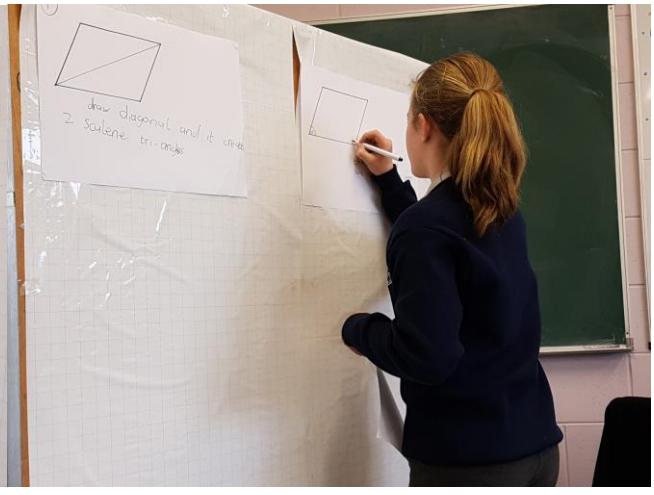

There are two 65° angles and two 112° angles
The top and bottom line is 8cm
The two side lines are 6cm
The diagonal line is 11.5cm

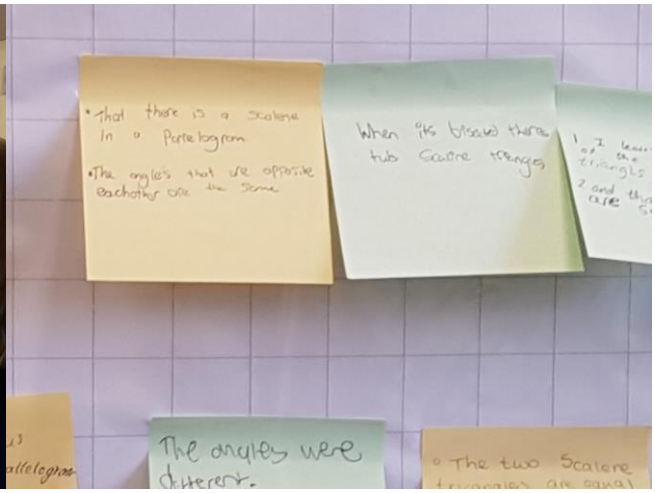


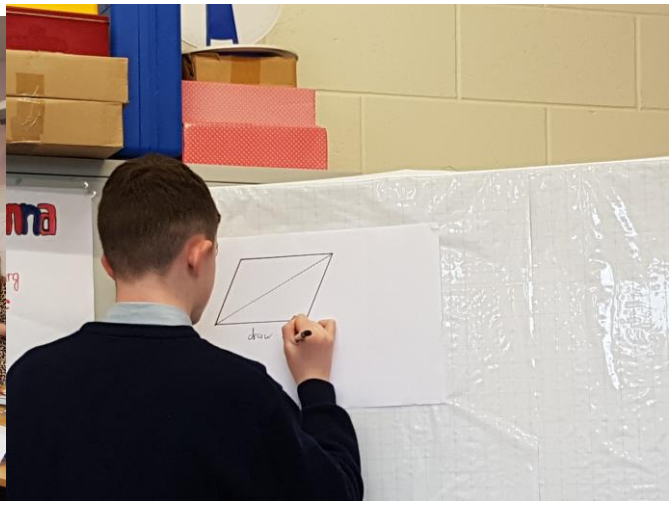

8cm
6cm
11.5cm
6cm
8cm
The lengths are 8cm. The width is 6cm. The angle in the top corner is 75°. The angle in the top right corner is 65°. The angle in left is 65° The angle


Learning Objectives


Learning Objectives


draw diagonal and it



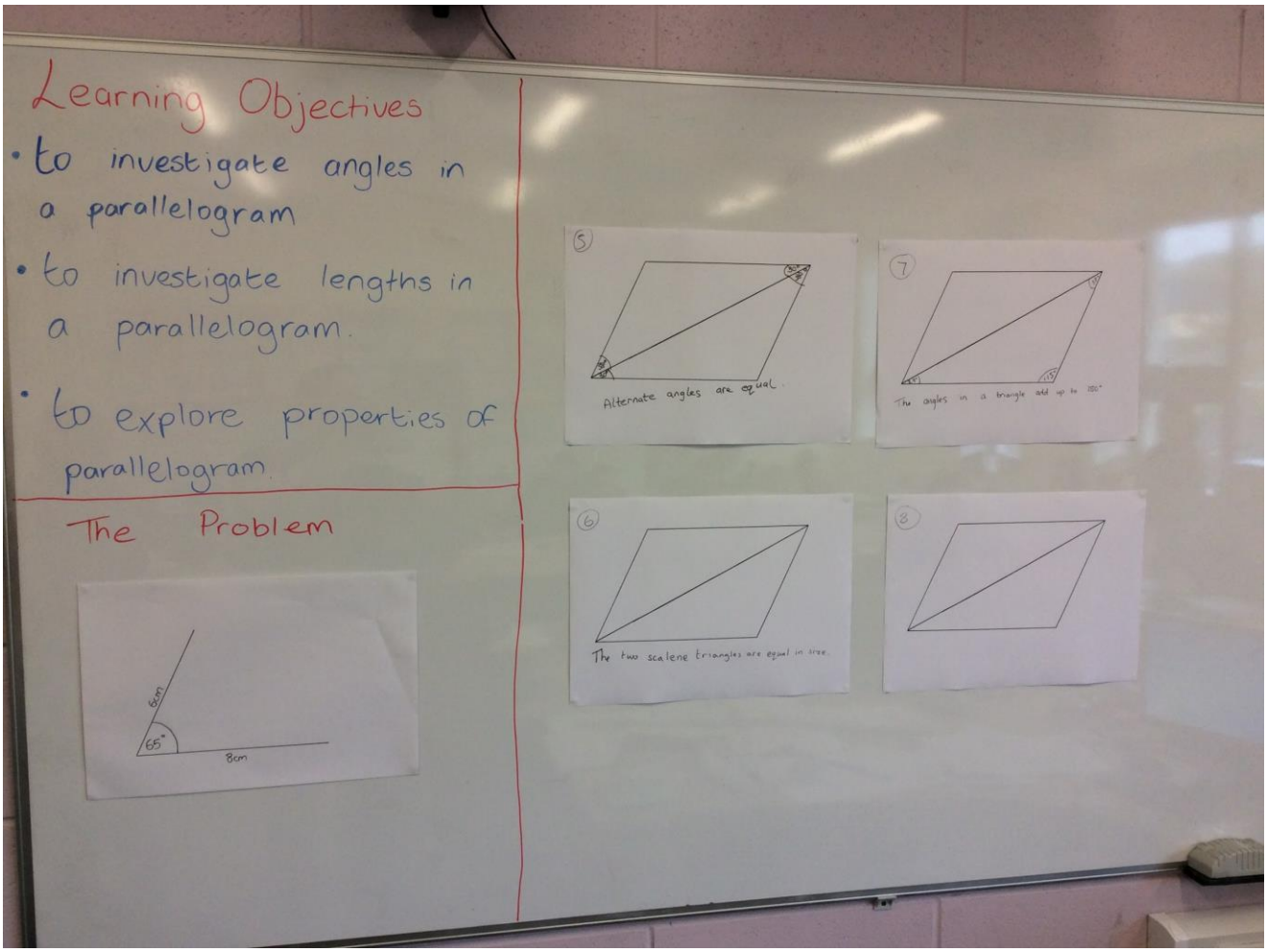
### 10. Board Plan

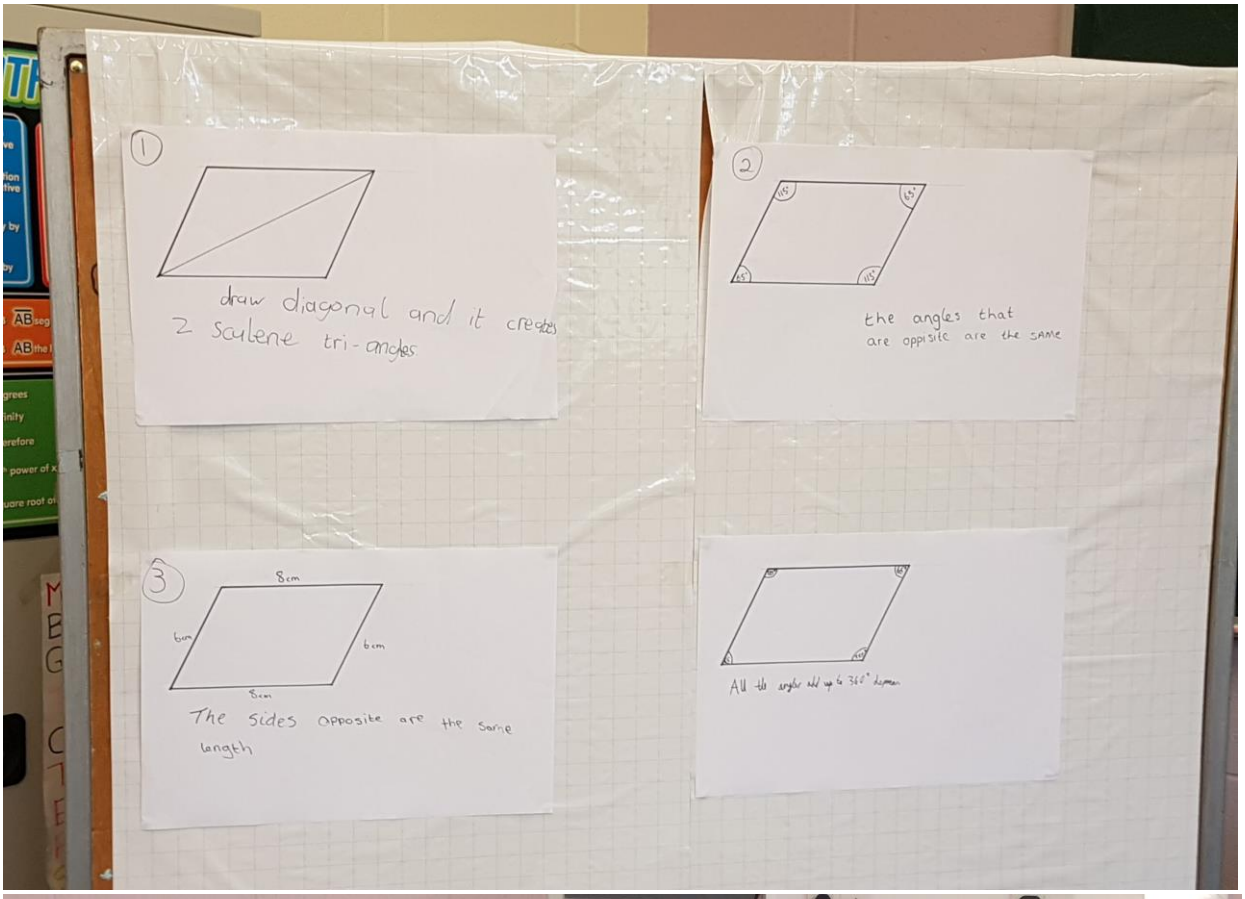
The primary aim of our board work was to provide a clear record for students of how learning progressed throughout and to highlight connections across seemingly disparate areas of mathematics. We planned to arrange solutions in order from least sophisticated to most sophisticated, ensuring we displayed at least one of each of the main types of solutions, along with some variety in approaches such as direct measurement or use of cutouts:

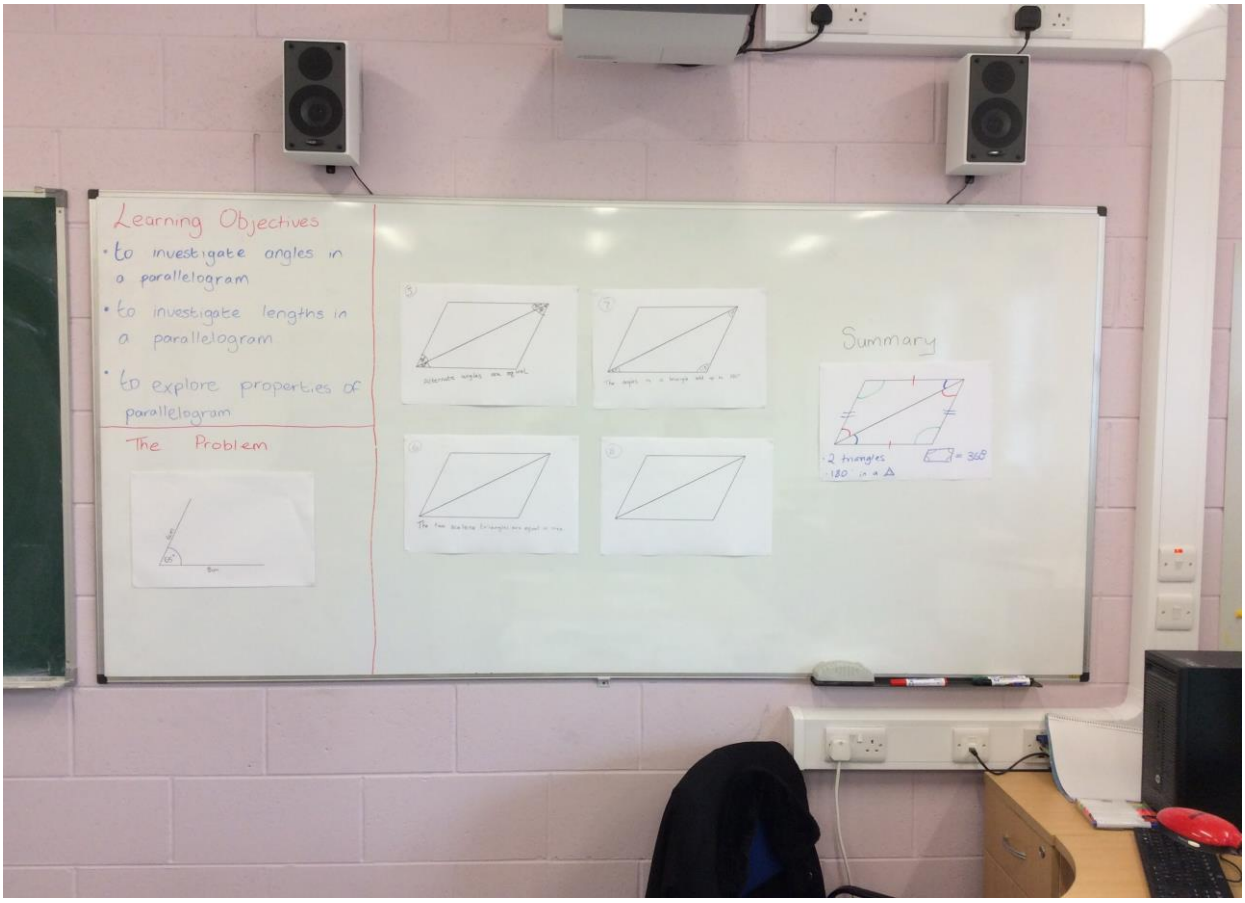
- opposite sides equal
- opposite angles equal
- sum of angles in triangles or parallelograms
- diagonals dividing parallelograms into triangles of equal area, or identical (congruent) triangles
- alternate angles equal.

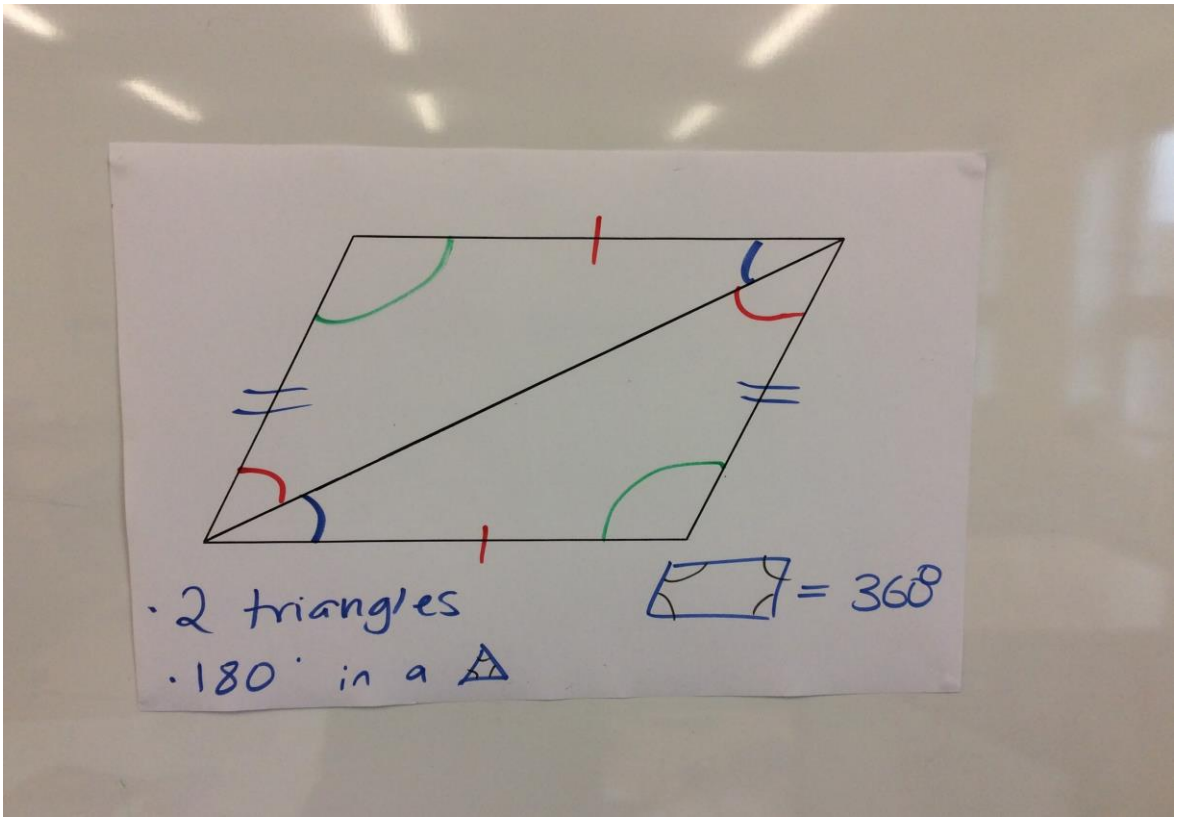
We prepared a basis of each anticipated solution to put up on the board as a starting point for each student to work with. We practiced a number of solutions in advance, emphasising effective questioning to draw out student explanation and communication while at the board.

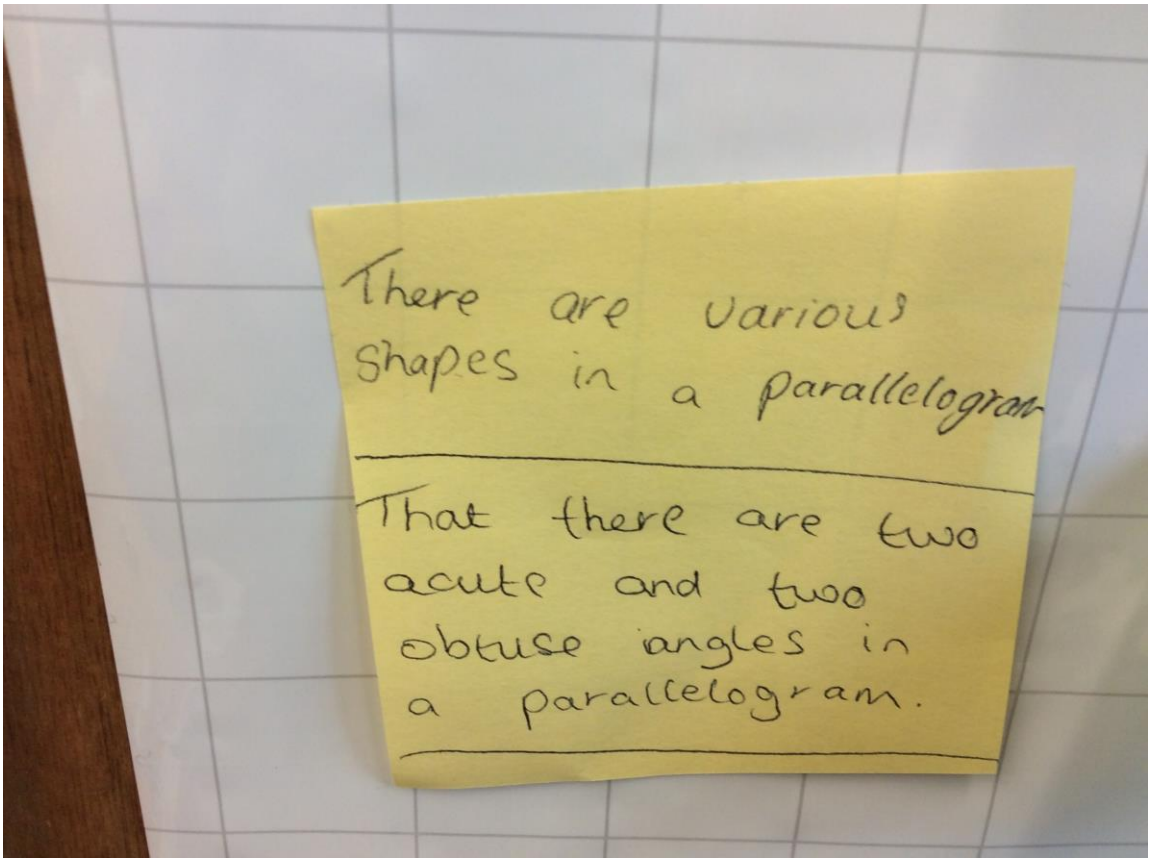
Board Plan

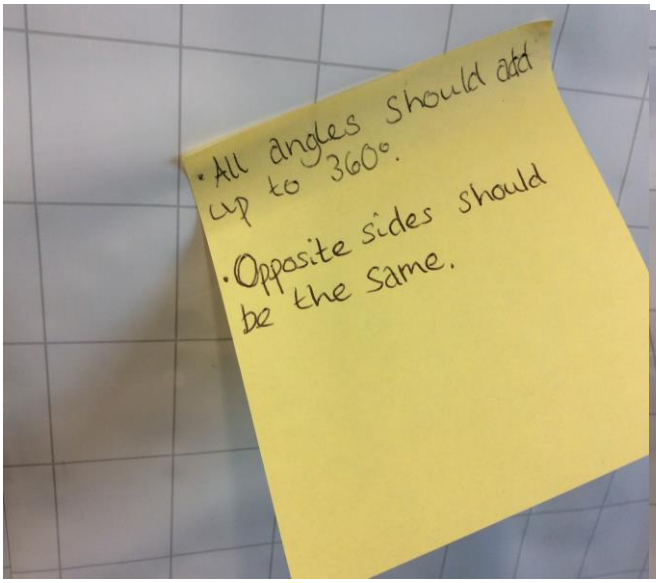

Learning Objectives
• to investigate angles in a parallelogram
• to investigate lengths in a parallelogram.
• to explore properties of parallelogram.
The Problem
6cm
65°
8cm
5
Alternate angles are equal.
7
The angles in a triangle add up to 180°
6
The two scalene triangles are equal in area.
8

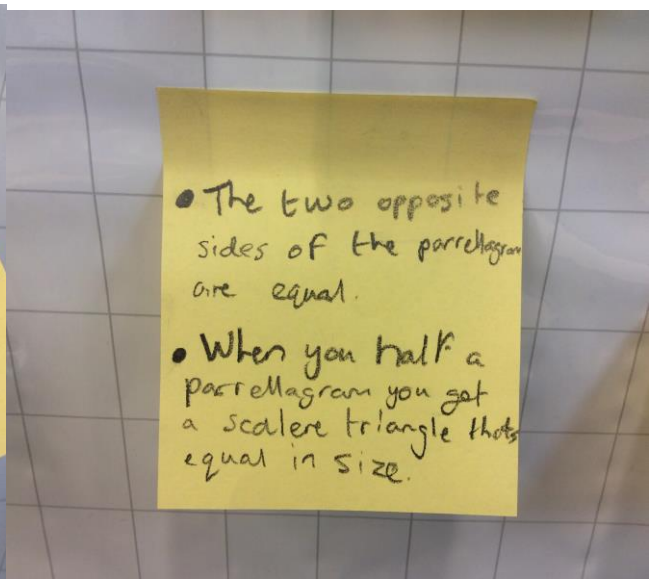

1
draw diagonal and it creates 2 scalene tri-angles
2
the angles that are opposite are the same
3
8cm
6cm
6cm
8cm
The sides opposite are the same length

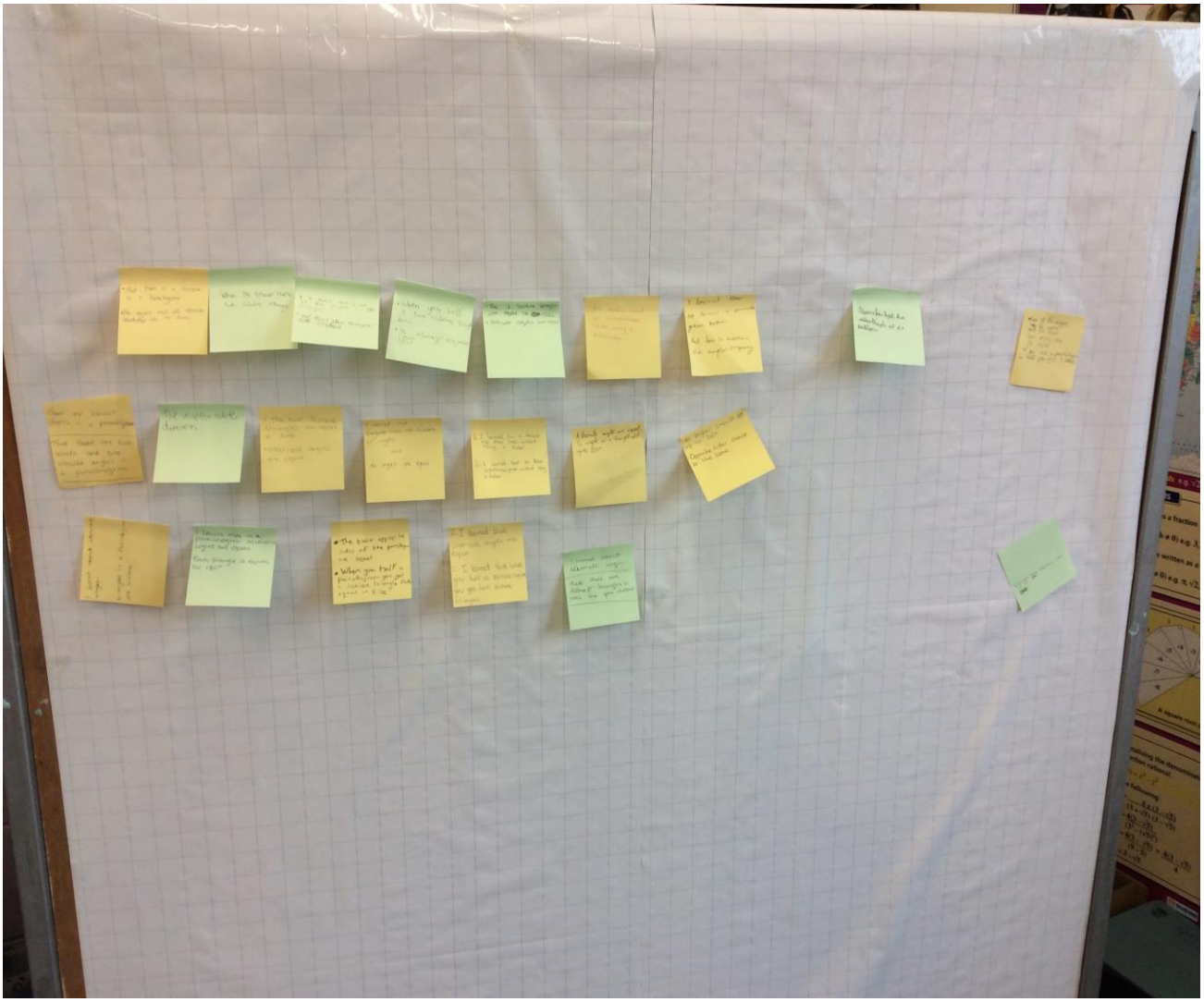

Learning Objectives
• to investigate angles in a parallelogram
• to investigate lengths in a parallelogram
• to explore properties of parallelogram
The Problem
Summary


• 2 triangles
• 180° in a
= 360°


There are various shapes in a parallelogram
That there are two acute and two obtuse angles in a parallelogram.


• All angles should add up to 360°.
• Opposite sides should be the same.


• The two opposite sides of the parrellagram are equal.
• When you half a parrellagram you get a scalere triangle thats equal in size.



## 11. Evaluation

Overall, we were satisfied that the goals of the lesson were largely achieved. Pupils investigated the lengths of sides, size of angles and other properties in various ways. They used constructions, tracing paper and cut outs. All of these helped pupils to discover properties at different stages of the class. In particular we found the use of tracing paper to be very productive. By cutting out parallelograms and triangles, students became engaged in examining and discussing features of the shapes. We will certainly use these ideas going forward.

While they were working individually and in pairs, the teacher used appropriate suggestions to help guide students to explore further properties, in particular to look at alternate angles. A number of interesting misconceptions arose. Some students had difficulty in drawing parallel lines and quite a few students used a protractor incorrectly, leading to a clearly obtuse angle being measured at $65^{o}$. As they had time to explore issues further in the class, most students were able to correct their own misunderstandings, sometimes with direction from the teacher. Most students understood that some level of discrepancy was acceptable in construction, but beyond a certain level of error, the investigations would not match the properties.

Initially, some pairs were very quiet or did not stay on the task for long. Most pairs eventually worked well and helped to formulate properties clearly. The use of the active methodology of tracing and cutting out was particularly helpful here as it helped them to observe more difficult properties. It was also fun. There was an interesting progression in learning from the start of individual work to the end of paired work. There was further, deeper exploration. Explanation and communication improved, where students used key terms to share their observations. They asked questions such as " I got X. Did you get that?"; "How did you get that angle?"; "Should they not be the same?"; "Mine didn't add up to 360". One group had a full row about odd and even totals, which was great to watch and full of learning. This was helped by good suggestions from the teacher as she observed the pairs, picking up difficulties and possible new ideas.

Paired work facilitated communication after the individual work stage, insofar as students had the opportunity to explain their work and question their colleagues in a smaller, less intimidating setting. It was almost like a pre-Ceardaíocht for both. We felt afterwards that paired work would be better than groups of four, as in groups some students can become slightly detached. Paired work did promote differentiation of levels of work, with weaker students having to stand over their solutions, even allowing for less sophisticated approaches. This was a central lesson goal. In a group setting some students are happy to hold on to their own individual answers and not fully engage in discussion. From last year we saw the need for effective paired work (e.g. requiring regular checking of what others are doing). Generally the preparation for effective questioning during the Ceardaíocht developed by the teaching group helped to tease out key ideas during the lesson.

After last year's lesson we felt that students who were presenting solutions at the board needed to communicate more effectively. If allowed to do so, students at the board will just write. They need to turn around and explain their thinking out loud to the classroom. Students didn't understand initially that, when a pupil was finished explaining a solution at the board, other pupils in the group could add to the explanation. The teacher clarified this early on and it produced good insights.
The board work was effective in that students could clearly see the progression of solutions during and at the end of the lesson. In their comments afterwards, students showed that they understood the main properties of parallelograms, along with some interesting extra issues (e.g. "learn to use a protractor properly"). The board work was clear and visible to all in the class. It was clear from the student feedback at the end of the lesson (where students noted briefly on post-its' the main learning they

enjoyed and difficulties they faced) that the main aims of the lesson were largely achieved. This feedback is very useful for informing future lessons with this class and also for our Mathematics Department in future planning of Lesson Study.

Some extended learning was immediately promising, such as a suggestion from one student that "one half folds onto the other along the diagonal". This opened the possibility of looking at axes of symmetry in polygons and deceptive cases.

## 12. Reflection

The main features of the lesson which stand out for us are the huge benefits of using a well-planned problem-solving approach to examining mathematical situations. Enabling students to be truly actively engaged in thinking about how to approach problems, in exploring multiple solutions to the same problem and in considering links between various strategies really enhanced understanding and confidence. We had hoped that students would demonstrate a good understanding of the key elements of parallelograms as the lesson progressed. This certainly happened. Good questioning by the teacher and well-structured board work aided this.

As a group of teachers planning a Lesson Study class, we must continue to consider a number of important issues for the future. Firstly, at the planning stage, we must practice our board work more to emphasise the importance of students genuinely communicating as they present answers at the board. The development of this skill takes time and practice for both the teacher and students. This communication cannot be perfunctory. It is a central objective of the process and must be thorough and searching. We must also prepare strategies to get the most out of paired work, ensuring all students are fully engaged. The most important part of the whole Lesson Study concept is the Ceardaíocht stage. We learned during the lesson that, where we considered this discussion in depth in advance of the class and practiced extensively the type of questioning we judged to be essential, the outcome of our Ceardaíocht was more productive. For any anticipated solutions where we did less planning on what discussion may arise, we found that we got less from the subsequent interaction in the class. The potential benefits of such student work is too valuable to miss out on. Our teaching group recognises that our preparation must be directed to get the most from this stage. Sufficient time at planning should be spent on core elements of the Lesson Study process. These core elements arise from the research theme and the goals of the lesson, which we must keep in mind when allocating our planning time.

Engaging in the process of Lesson Study over the last four years has brought real benefits to the students and Maths teachers of Deele College. We have found that the supports developed by the Project Maths team over that time have enabled us to make much better use of the process. The learning outcomes for students in terms of better communication and deeper understanding of concepts are clear. Collaborative planning, critical evaluation and peer observation embedded in Lesson Study have offered our teachers valuable opportunities to reflect on and improve the quality of Maths teaching in our school. Implementing the lessons learned from previous years of Lesson Study helped our work this year and will continue to form part of our development as teachers and a Maths department.


• to investigate angles in a parallelogram
• to investigate lengths in a parallelogram.
• to explore properties of
The
Alternate angles are equal

***APPENDIX 1***

Observation Template

# Observation Template

**School: Deele College Raphoe**

**Title of Lesson: What about parallelograms?**

**Date: 6th March 2019**

| | |
|---|---|
| **Introduction and Prior Knowledge** | |
| **Posing the Task** | |

| | |
|---|---|
| **Student Individual Work** | • What methods did the students use to solve the problem?<br>• Were any anticipated solutions not used?<br>• What comments or questions did the students have?<br>• Did students show a good understanding of the key features associated with linear patterns?<br>• To what extent were students able to use various approaches and link them?<br>• What were the common misconceptions and misunderstandings? |

| | |
|---|---|
| **Collaborative Group work** | • How and when did students' understanding change?<br>• Did any students demonstrate a continued lack of understanding or progress? How could this be addressed?<br>• How well did students collaborate at group work stage?<br>• How successful was differentiation of students' work? |
| **Unexpected Outcomes** | |

| | |
|---|---|
| | |
| **Ceardaíocht** | • Did the students' presentation and discussion promote their thinking and learning?<br>• How well did students communicate solutions and answers during discussion at Ceardaíocht stage? |

| | |
|---|---|
| | |
| **Extending Students' Learning** | |

| | |
|---|---|
| | |
| **Summing Up** | • Were the goals and research theme of the lesson met?<br>• What did students learn?<br>• What teacher approaches worked well?<br>• What teacher approaches did not work well?<br>• Was time used well?<br>• How can we improve our approaches and methodologies? |
| **Student Reflection and Feedback** | |

| | |
|---|---|
| | |
| **Evaluation** | • To what extent were students able to use various approaches and link them?<br>• What were the common misconceptions and misunderstandings?<br>• How and when did students' understanding change?<br>• Did any students demonstrate a continued lack of understanding or progress? How could this be addressed?<br>• Was the flow of the lesson coherent?<br>• Did the students display a positive disposition?<br>• Did the activities support the goals? |
| **Other Observations** | |

***APPENDIX 2***

Resources to be used:

- Project Maths Workshops 2, 3, 6, 9, 10
- NCCA Junior Cert Maths Specification 2018
- NCCA J.C 2016, L.C 2015 Syllabuses
- Project Maths Teacher Handbooks
- ICT - Geogebra
- Construction instruments
- Geostrips
- Tracing paper and scissors
- "Active Maths 1" Textbook
- "Text and Tests 1" Textbook
- Google Searches on Multiple Approach Problem Solving in Geometry
- www.projectmaths.ie
- www.pdst.ie (Maths material)
- Geometry Teaching and Learning Plans on www.projectmaths.ie