

## WS08.02 Informal Introduction to Limits and Continuity of Functions

### Student Activity: Concept of a Limit

**Aim:** To investigate the trends in the values of different functions as  $x$  approaches  $x_0$

Given:  $f(x) = x + 3$ ,  $g(x) = \frac{x^2 - 9}{x - 3}$  and  $h(x) = \frac{1}{x - 3}$ .

1. Fill in the following table:

$x$	2.9	2.99	2.999	2.9999	3	3.0001	3.001	3.01	3.1
$f(x) = x + 3$									
$g(x) = \frac{x^2 - 9}{x - 3}$									
$h(x) = \frac{1}{x - 3}$									

2. When  $x$  approaches 3 (but  $x \neq 3$ ) does the value of each of the following functions approach a fixed value?

If so, find that value.

If not, can you describe what you think is happening?

(i)  $f(x) = x + 3$

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(ii)  $g(x) = \frac{x^2 - 9}{x - 3}$

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(iii)  $h(x) = \frac{1}{x - 3}$

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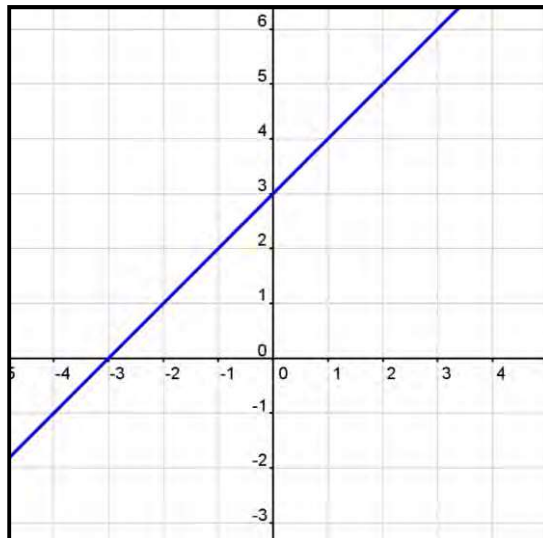


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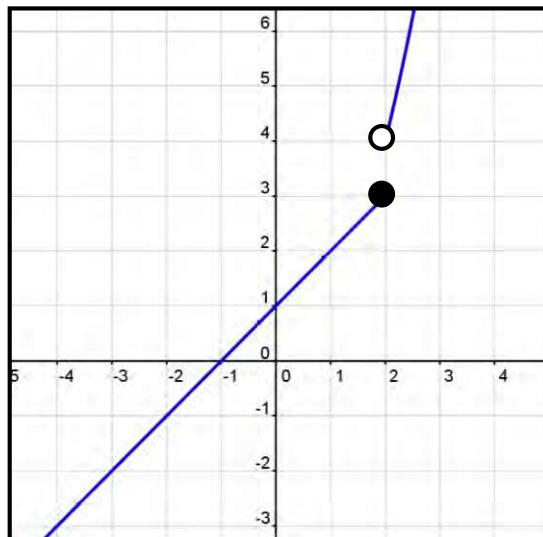
**Task 1**

Analyse each graph below and determine whether the function it represents is continuous:  
If not state why the function is discontinuous.

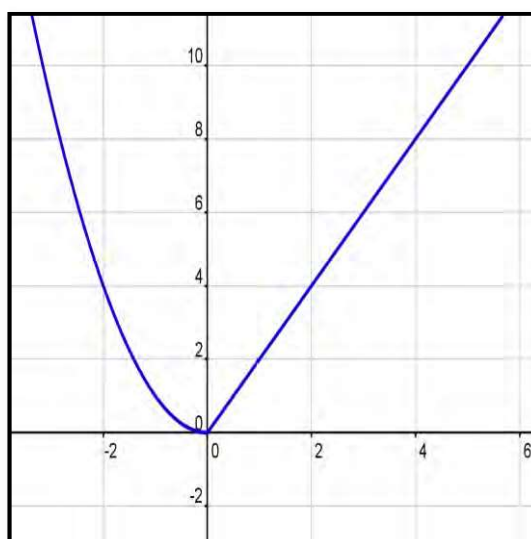
(i)  $f: x \mapsto x+3$



(ii)  $y = \begin{cases} x+1, & \text{if } x \leq 2 \\ x^2, & \text{if } x > 2 \end{cases}$



(iii)  $g(x) = \begin{cases} x^2, & \text{if } x < 0 \\ 2x, & \text{if } x \geq 0 \end{cases}$



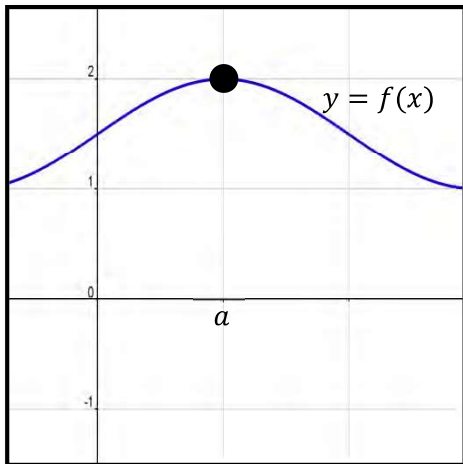
## Task 2

According to each of the following graphs of functions  $f(x)$ :

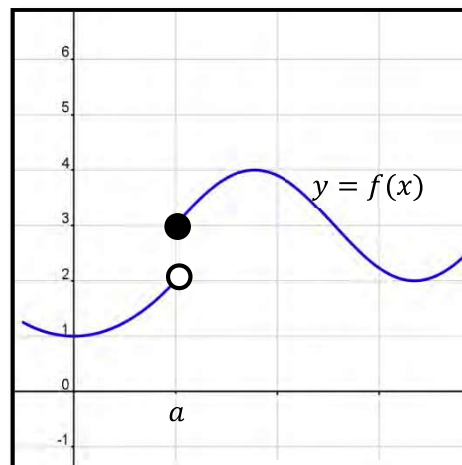
(i) Determine whether  $f(x)$  is continuous at  $x = a$ .

(ii) Find  $\lim_{x \rightarrow a} f(x)$ .

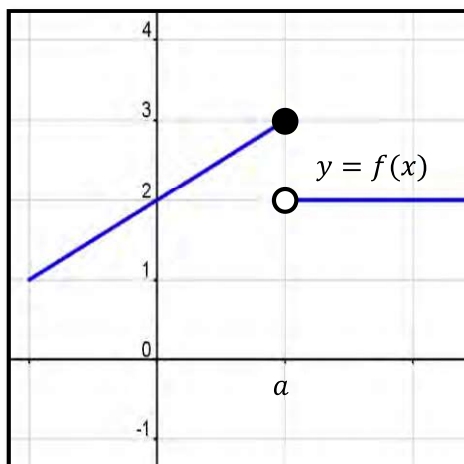
(a)



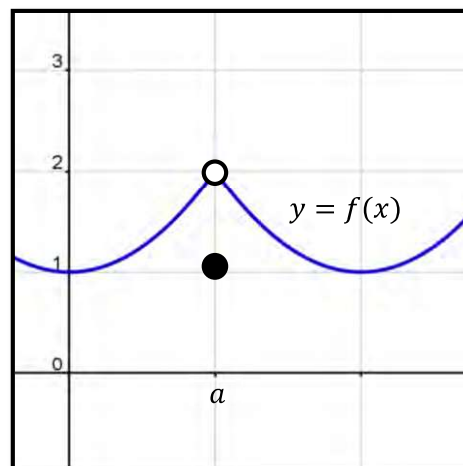
(b)



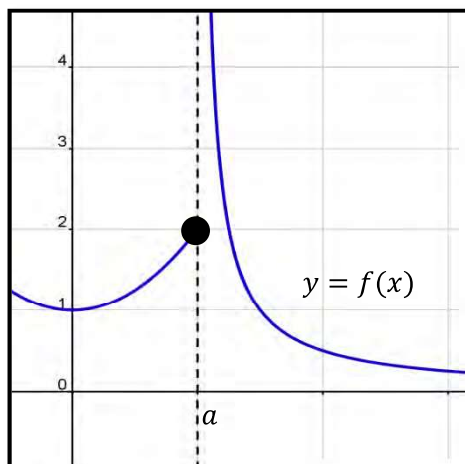
(c)



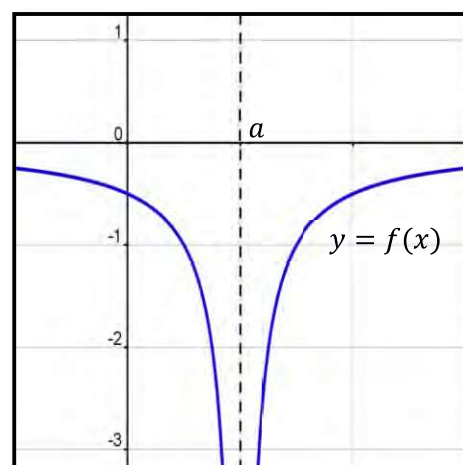
(d)



(e)



(f)



Solutions to **(a)** to **(f)** above:

- |            |            |                        |             |                      |
|------------|------------|------------------------|-------------|----------------------|
| <b>(a)</b> | <b>(i)</b> | Yes continuous at a    | <b>(ii)</b> | Limit is 2           |
| <b>(b)</b> | <b>(i)</b> | No not continuous at a | <b>(ii)</b> | Limit does not exist |
| <b>(c)</b> | <b>(i)</b> | No not continuous at a | <b>(ii)</b> | Limit does not exist |
| <b>(d)</b> | <b>(i)</b> | No not continuous at a | <b>(ii)</b> | Limit is 2           |
| <b>(e)</b> | <b>(i)</b> | No not continuous at a | <b>(ii)</b> | Limit does not exist |
| <b>(f)</b> | <b>(i)</b> | No not continuous at a | <b>(ii)</b> | Limit does not exist |