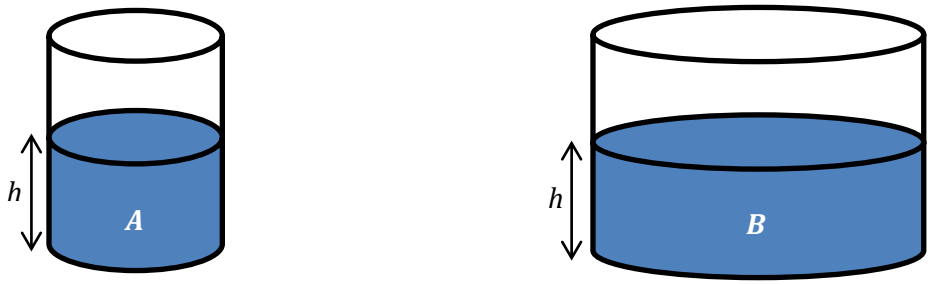
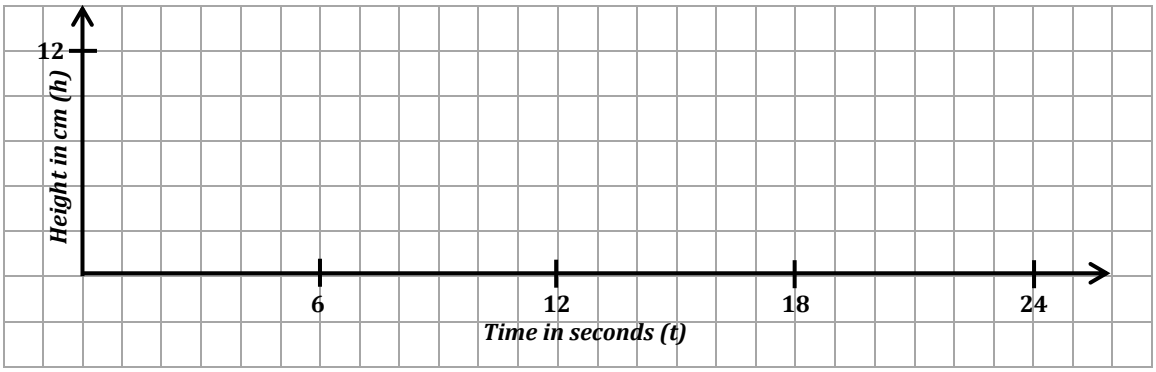
Section A: Student Activity 3 – Part 1

1. (i) Two cylindrical containers, *A* and *B* are being filled with water.
The volume of water increases at the same rate in both and the height of both containers is 12 cm. Sketch a graph to show the rate at which the height of the water level changes with time for both containers. Put both containers on one graph.
Container *A* is full after 6 seconds and container *B* is full after 24 seconds.

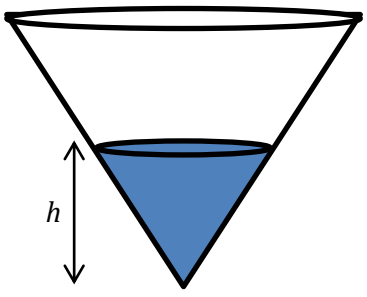


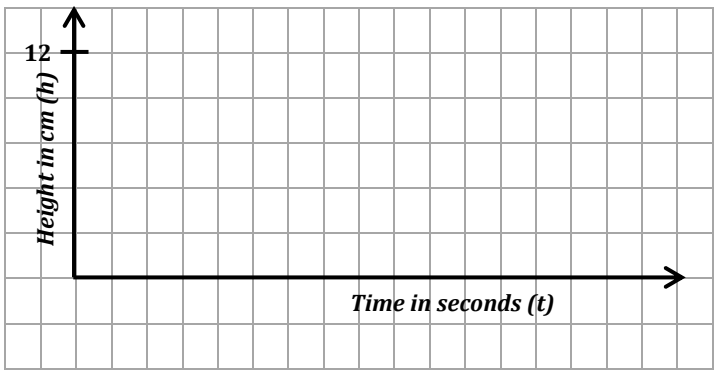


(ii) Why does it take container *B* longer to fill?

2. (i) Water flows into a vessel in the shape of an inverted cone as shown below.
The volume of water increases at the same rate as for the two cylinders above.
The vessel has the same height and radius as container *B*.
How long will it take to fill the vessel?

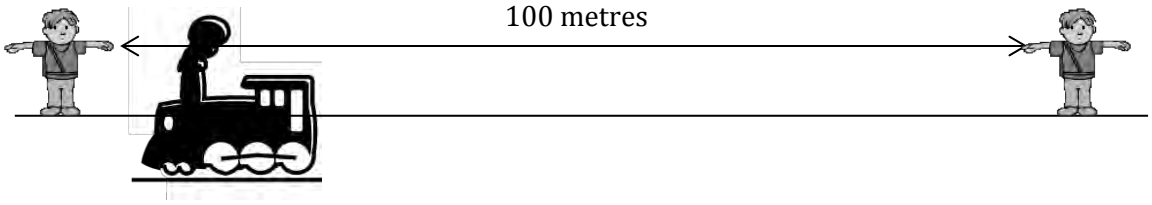
(ii) As water is poured into the vessel, sketch a rough graph to show how the height of the water level changes with time.

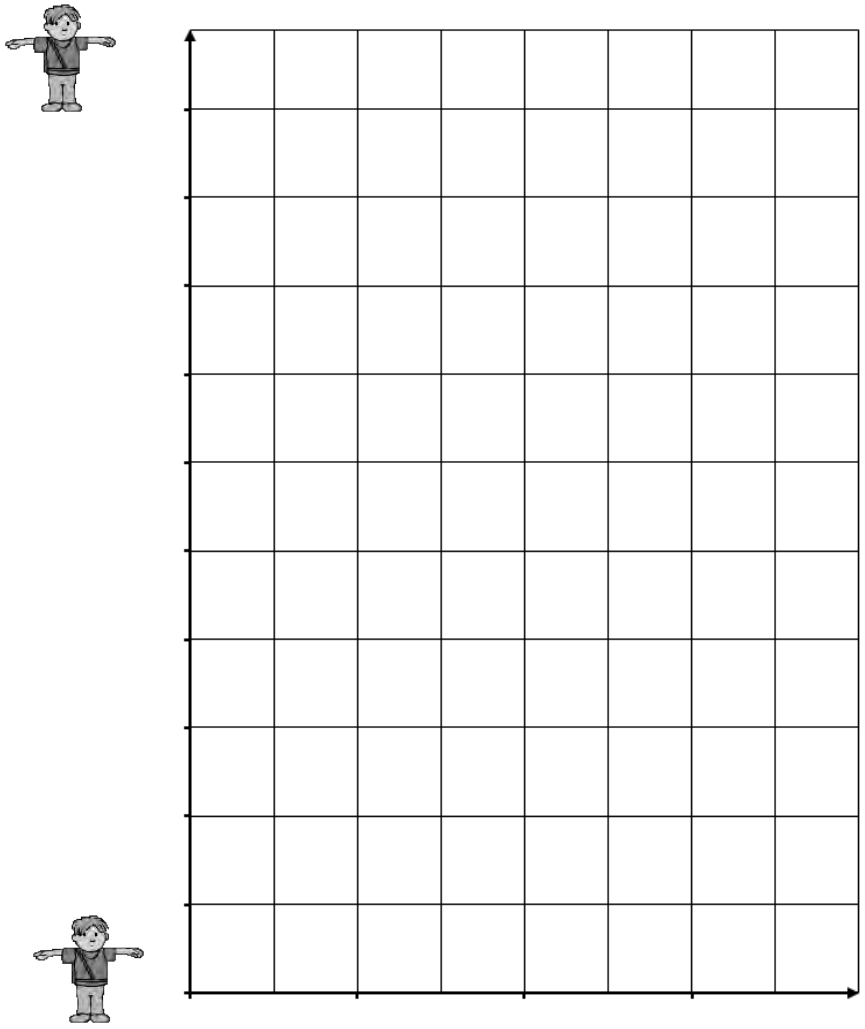




Section A: Student Activity 4

Some Transition Year students decide to carry out an experiment on constant speed. They have a class discussion on where they might see a model for constant speed. They decide that if they go to a train station and choose a train that is not scheduled to stop there, that the train will most likely pass them at a constant speed. Two students from the class arrange to stand 100 meters apart at either end of the platform and time the train between these two positions.





2. At what speed does the train pass the two students in km/hour?

_____

_____

3. If the teacher was standing half way between the students during the experiment to supervise, at what speed do you think the train passed the teacher? Give a reason for your answer.

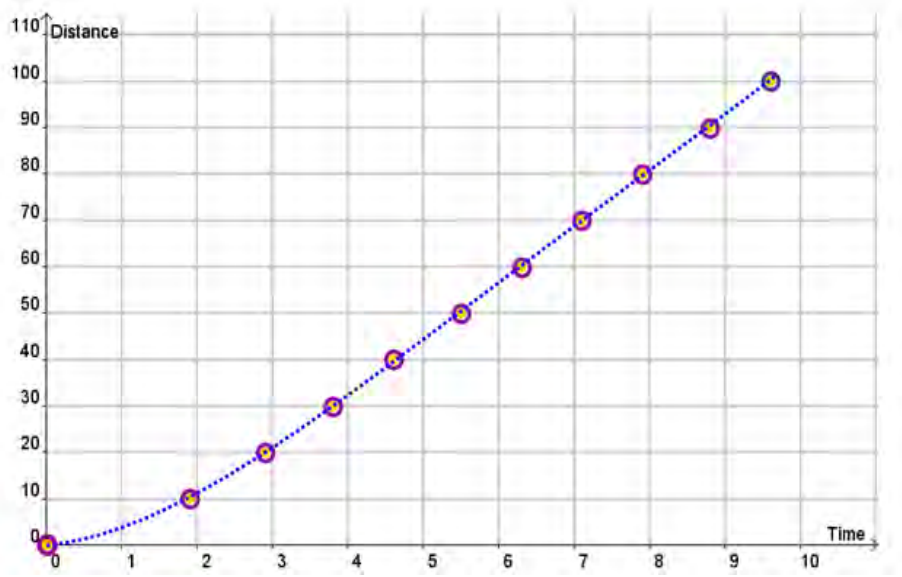
_____

_____

Section A: Student Activity 5

In the 2009 World Championships in Berlin, Usain Bolt set the World Record for the Men's 100 *m* sprint, running it in 9.58 seconds.
Below is a table of Usain Bolt's split times every 10 metres during the race.

Distance *(m)*	10	20	30	40	50	60	70	80	90	100
Time *(s)*	1.89	2.88	3.78	4.64	5.47	6.29	7.10	7.92	8.75	9.58

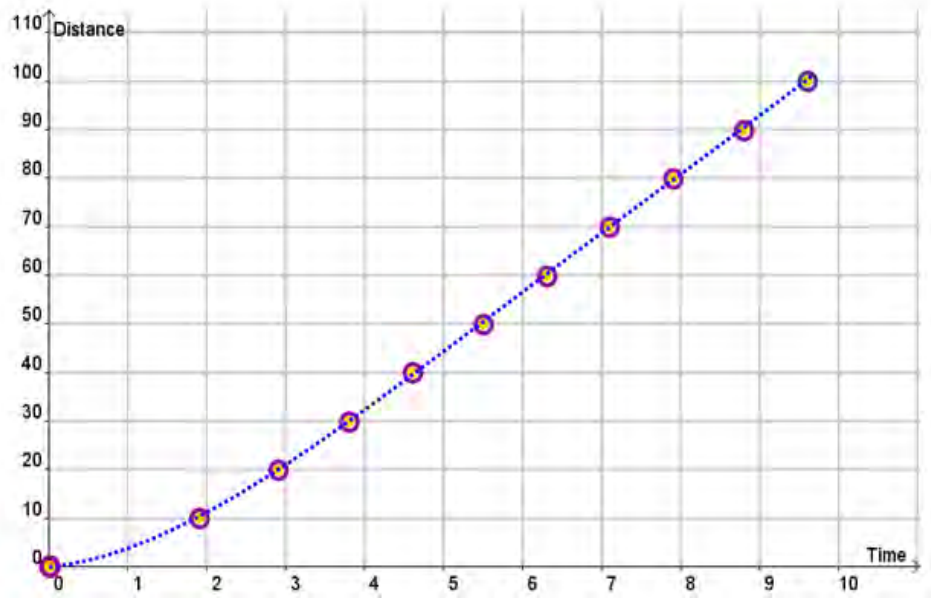


1. How fast do you think Usain Bolt ran during the race? Give your answer correct to 2 decimal places in m/sec.

2. Do you think he ran at this speed throughout the whole race? Give *two* reasons for your answer.

3. What do you think your answer for Question 1 represents?

4. **(i)** Using a ruler, join the points (0, 0) and (9.58, 100) on the graph below.



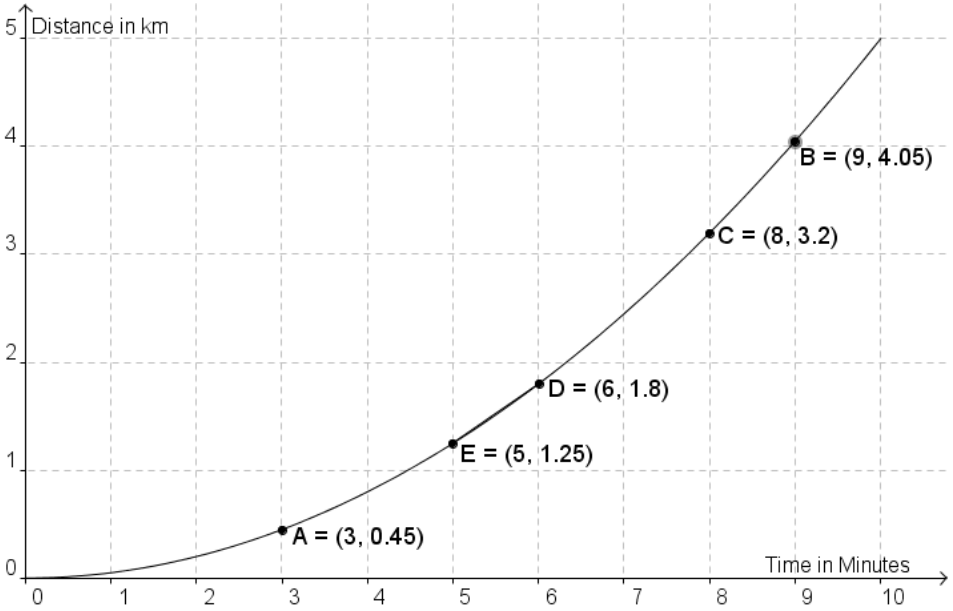
(ii) Find the slope of this line.

(iii) The line that joins (0,0) to (9.58,100) has a special name.
It is called a *secant line* to the above curve.
What observation can you make about the slope of this secant line?

5. How do you think we could calculate Usain's speed at *precisely* 1 second into the race?

Section A: Student Activity 6 – Part 1

Below is a distance-time graph of the first ten minutes of a warm up cycle by Olympic Gold medallist Victoria Pendleton.



1. Over these 10 minutes, what is Victoria Pendleton's average speed in km/min?

2. The coach wants to know what her speed is at exactly 3 minutes during this warm up. To help answer this question do the following:

(i) Using your ruler, draw in the secants $[AB]$, $[AC]$, $[AD]$, $[AE]$.

(ii) Fill in the following table. Answers correct to 2 decimal places.

Slope of Secant $[AB] =$	Average speed between A and $B =$
Slope of Secant $[AC] =$	Average speed between A and $C =$
Slope of Secant $[AD] =$	Average speed between A and $D =$
Slope of Secant $[AE] =$	Average speed between A and $E =$

3. The slope of which secant is the nearest estimate to Victoria's speed after exactly 3 minutes?

_____

4. How might you find a better estimate for Victoria's speed after exactly 3 minutes?

_____

WS08.04 Exploring the Slope of Tangents and Rates of Change

1. Slides with the various functions will be shown. Fill in the following table.

Function	Slope at all points on the function	Equation of the slope function
$f(x) = 3$		$f'(x) =$
$g(x) = 1$		$g'(x) =$
$h(x) = -1$		$h'(x) =$
$y = -3$		$\frac{dy}{dx} =$
⋮	⋮	⋮
$f(x) = n$		$f'(x) =$

The derivative of a constant is _____.

2. Slides with the various functions will be shown. Fill in the following table.

Function	Slope at all points on the function	Equation of the slope function
$f(x) = x$		$f'(x) =$
$g(x) = x + 2$		$g'(x) =$
$h(x) = \frac{3}{2}x$		$h'(x) =$
$y = \frac{3}{2}x + 3$		$\frac{dy}{dx} =$
$p(x) = -x$		
$k(x) = -x + 3$		
⋮	⋮	⋮
$f(x) = nx$ (where *n* is a constant)		
$f(x) = nx + c$ (where *n* and *c* are constants)		

The derivative of $f(x) = nx$ is _____.

The derivative of $f(x) = nx + c$ is _____.

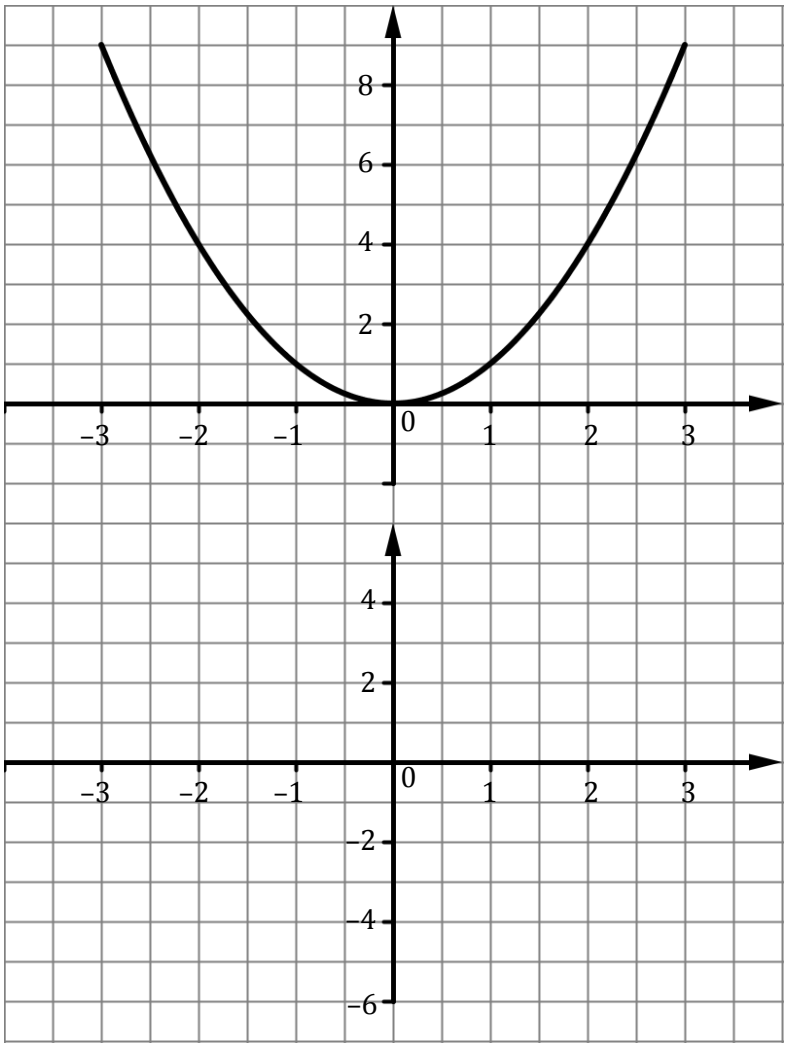
3. Page 14 has a graph of the function $f(x) = x^2$.

(a) Calculate the slopes of the tangents at the points indicated and enter your answers in the table below.

Hint: A ruler will help you read the slopes of the tangents.

(b) Graph the values from the table ***(on the lower graph)*** in the space provided.

x	Slope of Tangent
–3	
–2	
–1	
0	
1	
2	
3	

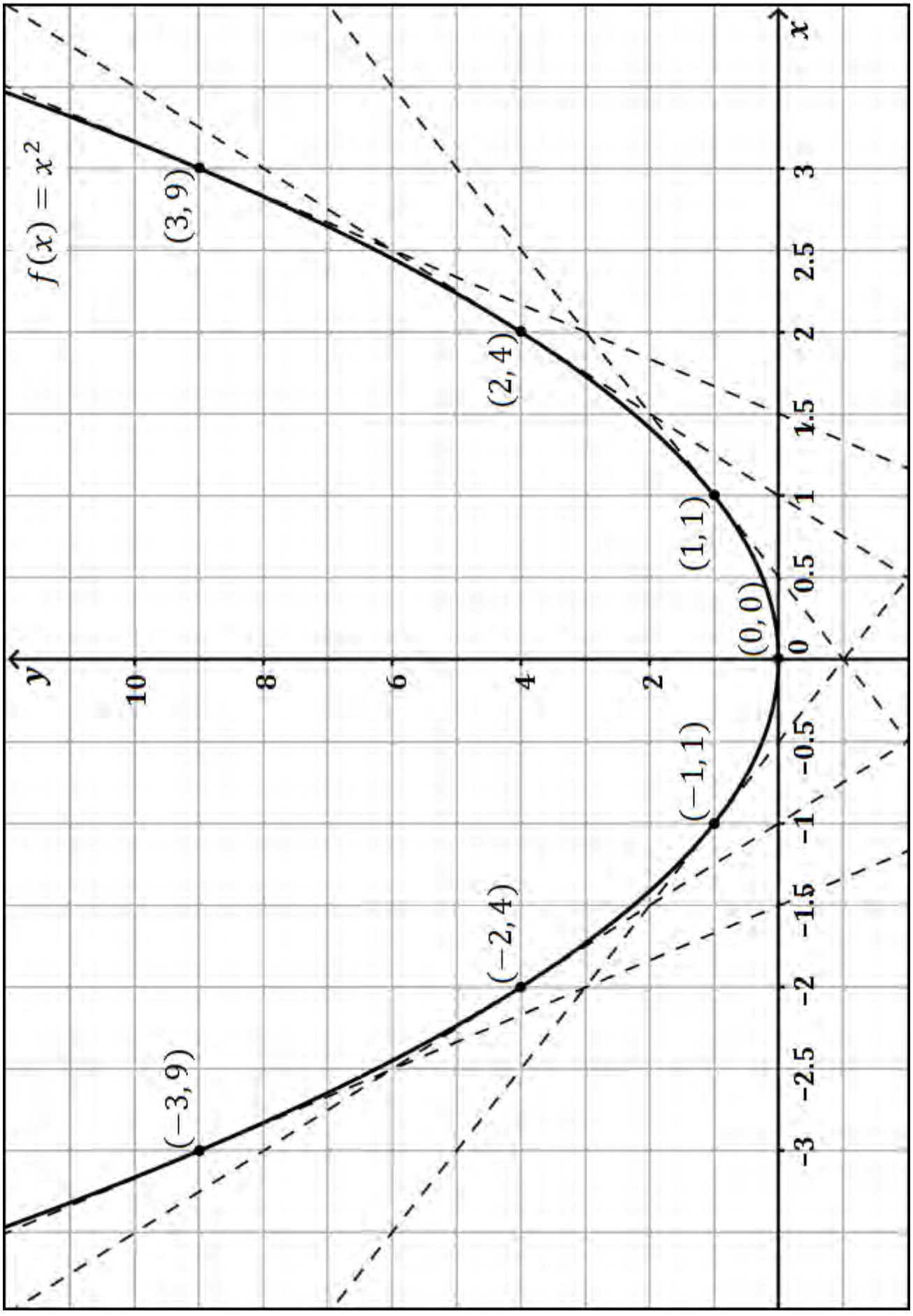


(c) Which of these represents the slope function?

(i) $f'(x) = x + 2$ **(ii)** $f'(x) = 2$ **(iii)** $f'(x) = 2x$

Give a reason for your answer.

Graph of $f(x) = x^2$

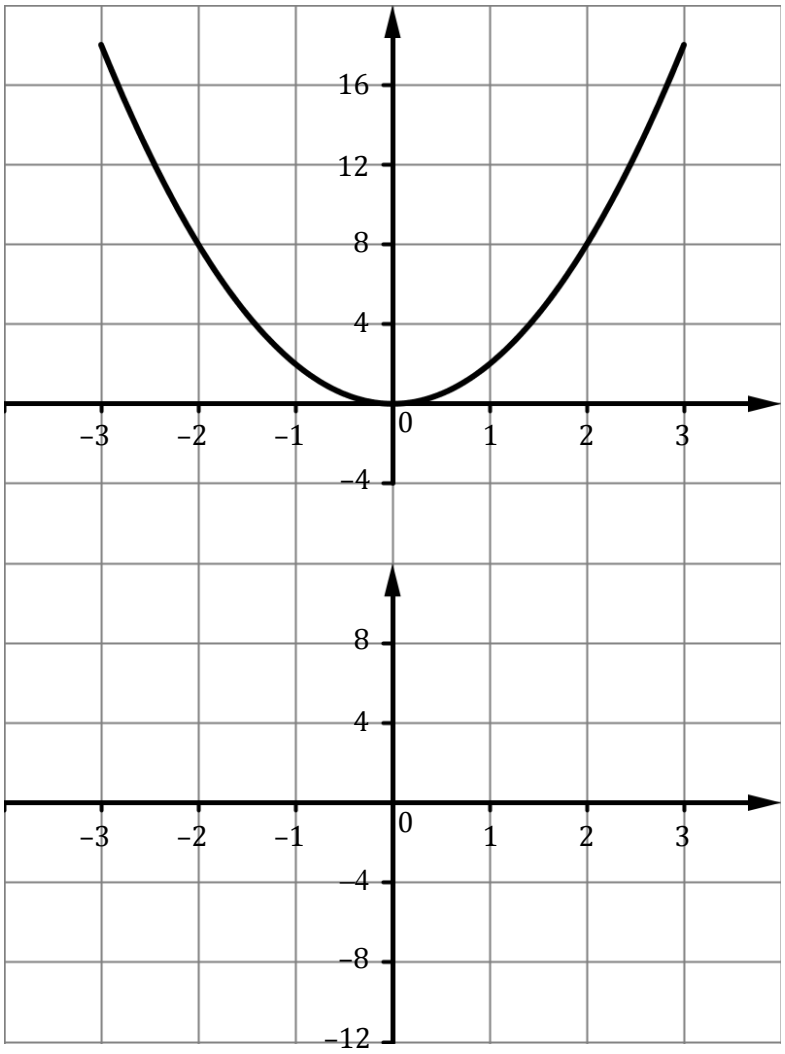


4. To complete the exercise below refer to the graph of the function $g(x) = 2x^2$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated and enter your answers in the table below.

(b) Graph the values from the table ***(on the lower graph)*** in the space provided.

x	Slope of Tangent
–3	
–2	
–1	
0	
1	
2	
3	



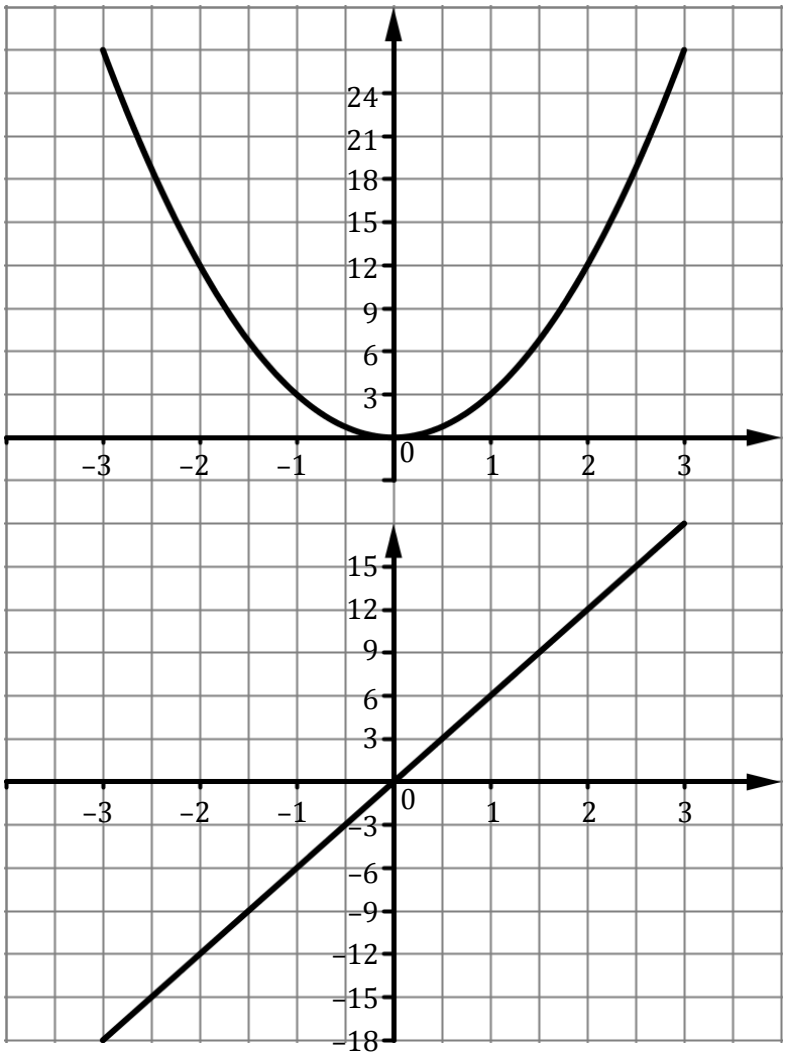
(c) The equation of the slope function, $g'(x)$, is:

5. To complete the exercise below refer to the graph of the function $h(x) = 3x^2$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table ***(on the lower graph)*** in the space provided.

x	Slope of Tangent
–3	–18
–2	–12
–1	–6
0	0
1	6
2	12
3	18



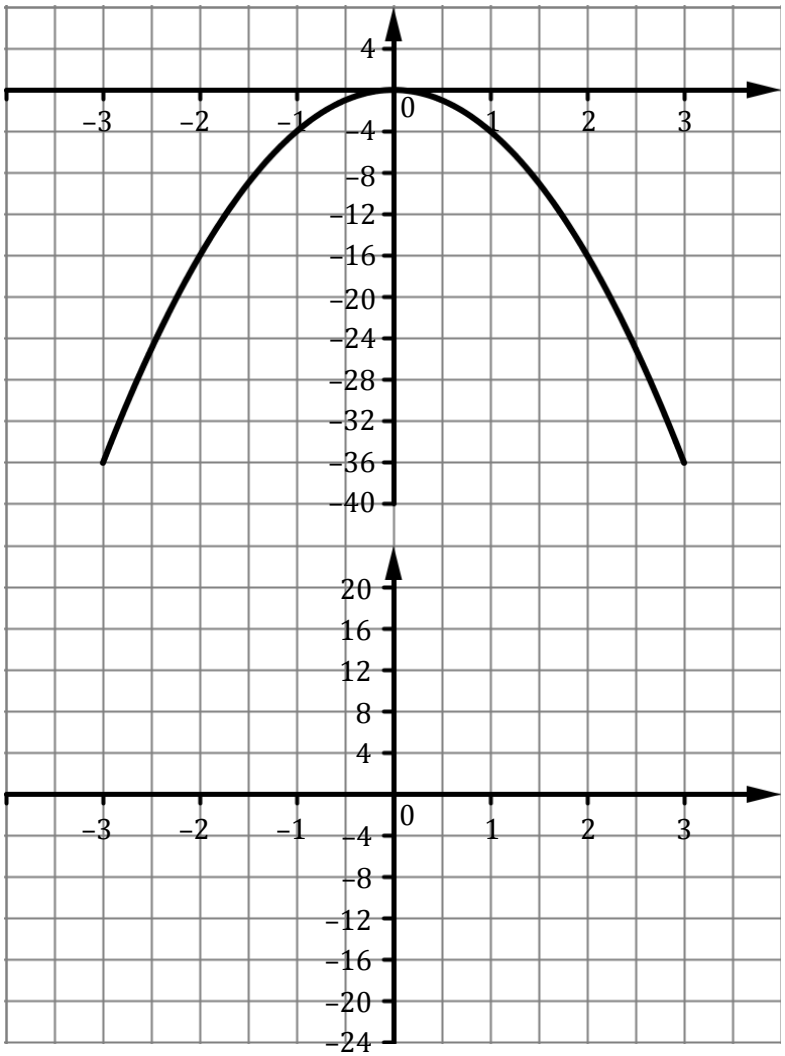
(c) The equation of the slope function, $h'(x)$, is: $h'(x) = 6x$

6. To complete the exercise below refer to the graph of the function $y = -4x^2$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table ***(on the lower graph)*** in the space provided.

x	Slope of Tangent
−3	
−2	
−1	
0	
1	
2	
3	



(c) The equation of the slope function, $\frac{dy}{dx}$, is:

7. Refer to questions 3, 4, 5, and 6 in order to fill in the table below.

Function, $f(x)$	Slope of Function or $f'(x)$
$f(x) = x^2$	$f'(x) =$
$g(x) = 2x^2$	$g'(x) =$
$h(x) = 3x^2$	$h'(x) =$
$y = -4x^2$	$\frac{dy}{dx} =$
$k(x) = 10x^2$	$k'(x) =$
⋮	⋮
$f(x) = ax^2$	$f'(x) =$

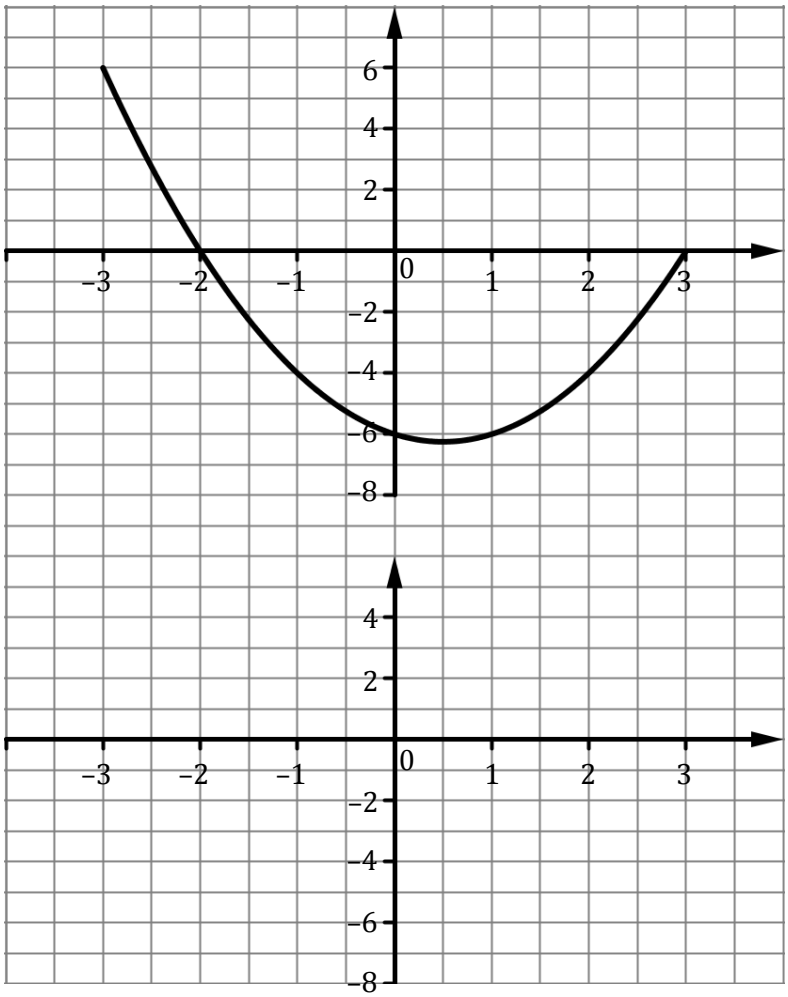
8. To complete the exercise below refer to the graph of the function $f(x) = x^2 - x - 6$ which is on the board.

(a) What do you think the equation of the slope function, $f'(x)$, is?_____

(b) Fill in the slopes of the tangents at the points indicated.

(c) Graph the values from the table ***(on the lower graph)*** in the space provided.

x	Slope of Tangent
–3	
–2	
–1	
0	
1	
2	
3	



(d) The equation of the slope function, $f'(x)$, is:

(e) Compare this to your prediction in part (a).
What conclusion can be drawn from finding the slope function of this last function?

(f) Patrick wants to know what the slope of $f(x)$ when $x = 1.5$
There are a few ways of estimating/calculating the slope of $f(x)$ when $x = 1.5$.

(i) From the graph of $f(x)$ in the GeoGebra file what would be a good estimate of the slope of $f(x)$ when $x = 1.5$?

(ii) From the list of slopes in the table what would be a good estimate of the slope of $f(x)$ when $x = 1.5$?

(iii) From the graph of $f'(x)$ what would be a good estimate of the slope of $f(x)$ for $x = 1.5$? (Show your workings on the graph)

(iv) From the equation $f'(x)$ what is the slope of $f(x)$ when $x = 1.5$?

9. **(a)** What is the equation of the slope function $(f'(x))$ of $f(x) = x^2 - x$?

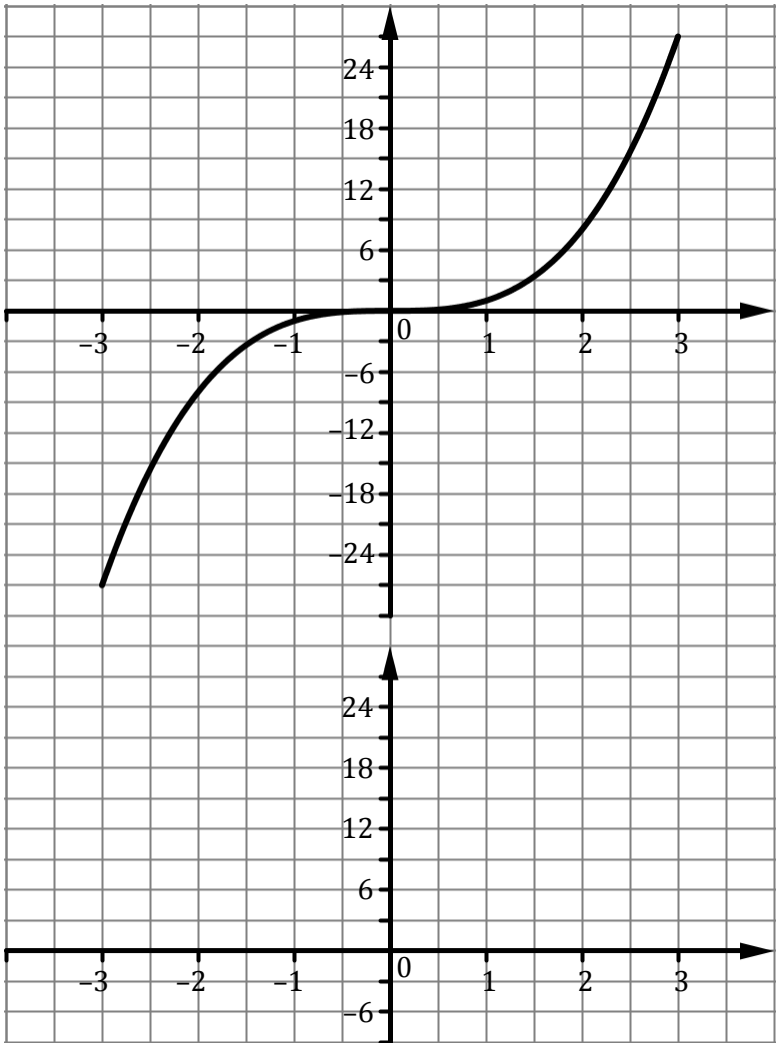
(b) What is the slope of the tangent of the function $f(x) = x^2 - x$ when $x = 3$?

10. To complete the exercise below refer to the graph of the function $f(x) = x^3$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table ***(on the lower graph)*** in the space provided.

x	Slope of Tangent
–3	
–2	
–1	
0	
1	
2	
3	



(c) What is the shape of the slope function?_____

(d) How could you confirm this?_____

The parent function for quadratics is $y = x^2$.

$y = x^2$contains the points: $(-3, 9),\ (-2, 4),\ (-1, 1),\ (0, 0),\ \ (1, 1),\ \ (2, 4),\ \ (3, 9).$

Your slope function contains the points: $(-3,\ \ ),\ (-2,\ \ ),\ (-1,\ \ ),\ (0,\ \ ),\ \ (1,\ \ ),\ \ (2,\ \ ),\ \ (3,\ \ ).$

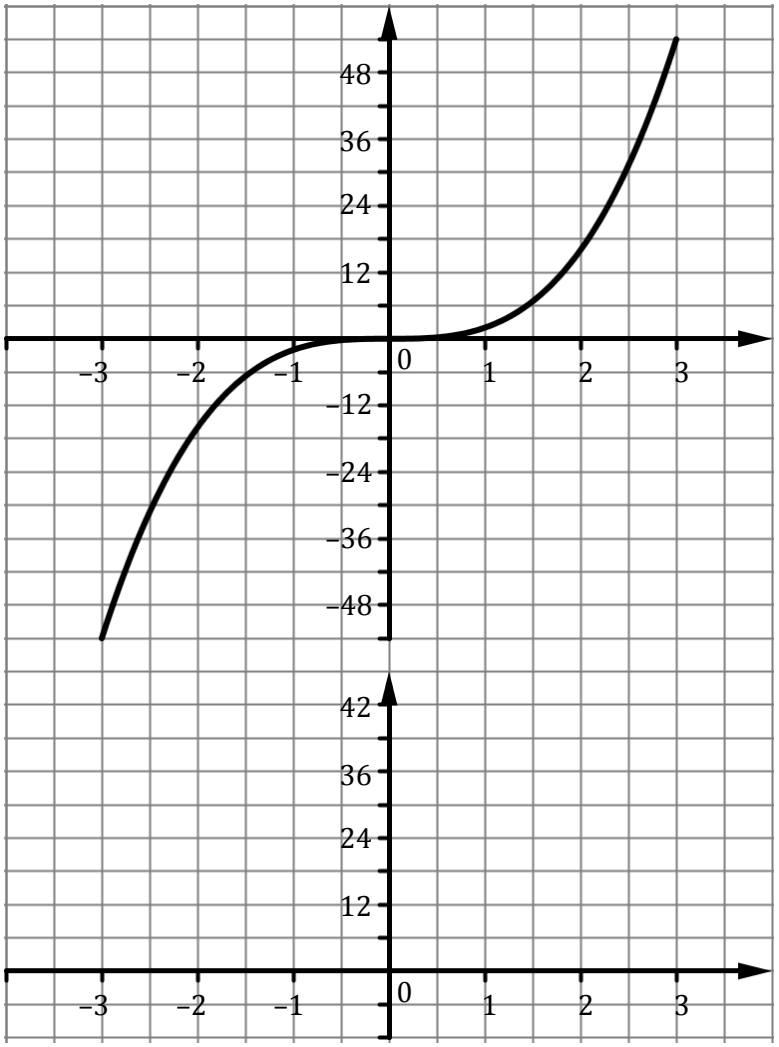
The equation of the slope function, $f'(x)$, is: _____

11. To complete the exercise below refer to the graph of the function $g(x) = 2x^3$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table ***(on the lower graph)*** in the space provided.

x	Slope of Tangent
–3	
–2	
–1	
0	
1	
2	
3	



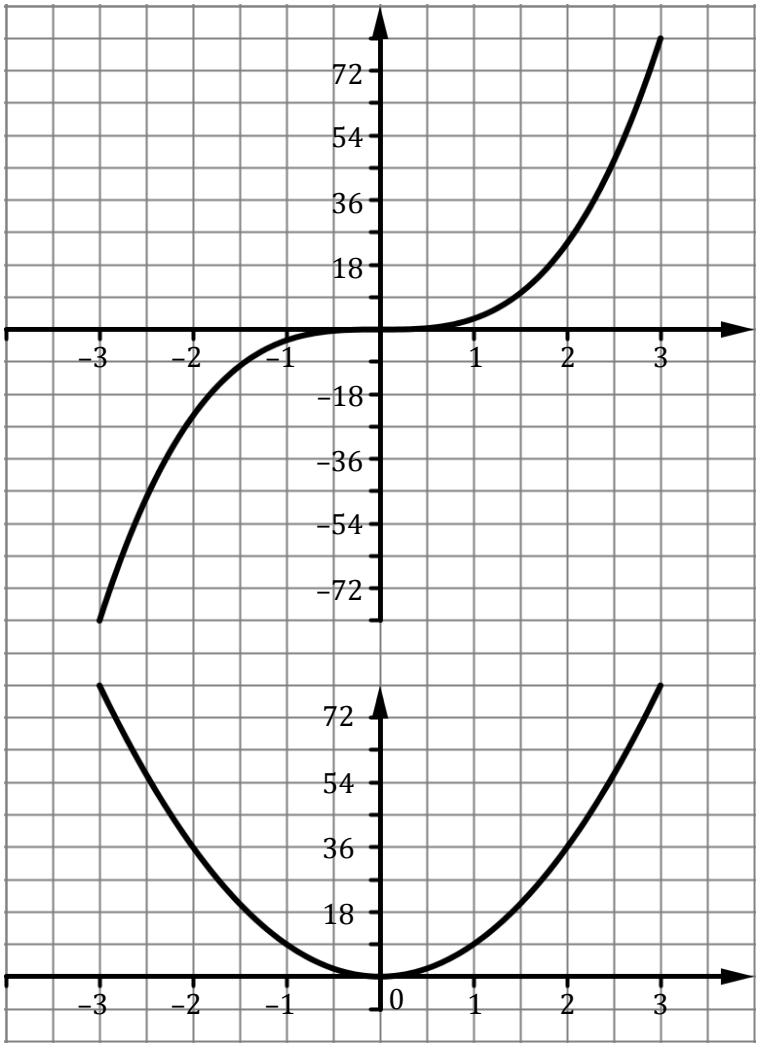
(c) The equation of the slope function, $g'(x)$, is:

12. To complete the exercise below refer to the graph of the function $h(x) = 3x^3$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table ***(on the lower graph)*** in the space provided.

x	Slope of Tangent
–3	81
–2	36
–1	9
0	0
1	9
2	36
3	81



(c) The equation of the slope function, $h'(x)$ is: $9x^2$

13. Refer to questions 10, 11, and 12 in order to fill in the table below.

Function, $f(x)$	Slope Function or $f'(x)$
$f(x) = x^3$	$f'(x) =$
$g(x) = 2x^3$	$g'(x) =$
$h(x) = 3x^3$	$h'(x) =$
⋮	⋮
$f(x) = ax^3$	$f'(x) =$

14. What are the slopes of the functions:

(i) $h(x) = x^8$

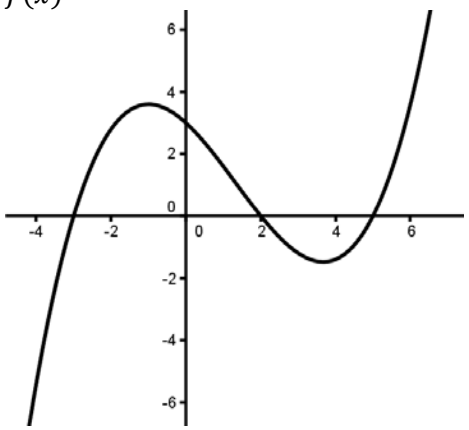
(ii) $g(x) = 3x^{10}$

(iii) $f(x) = 5x^2 - 3x - 6$?

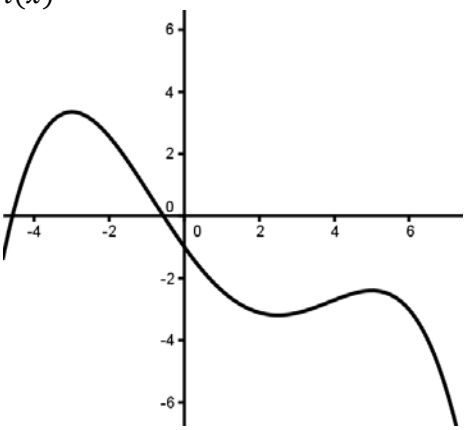
Describing Curves

Pair up. One person describes a function. The other person draws the function on their whiteboard using only the descriptions their partner gives them i.e. they cannot look at this page.

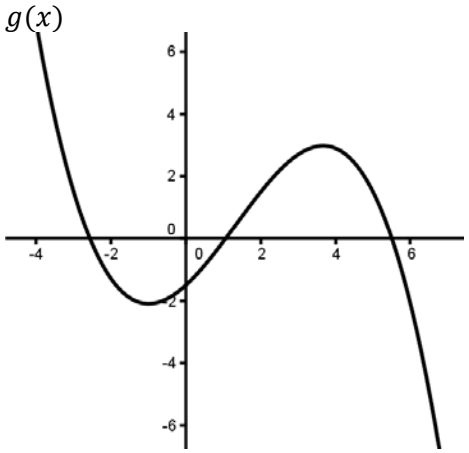
$f(x)$



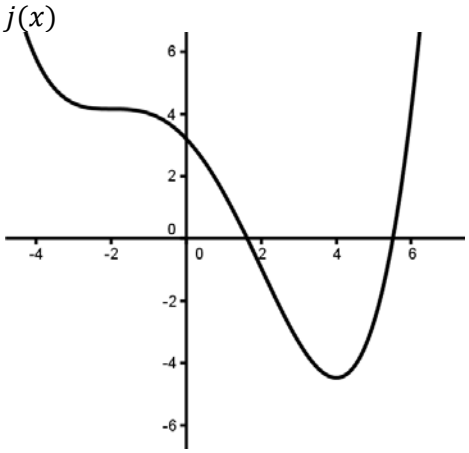
$i(x)$



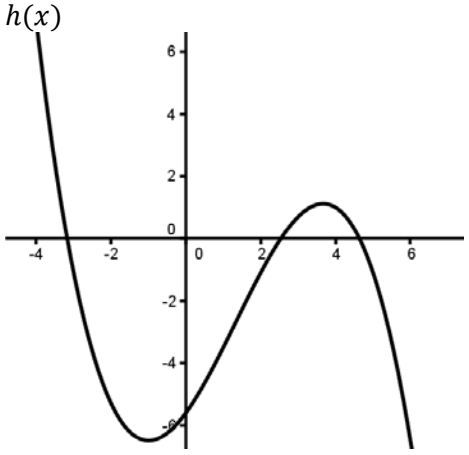
$g(x)$



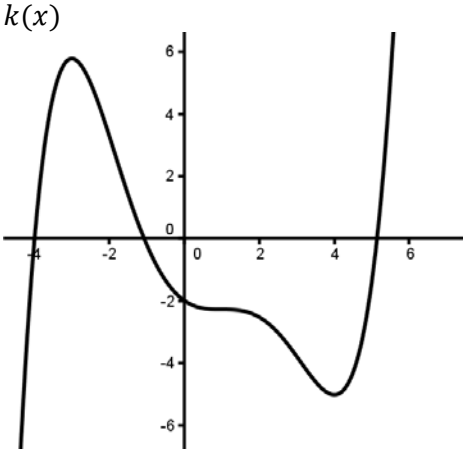
$j(x)$



$h(x)$



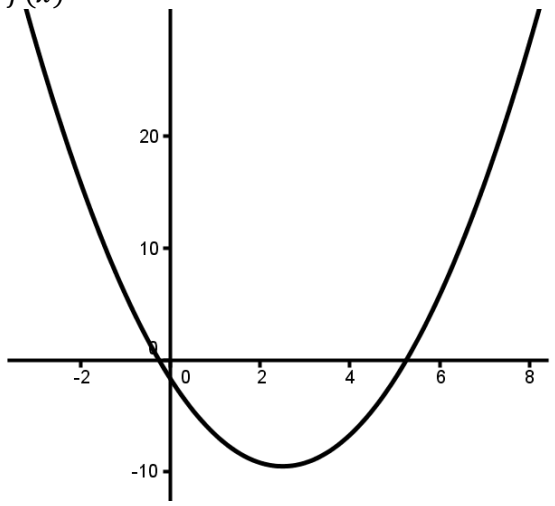
$k(x)$



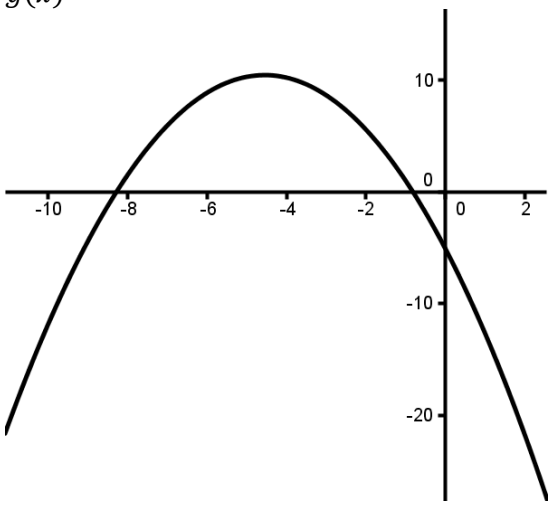
Curve Sketching Derivatives

Sketch the graphs of the first and second derivatives of each function below:

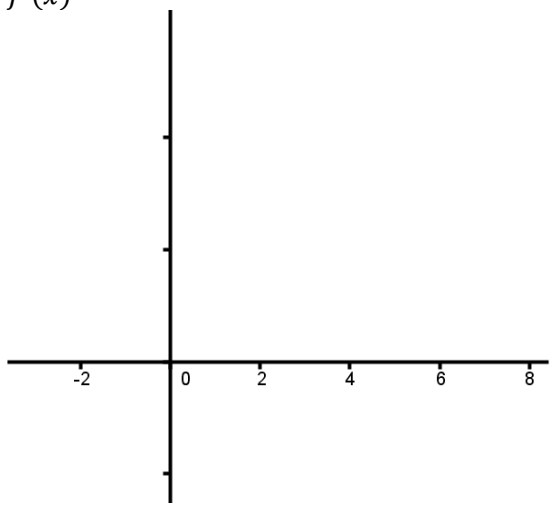
$f(x)$



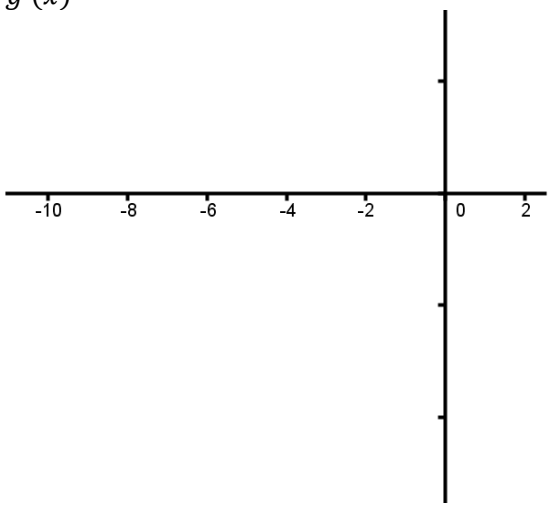
$g(x)$



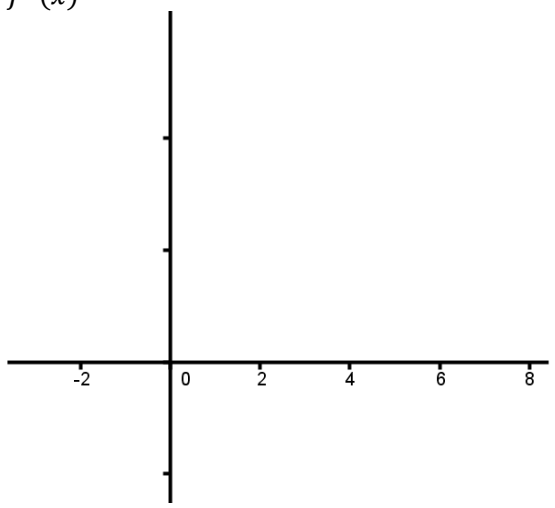
$f'(x)$



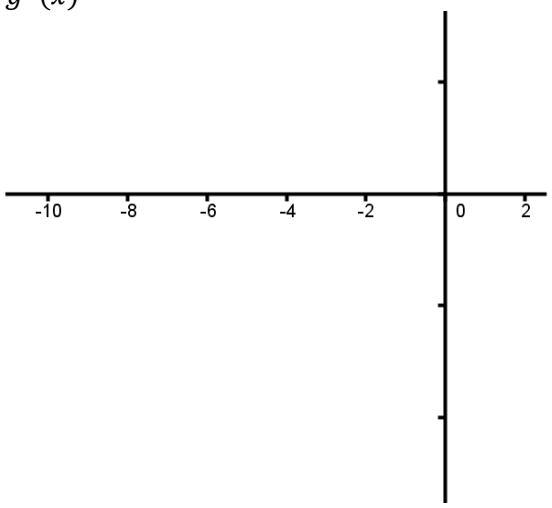
$g'(x)$



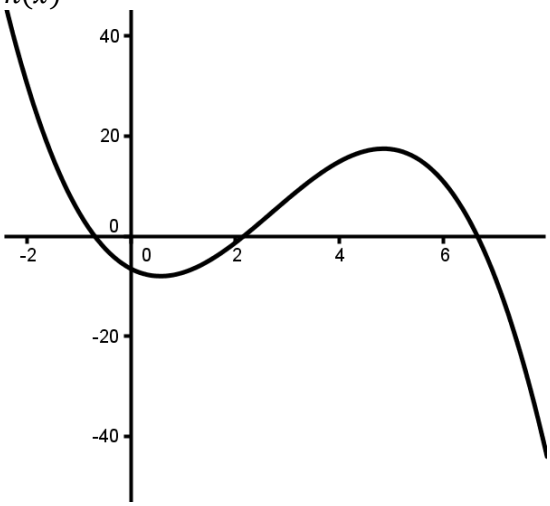
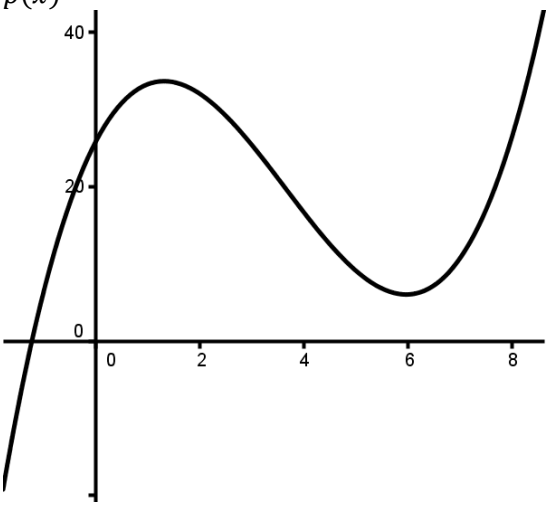
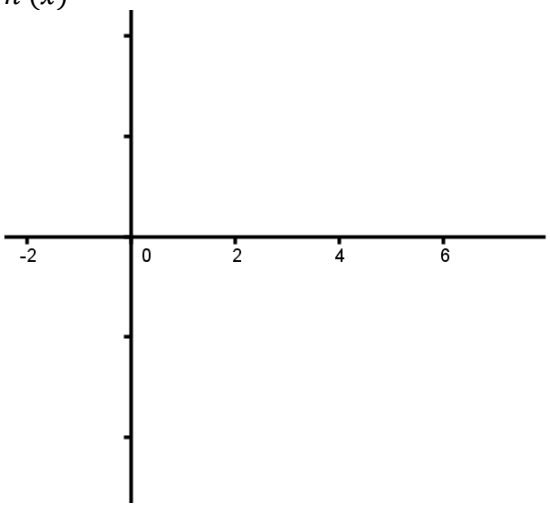
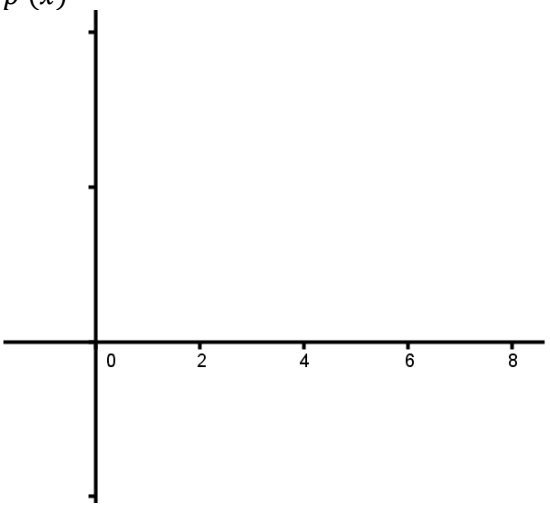
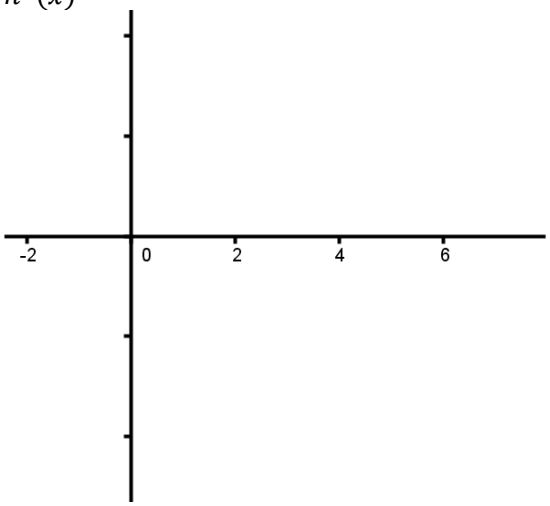
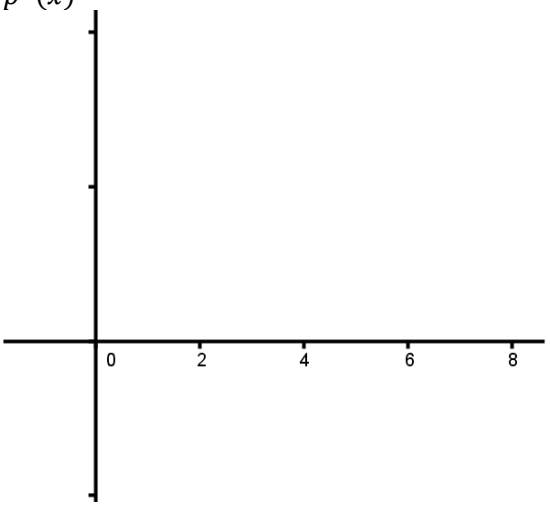
$f''(x)$



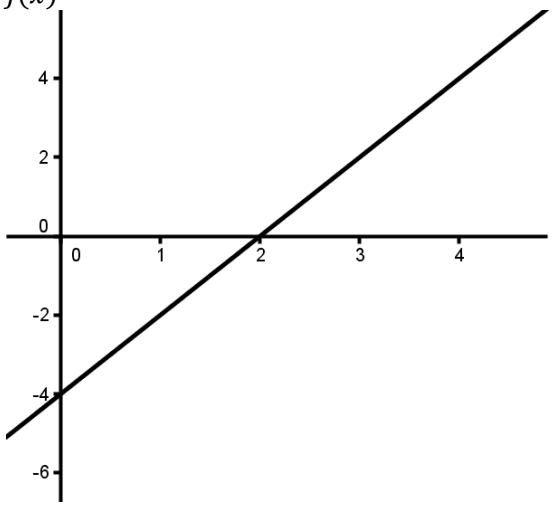
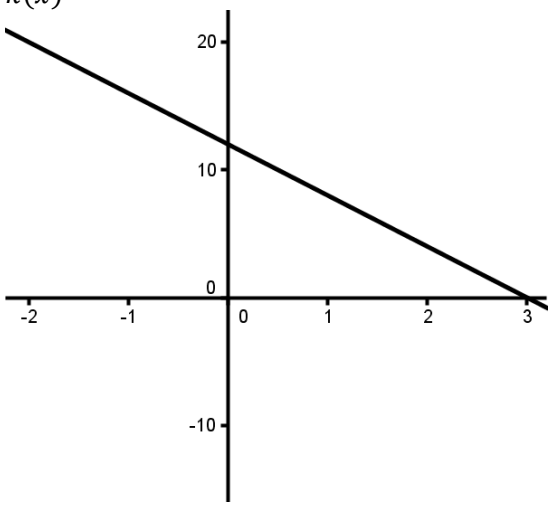
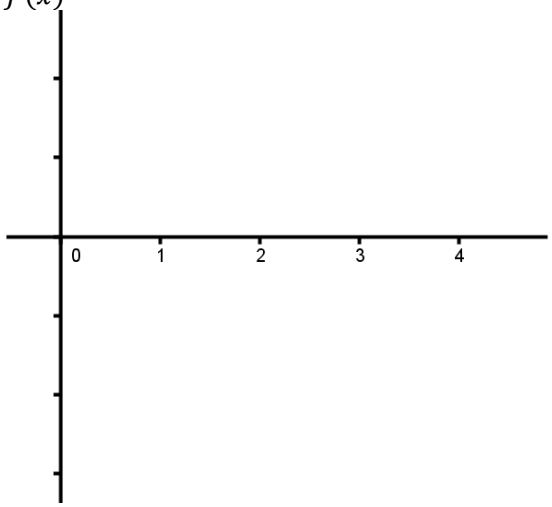
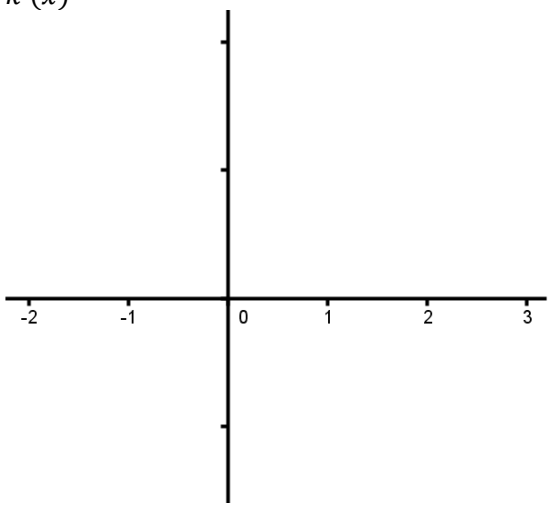
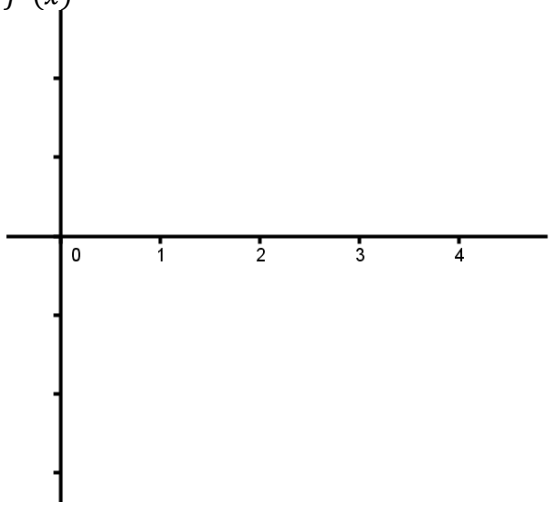
$g''(x)$



Sketch the graphs of the first and second derivatives of each function below:

$h(x)$ 	$p(x)$ 
$h'(x)$ 	$p'(x)$ 
$h''(x)$ 	$p''(x)$ 

Sketch the graphs of the first and second derivatives of each function below:

$j(x)$ 	$k(x)$ 
$j'(x)$ 	$k'(x)$ 
$j''(x)$ 	$k''(x)$