WS08.04   *Exploring the Slope of Tangents and Rates of Change*

1.   Slides with the various functions will be shown. Fill in the following table.

Function	Slope at all points on the function	Equation of the slope function
$f(x) = 3$		$f'(x) =$
$g(x) = 1$		$g'(x) =$
$h(x) = -1$		$h'(x) =$
$y = -3$		$\dfrac{dy}{dx} =$
$\vdots$	$\vdots$	$\vdots$
$f(x) = n$		$f'(x) =$

The derivative of a constant is _____.

2.   Slides with the various functions will be shown. Fill in the following table.

Function	Slope at all points on the function	Equation of the slope function
$f(x) = x$		$f'(x) =$
$g(x) = x + 2$		$g'(x) =$
$h(x) = \dfrac{3}{2}x$		$h'(x) =$
$y = \dfrac{3}{2}x + 3$		$\dfrac{dy}{dx} =$
$p(x) = -x$		
$k(x) = -x + 3$		
$\vdots$	$\vdots$	$\vdots$
$f(x) = nx$ (where n is a constant)		
$f(x) = nx + c$ (where n and c are constants)		

The derivative of $f(x) = nx$ is _____.
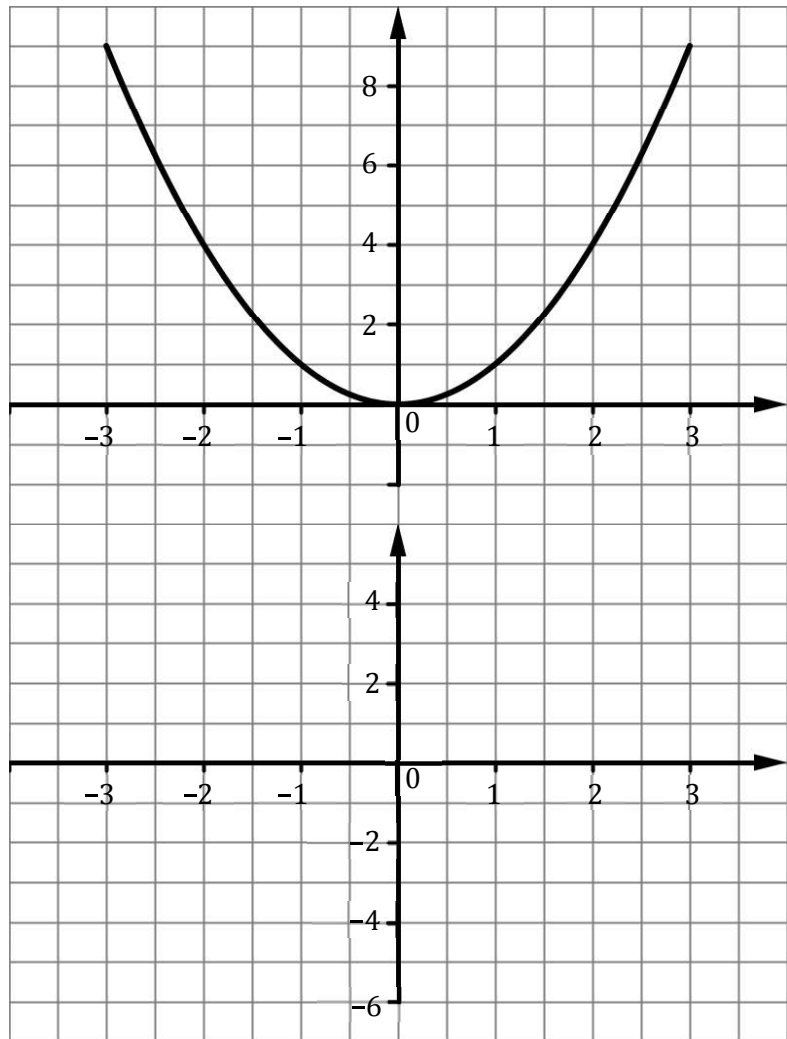
The derivative of $f(x) = nx + c$ is _____.

3. Page 14 has a graph of the function $f(x) = x^2$.

(a) Calculate the slopes of the tangents at the points indicated and enter your answers in the table below.

Hint: A ruler will help you read the slopes of the tangents.

(b) Graph the values from the table *(on the lower graph)* in the space provided.

x	Slope of Tangent
−3	
−2	
−1	
0	
1	
2	
3	

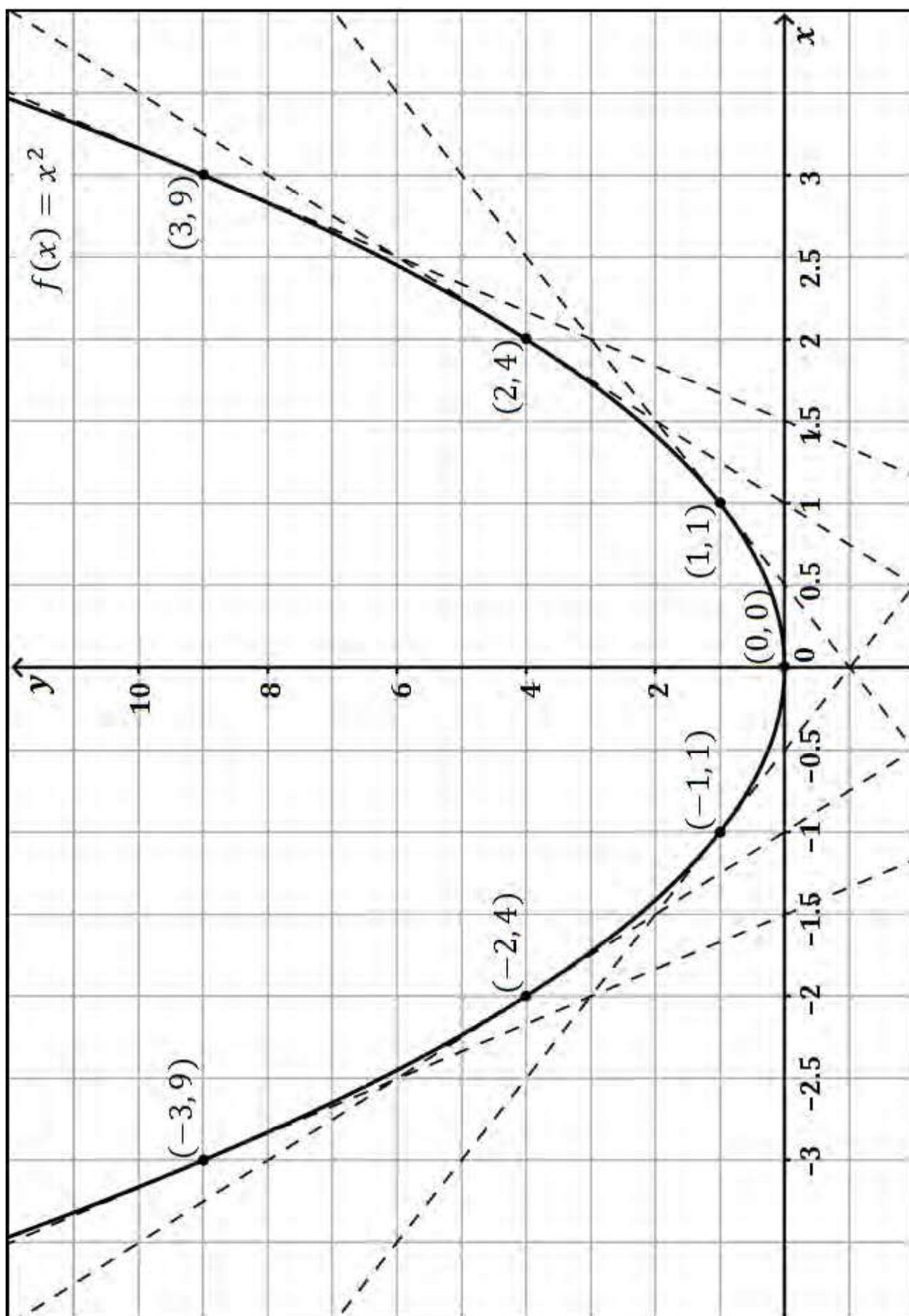**(c)** Which of these represents the slope function?

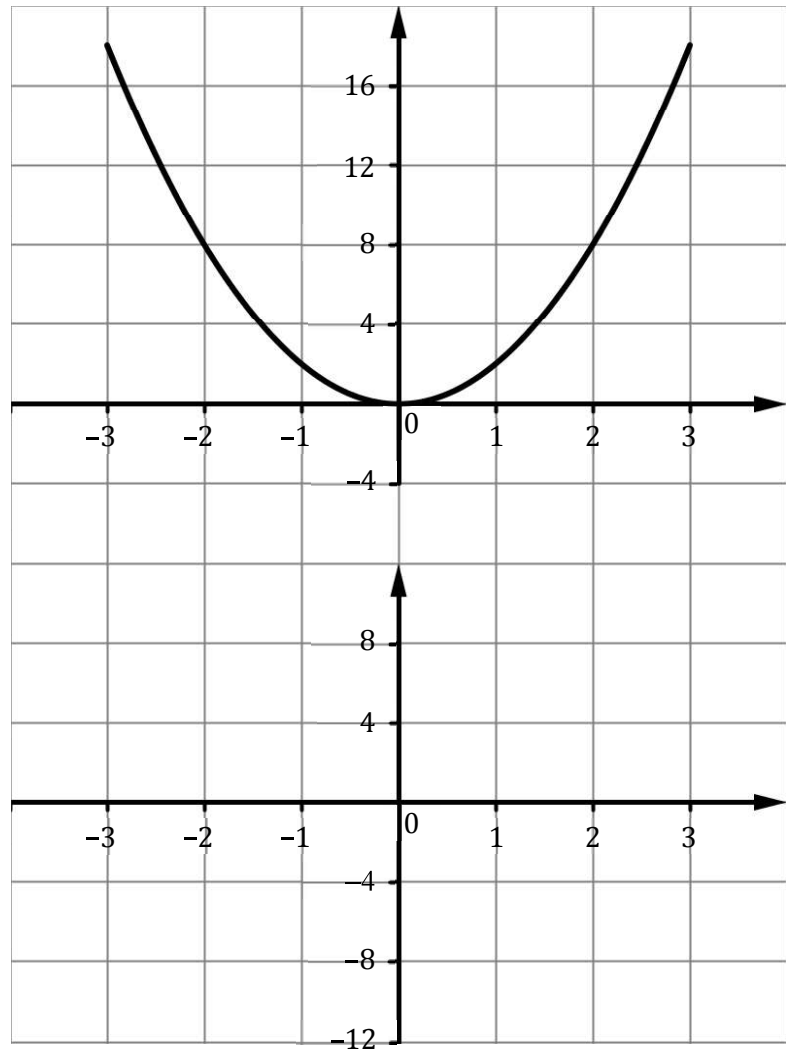**(i)** $f'(x) = x + 2$ **(ii)** $f'(x) = 2$ **(iii)** $f'(x) = 2x$

Give a reason for your answer.

Graph of $f(x) = x^2$

4. To complete the exercise below refer to the graph of the function $g(x) = 2x^2$ which is on the board.

    (a) Fill in the slopes of the tangents at the points indicated and enter your answers in the table below.

    (b) Graph the values from the table *(on the lower graph)* in the space provided.

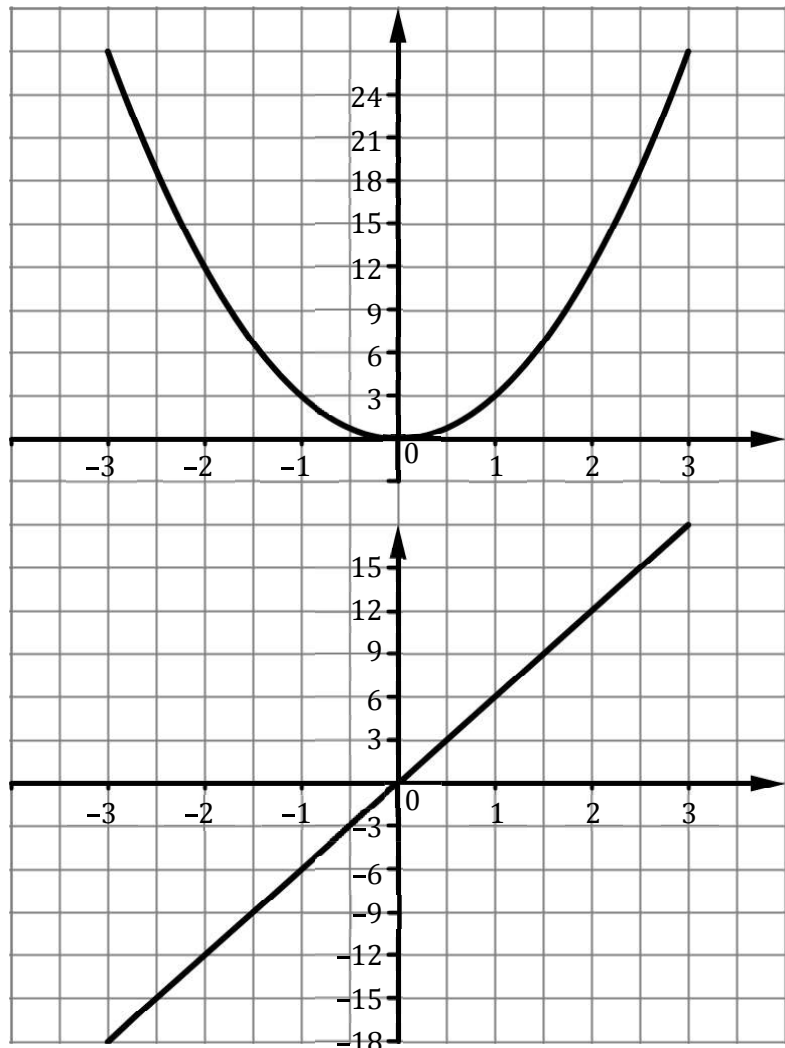| x	Slope of Tangent
−3	
−2	
−1	
0	
1	
2	
3	

(c) The equation of the slope function, $g'(x)$, is:

5. To complete the exercise below refer to the graph of the function $h(x) = 3x^2$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table *(on the lower graph)* in the space provided.

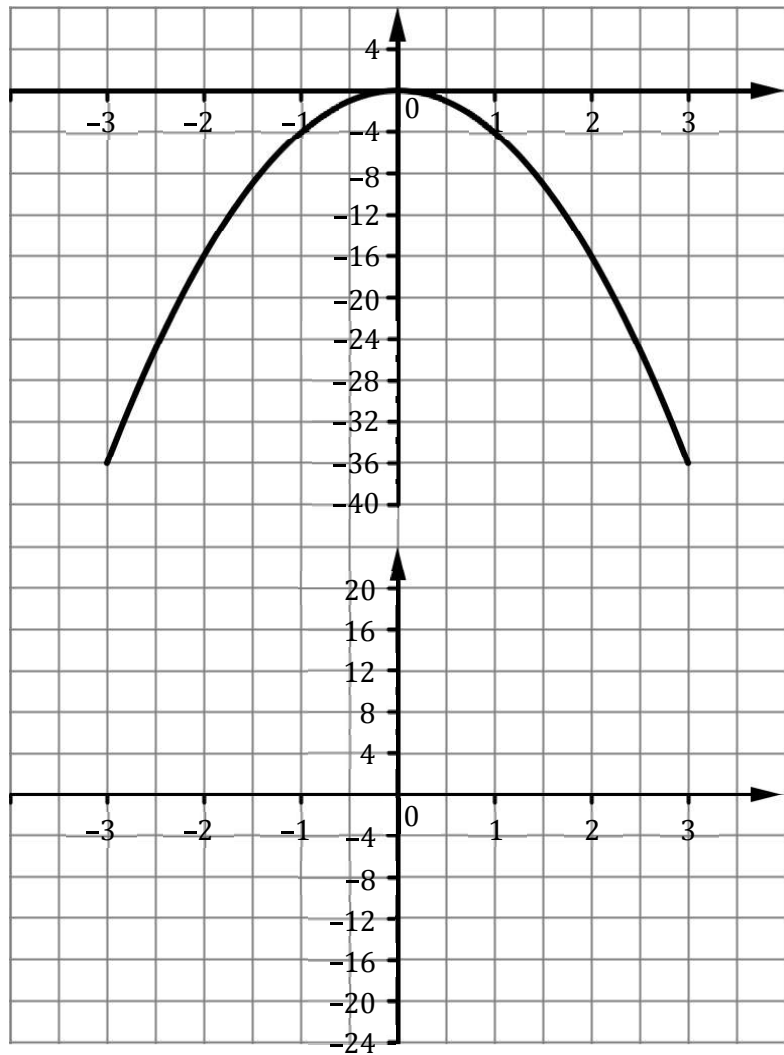| x	Slope of Tangent
−3	−18
−2	−12
−1	−6
0	0
1	6
2	12
3	18

(c) The equation of the slope function, $h'(x)$, is: $h'(x) = 6x$

6. To complete the exercise below refer to the graph of the function $y = -4x^2$ which is on the board.
  (a) Fill in the slopes of the tangents at the points indicated.
  (b) Graph the values from the table *(on the lower graph)* in the space provided.

x	Slope of Tangent
−3	
−2	
−1	
0	
1	
2	
3	

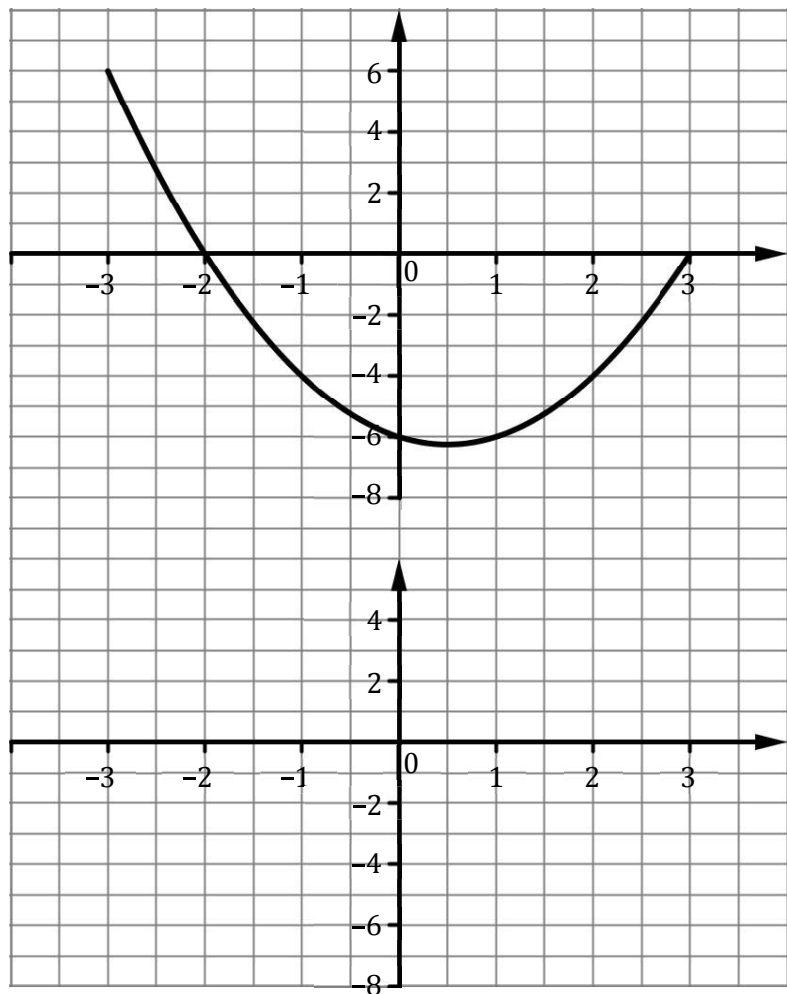 **(c)** The equation of the slope function, $\frac{dy}{dx}$, is:

7.   Refer to questions 3, 4, 5, and 6 in order to fill in the table below.

Function, $f(x)$	Slope of Function or $f'(x)$
$f(x) = x^2$	$f'(x) =$
$g(x) = 2x^2$	$g'(x) =$
$h(x) = 3x^2$	$h'(x) =$
$y = -4x^2$	$\dfrac{dy}{dx} =$
$k(x) = 10x^2$	$k'(x) =$
$\vdots$	$\vdots$
$f(x) = ax^2$	$f'(x) =$

8. To complete the exercise below refer to the graph of the function $f(x) = x^2 - x - 6$ which is on the board.

(a) What do you think the equation of the slope function, $f'(x)$, is?_____

(b) Fill in the slopes of the tangents at the points indicated.

(c) Graph the values from the table *(on the lower graph)* in the space provided.

x	Slope of Tangent
−3	
−2	
−1	
0	
1	
2	
3	

(d) The equation of the slope function, $f'(x)$, is:

(e) Compare this to your prediction in part (a).
What conclusion can be drawn from finding the slope function of this last function?

(f) Patrick wants to know what the slope of $f(x)$ when $x = 1.5$
There are a few ways of estimating/calculating the slope of $f(x)$ when $x = 1.5$.
(i) From the graph of $f(x)$ in the GeoGebra file what would be a good estimate of the slope of $f(x)$ when $x = 1.5$?
(ii) From the list of slopes in the table what would be a good estimate of the slope of $f(x)$ when $x = 1.5$?
(iii) From the graph of $f'(x)$ what would be a good estimate of the slope of $f(x)$ for $x = 1.5$? (Show your workings on the graph)
**(iv)** From the equation $f'(x)$ what is the slope of $f(x)$ when $x = 1.5$?

9. **(a)** What is the equation of the slope function $(f'(x))$ of $f(x) = x^2 - x$?

(b) What is the slope of the tangent of the function $f(x) = x^2 - x$ when $x = 3$?

10. To complete the exercise below refer to the graph of the function $f(x) = x^3$ which is on the board.
   (a) Fill in the slopes of the tangents at the points indicated.
   (b) Graph the values from the table *(on the lower graph)* in the space provided.

x	Slope of Tangent
–3	
–2	
–1	
0	
1	
2	
3	
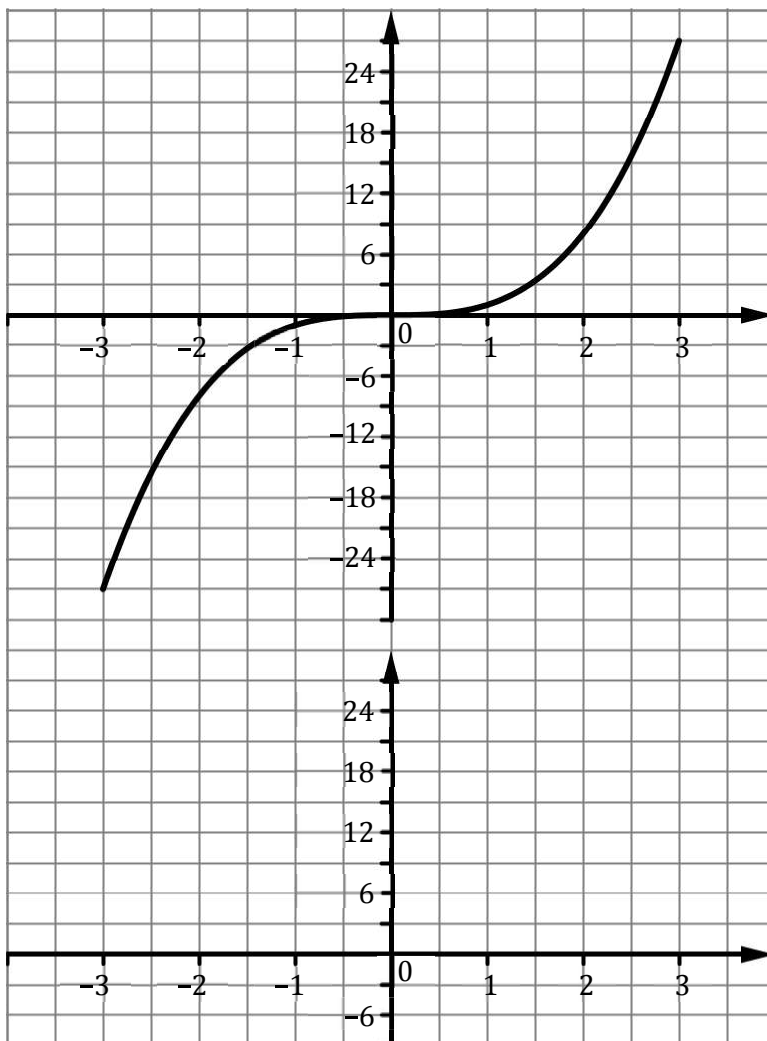
   (c) What is the shape of the slope function?_____

   (d) How could you confirm this?_____

The parent function for quadratics is $y = x^2$.

$y = x^2$ contains the points: $(-3, 9)$, $(-2, 4)$, $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(2, 4)$, $(3, 9)$.

Your slope function contains the points: $(-3, \ )$, $(-2, \ )$, $(-1, \ )$, $(0, \ )$, $(1, \ )$, $(2, \ )$, $(3, \ )$.
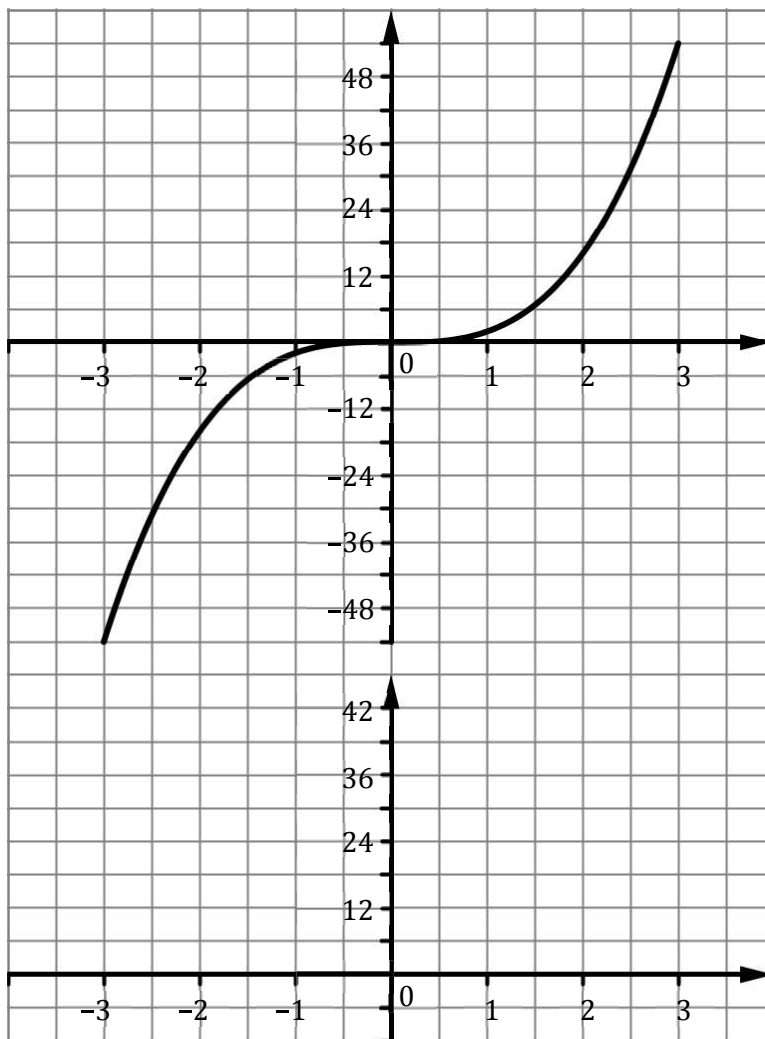
The equation of the slope function, $f'(x)$, is: _____

11. To complete the exercise below refer to the graph of the function $g(x) = 2x^3$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table *(on the lower graph)* in the space provided.

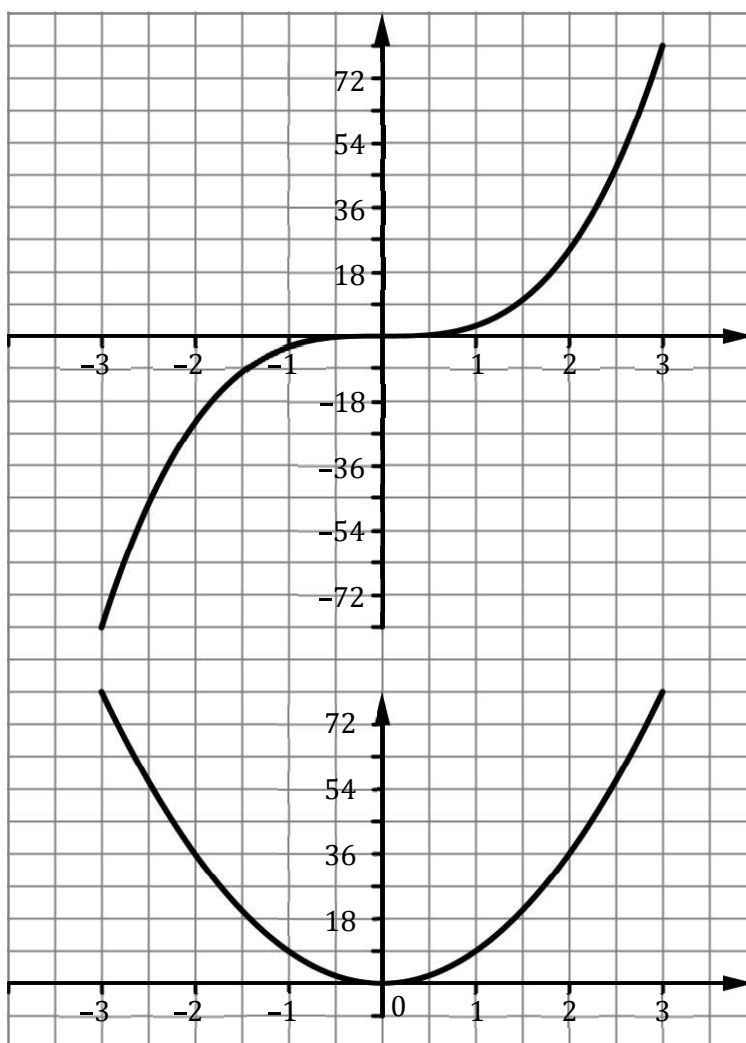| x	Slope of Tangent
−3	
−2	
−1	
0	
1	
2	
3	

(c) The equation of the slope function, $g'(x)$, is:

12. To complete the exercise below refer to the graph of the function $h(x) = 3x^3$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table *(on the lower graph)* in the space provided.

x	Slope of Tangent
−3	81
−2	36
−1	9
0	0
1	9
2	36
3	81

(c) The equation of the slope function, $h'(x)$ is: $9x^2$

13. Refer to questions 10, 11, and 12 in order to fill in the table below.

Function, $f(x)$	Slope Function or $f'(x)$
$f(x) = x^3$	$f'(x) =$
$g(x) = 2x^3$	$g'(x) =$
$h(x) = 3x^3$	$h'(x) =$
$\vdots$	$\vdots$
$f(x) = ax^3$	$f'(x) =$

14. What are the slopes of the functions:
  (i) $h(x) = x^8$

  (ii) $g(x) = 3x^{10}$

  (iii) $f(x) = 5x^2 - 3x - 6$?