WS07.02 Exponential Functions

Aim: To study the properties of exponential functions and learn the features of their graphs

Section A - Activity 1: The Exponential Function, $f(x) = 2^x$.

1. For $f(x) = 2^x$:

(i) The base of $f(x) = 2^x$ is _____

(ii) The exponent of $f(x) = 2^x$ is _____

(iii) What is varying in the function $f(x) = 2^x$? _____

(iv) What is constant in the function $f(x) = 2^x$? _____

2. For $f(x) = 2^x$:
What are the possible inputs i.e. values for x (the domain)?

Natural numbers	☐	Irrational numbers	☐
Integers	☐	Real numbers	☐
Rational numbers	☐		

3. Set up a table of values and draw the graph of $f(x) = 2^x$ on your whiteboard:

X	2^x	$y = f(x)$
-4	2^{-4}	$\frac{1}{16}$
-3		
-2		
-1		
0		
1		
2		
3		
4		

4. Describe the graph of $f(x) = 2^x$:

(i) Is it a straight line? _____

(ii) Is $y = f(x)$ increasing or decreasing as x increases? _____

(iii) From the table above, find the average rate of change over different intervals.
For example from -2 to -1 and 2 to 3.

What do you notice? _____

_____

(iv) Describe how the curvature/rate of change is changing. _____

_____

5. For $f(x) = 2^x$:

(i) What are the possible outputs (range) for $f(x) = 2^x$. _____

(ii) Is it possible to have negative outputs? Explain why? _____

_____

(iii) What happens to the output as x decreases? _____

_____

(iv) Is an output of 0 possible? Why do you think this is? _____

_____

(v) What are the implications of this for the x-intercept of the graph? _____

_____

(vi) What is the y-intercept of the graph of $f(x) = 2^x$? _____

_____

Section A – Activity 2: The Exponential Function, $g(x) = 3^x$.

1. For $g(x) = 3^x$:

(i) The base of $g(x) = 3^x$ is _____

(ii) The exponent of $g(x) = 3^x$ is _____

(iii) What is varying in the function $g(x) = 3^x$? _____

(iv) What is constant in the function $g(x) = 3^x$? _____

2. For $g(x) = 3^x$:
What are the possible inputs i.e. values for x (the domain)?

Natural numbers	☐	Irrational numbers	☐
Integers	☐	Real numbers	☐
Rational numbers	☐		

3. Set up a table of values and draw the graph of $g(x) = 3^x$ on your whiteboard:

X	3^x	$y = g(x)$
-4	3^{-4}	$\frac{1}{81}$
-3		
-2		
-1		
0		
1		
2		
3		
4		

4. In relation to the graph of $g(x) = 3^x$:

(i) Is it a straight line? _____

(ii) Is $y = g(x)$ increasing or decreasing as x increases? _____

(iii) From the table above, find the average rate of change over different intervals.
For example from -2 to -1 and 2 to 3.

What do you notice? _____

_____

(iv) Describe how the curvature/rate of change is changing. _____

_____

5. For $g(x) = 3^x$:

(i) What are the possible outputs (range) for $g(x) = 3^x$. _____

(ii) Is it possible to have negative outputs? Explain why? _____

_____

(iii) What happens to the output as x decreases? _____

_____

(iv) Is an output of 0 possible? Why do you think this is? _____

_____

(v) What are the implications of this for the x-intercept of the graph? _____

_____

(vi) What is the y-intercept of the graph of $g(x) = 3^x$? _____

_____

Section A – Activity 3: Compare the graph of $f(x) = 2^x$ with the graph of $g(x) = 3^x$.

1. How are they similar and how do they differ?

2. Consider the relations $\{(x,y) | x \in \mathbb{R},\ y \in \mathbb{R},\ y = 2^x\}$ and $\{(x,y) | x \in \mathbb{R},\ y \in \mathbb{R},\ y = 3^x\}$.

Are they functions? Explain.

3. What name do you think is given to this type of function and why do you think it is given this name?

Section A – Activity 4: Understand the characteristics of $f(x) = a^x$, $a > 1$.

1. What is the domain of $f(x) = a^x$, $a > 1$?

2. In relation to the graph of $f(x) = a^x$, $a > 1$.

(i) Is it a straight line?

(ii) Is $y = f(x)$ increasing or decreasing as x increases?

(iii) Does it have a maximum value?

(iv) Does it have a minimum value?

(v) Describe how its curvature/rate of change is changing.

3. What is the range of $f(x) = a^x$, $a > 1$?

4. What is the x–intercept of the graph $f(x) = a^x$, $a > 1$?

5. What is the y–intercept of the graph $f(x) = a^x$, $a > 1$?

Section B - Activity 1: The Exponential Function, $f(x) = \left(\frac{1}{2}\right)^x$.

1. For $f(x) = \left(\frac{1}{2}\right)^x$:

(i) The base of $f(x) = \left(\frac{1}{2}\right)^x$ is _____

(ii) The exponent of $f(x) = \left(\frac{1}{2}\right)^x$ is _____

(iii) What is varying in the function $f(x) = \left(\frac{1}{2}\right)^x$? _____

(iv) What is constant in the function $f(x) = \left(\frac{1}{2}\right)^x$? _____

2. For $f(x) = \left(\frac{1}{2}\right)^x$:

What are the possible inputs i.e. values for x (the domain)?

Natural numbers	☐	Irrational numbers	☐
Integers	☐	Real numbers	☐
Rational numbers	☐		

3.

Set up a table of values and draw the graph of $f(x) = \left(\frac{1}{2}\right)^x$ on your whiteboard:

X	$\left(1/2\right)^x$	$y = f(x)$
–4	$\left(1/2\right)^{-4}$	16
–3		
–2		
–1		
0		
1		
2		
3		
4		

4. In relation to the graph of $f(x) = \left(\frac{1}{2}\right)^x$:

(i) Is it a straight line? _____

(ii) Is $y = f(x)$ increasing or decreasing as x increases? _____

(iii) From the table above, find the average rate of change over different intervals. For example from -2 to -1 and 2 to 3.

What do you notice? _____

_____

(iv) Describe how the curvature/rate of change is changing. _____

_____

5. For $f(x) = \left(\frac{1}{2}\right)^x$:

(i) What are the possible outputs (range) for $f(x) = \left(\frac{1}{2}\right)^x$. _____

(ii) Is it possible to have negative outputs? Explain why? _____

_____

(iii) What happens to the output as x decreases? _____

_____

(iv) Is an output of 0 possible? Why do you think this is? _____

_____

(v) What are the implications of this for the x-intercept of the graph? _____

_____

(vi) What is the y-intercept of the graph of $f(x) = \left(\frac{1}{2}\right)^x$? _____

_____

Section B - Activity 2: The Exponential Function, $g(x) = \left(\frac{1}{3}\right)^x$.

1. For $g(x) = \left(\frac{1}{3}\right)^x$:

(i) The base of $g(x) = \left(\frac{1}{3}\right)^x$ is _____

(ii) The exponent of $g(x) = \left(\frac{1}{3}\right)^x$ is _____

(iii) What is varying in the function $g(x) = \left(\frac{1}{3}\right)^x$? _____

(iv) What is constant in the function $g(x) = \left(\frac{1}{3}\right)^x$? _____

2. For $g(x) = \left(\frac{1}{3}\right)^x$:

What are the possible inputs i.e. values for x (the domain)?

Natural numbers	☐	Irrational numbers	☐
Integers	☐	Real numbers	☐
Rational numbers	☐		

3.

Set up a table of values and draw the graph of $g(x) = \left(\frac{1}{3}\right)^x$ below on your whiteboard:

X	$\left(\frac{1}{3}\right)^x$	$y = g(x)$
–4	$\left(\frac{1}{3}\right)^{-4}$	81
–3		
–2		
–1		
0		
1		
2		
3		
4		

4. In relation to the graph of $g(x) = \left(\frac{1}{3}\right)^x$:

(i) Is it a straight line? _____

(ii) Is y increasing or decreasing as x increases? _____

(iii) From the table above, find the average rate of change over different intervals.
For example from -2 to -1 and 2 to 3.

What do you notice? _____

_____

(iv) Describe how the curvature/rate of change is changing. _____

_____

5. For $g(x) = \left(\frac{1}{3}\right)^x$:

(i) What are the possible outputs (range) for $g(x) = \left(\frac{1}{3}\right)^x$. _____

(ii) Is it possible to have negative outputs? Explain why? _____

_____

(iii) What happens to the output as x decreases? _____

_____

(iv) Is an output of 0 possible? Why do you think this is? _____

_____

(v) What are the implications of this for the x-intercept of the graph? _____

_____

(vi) What is the y-intercept of the graph of $g(x) = \left(\frac{1}{3}\right)^x$? _____

_____

Section B - Activity 3: Compare the graph of $f(x)=\left(\frac{1}{2}\right)^x$ with the graph of $g(x)=\left(\frac{1}{3}\right)^x$.

1. How are they the same and how do they differ? _____

_____

_____

2. Consider the relations $\left\{(x,y)\middle| x\in\mathbb{R},\ y\in\mathbb{R},\ y=\left(\frac{1}{2}\right)^x\right\}$ and $\left\{(x,y)\middle| x\in\mathbb{R},\ y\in\mathbb{R},\ y=\left(\frac{1}{3}\right)^x\right\}$.

Are they functions? Explain. _____

_____

_____

3. What name do you think we give to this type of function and why do you think it is given this name?

_____

_____

Section B - Activity 4: Understand the characteristics of $f(x)=a^x$, $0<a<1$.

1. What is the domain of $f(x)=a^x$, $0<a<1$? _____

2. In relation to the graph of $f(x)=a^x$, $0<a<1$.

(i) Is it a straight line? _____

(ii) Is $y=f(x)$ increasing or decreasing as x increases? _____

(iii) Does it have a maximum value? _____

(iv) Does it have a minimum value? _____

(iv) Describe how its curvature/rate of change is changing. _____

_____

3. What is the range of $f(x)=a^x$, $0<a<1$? _____

4. What is the x-intercept of the graph $f(x)=a^x$, $0<a<1$? _____

5. What is the y-intercept of the graph $f(x)=a^x$, $0<a<1$? _____

Note: For all the following, you should assume that the domain is $\mathbb{R}$.

Section C - Activity 1: Compare the graph of $f(x) = 2^x$ with the graph of $f(x) = \left(\frac{1}{2}\right)^x$.

1. How are the graphs similar? _____

_____

2. How are the graphs different? _____

_____

3. Rewrite $f(x) = \left(\frac{1}{2}\right)^x$ in the form $f(x) = 2^k$. _____

4. What transformation maps the graph of $f(x) = 2^x$ onto the graph of $f(x) = \left(\frac{1}{2}\right)^x$?

_____

Section C - Activity 2: Compare the graph of $g(x) = 3^x$ with the graph of $g(x) = \left(\frac{1}{3}\right)^x$.

1. How are the graphs similar? _____

_____

2. How do the graphs differ? _____

_____

3. Rewrite $g(x) = \left(\frac{1}{3}\right)^x$ in the form $g(x) = 3^k$. _____

4. What transformation maps the graph of $g(x) = 3^x$ onto the graph of $g(x) = \left(\frac{1}{3}\right)^x$?

_____

1. If $f(x) = a^x$, $a \in \mathbb{R}$, $a > 1$, then the ***properties*** of the exponential ***function*** are:

2. If $f(x) = a^x$, $a \in \mathbb{R}$, $a > 1$, then the ***features*** of the exponential ***graph*** are:

3. If $f(x) = a^x$, $a \in \mathbb{R}$, $0 < a < 1$, then the ***properties*** of the exponential ***function*** are:

4. If $f(x) = a^x$, $a \in \mathbb{R}$, $0 < a < 1$, then the ***features*** of the exponential ***graph*** are:

Section C - Activity 4: Which of the following equations represent exponential functions?

Function	Is it an exponential Function? Yes/No	Reason
$f(x) = \left(\frac{1}{2}\right)^x$		
$f(x) = x^2$		
$f(x) = (-2)^x$		
$f(x) = 2(3)^x$		
$f(x) = -2^x$		
$f(x) = 3(x)^{\frac{1}{2}}$		
$f(x) = (0.9)^x$		

Note: Extension Activities are required to strengthen students' abilities in the following areas from the syllabus:

Level	*Syllabus*	*Page*
JCHL	$f(x) = a2^x$ and $f(x) = a3^x$, where $a \in \mathbb{N}, x \in \mathbb{R}$.	Page 31
LCFL	$f(x) = a2^x$ and $f(x) = a3^x$, where $a \in \mathbb{N}, x \in \mathbb{R}$.	Page 32
LCOL	$f(x) = ab^x$, where $a \in \mathbb{N}$, $b, x \in \mathbb{R}$.	Page 32
LCHL	$f(x) = ab^x$, where a, b, $x \in \mathbb{R}$.	Page 32

1. A cell divides itself into two every day. The number of cells C after D days is obtained from the function:

$$C = 2^D$$

(a) Draw a graph of the function for $0 \leq D \leq 6$.
(b) Find the number of cells after 15 days.

2. The value of a mobile phone M (in cents) after T years can be obtained from the following function:

$$M = k\left(\frac{1}{2}\right)^T, \text{where } k \text{ is a constant.}$$

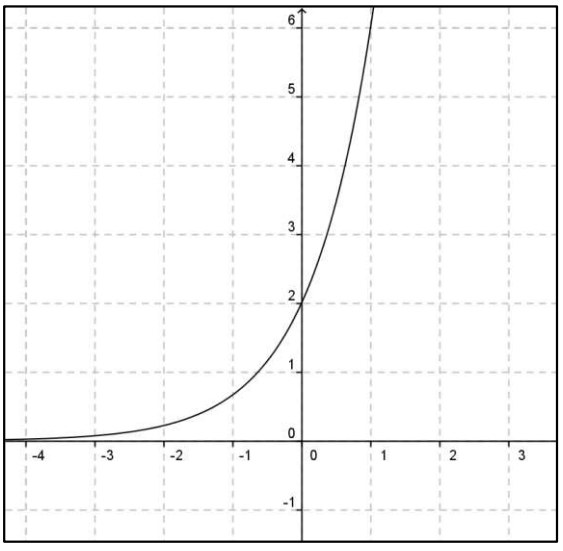
(a) Draw a graph of the function for $0 \leq T \leq 6$.
(b) Find the value of k given that the value of the mobile phone after 3 years is €100.
(c) Find the value of the phone after 7 years.

3. The number of bacteria B in a sample after starting an experiment for m minutes is given by:

$$B = 50(3)^{0.04m}$$

(a) Find the number of bacteria in the sample at the start of the experiment.
(b) Find the number of bacteria in the sample after starting the experiment for 3 hours.

4. The graph of $f(x) = ka^x$ is shown:
(a) Find the value of k and a.
(b) Hence find the value of $f(x)$ when $x = 8$.



Q5. Olive finds that the number of bacteria in a sample doubles every 5 hours. Originally there are 8 bacteria in the sample.

Complete the table below:

Number of hours (*hrs.*)	Number of bacteria (*b*)
0	8
5	
10	
15	

(a) Express *b* in terms of *h*.

(b) Find the number of bacteria in the sample after 13 hours.

(c) How many hours later will the number of bacteria be more than 100.

Q6. When a microwave oven is turned on for x minutes the relationship between the temperature C° inside the oven is given by $C(x) = 500 - 480(0.9)^x$ where $x \geq 0$.

(a) Find the value of $C(0)$.

(b) Explain the meaning of $C(0)$.

(c) Can the temperature inside the microwave oven reach $550\,C^\circ$?

Answers

Q1 (b) 32,768 cells, **Q2 (b)** €800 **(c)** €6.25, **Q3 (a)** 50 **(b)** 136,220 bacteria, **Q4 (a)** $k = 2$, $a = 3$ **(b)** 13,122, **Q5 (b)** $b = 8(2)^{\frac{h}{5}}$ **(c)** 48 bacteria **(d)** 18.22 hrs., **Q6 (a)** 20° C **(c)** No