

Student Activities for Theorem 15: Converse of Pythagoras' Theorem

Target Students: Junior Cycle
Ordinary/Higher.

Prior Knowledge: Theorems 1 - 14

Duration: 40 minutes.

Lesson Introduction:

In this lesson, we are going to explore further the Theorem of Pythagoras which we met in the last lesson. This theorem stated that the square on the hypotenuse in a right-angled triangle is equal to the sum of the squares on the other two sides.

Today we want to explore whether the theorem holds true in reverse. In other words:

Suppose the three sides of a triangle are such that the square of the biggest = the sum of the squares on the other two sides. Does this mean that we know for certain that this is a right-angled triangle?

Do you think so?

Is it possible that it might not be true?

It is important for you to realise that some statements when "put in reverse" as it were, are not true. In such cases one statement is known as the converse of the other.

Example: If I say that "All dogs are animals" does that mean that "All animals are dogs"?
[NO]

Example: If I say that "All rooks are black birds" does that mean that "All black birds are rooks"?
[NO]

Student Activity 1 (i) Statement: All congruent triangles have equal areas

State the converse statement.

Is it true?

(ii) Statement: If a quadrilateral is a rhombus then its diagonals are perpendicular.

State the converse.

Is it true?

(iii) All squares are rectangles.

State the converse.

Is it true?

So, let us explore whether the converse of Pythagoras is true.

Student Activity 2: Working in pairs, I want you to get three numbers, one bigger than the other two (the smaller ones could be equal, but probably not), so that the *(big no)*² = sum of the squares of the other two numbers. E.g. try this: *Is* $(1)^2 + (2)^2 = 3^2$?No

Now try some more.

Any answers, please? 3,4,5 Good. $(3)^2 + (4)^2 = 9 + 16 = 25 = 5^2$

Any other answers: 6,8,10 Good $(6)^2 + (8)^2 = 36 + 64 = 100 = 10^2$

Any more? 5,12,13 yes $(5)^2 + (12)^2 = 25 + 144 = 169 = 13^2$

Student Activity 3: Now I want you to use your geometry instruments to draw the three triangles with the three sets of dimensions you have just worked with.

Now measure the biggest angle in each triangle.

What size is it? [A right angle each time].

Going by these results, what do we think? Is the converse of Pythagoras true?

It would appear to be so.

We could investigate further, by getting more and more triangles. Let us try a few more.

Can we come up with any more triples? How many are there anyway?

Well, start again at 3,4,5. Here we have $(3)^2 + (4)^2 = 5^2$ which is correct as we know.

Now multiply each by 3. We get 9, 12, 15 $(9)^2 + (12)^2 = 81 + 144 = 225 = 15^2$

Now multiply each by 4. We get 12, 16, 20 $(12)^2 + (16)^2 = 144 + 256 = 400 = 20^2$

In fact we only have to multiply one triple by any number and we get a new one.

How many all together? An infinity of them. You can make as many as you like. In fact Euclid proved this fact around 300 B.C.

Student Activity 4: Now I want you to use your geometry instruments to draw the two triangles with these three sets of dimensions. i.e. (a) 9cm., 12cm., 15cm. and
(b) 12cm., 16cm., and 20cm.

Now measure the biggest angle in each triangle.

What size is it? A right angle each time.

The teacher could now take one of two optional courses in the lesson depending on the student's understanding of the material done so far.

Option 1 is for the students who have full understanding and need no further examples. This option gives these students some insights into the ways that maths can be so mysterious when a 2 becomes a 3 or a 4 or any other natural number bigger than it. i.e. the fact that $x^2 + y^2 = z^2$ has an infinite number of solutions but that $x^n + y^n = z^n$ for $n > 2$ has no solutions at all.

The human element of maths can also be shown in the story of Andrew Wiles and his lonely journey towards a proof which had escaped mankind for three and a half centuries.

Option 1:

Story of Fermat's Last Theorem: Fermat 1637 to Andrew Wiles 1995

Option 2: This option would be more suited to the student who needs further work towards an understanding of the theorem.

Student Activity 5

Q.1 Consider the numbers 8, 15.

Calculate $(8)^2 + (15)^2$

What is the number n where $(8)^2 + (15)^2 = n^2$? _____

Now construct the triangle with sides 8 cm, 15 cm and n cm.

What is the size of the biggest angle? _____

Q.2

Consider the two numbers 2.5 and 6.

Calculate $(2.5)^2 + (6)^2$

What is the number n where $(2.5)^2 + (6)^2 = n^2$? _____

Now construct the triangle with sides 2.5 cm, 6 cm and n cm.

What is the size of the biggest angle? _____