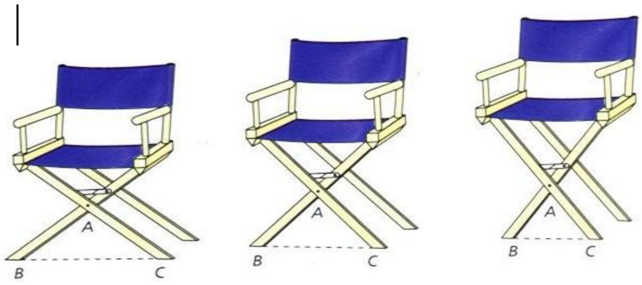
Student Activities for Theorem 7

The angle opposite the greater of two sides is greater than the angle opposite the lesser.

Resources needed: Compass, Ruler, Protractor and sharp pencil



Which of these chairs is the most stable? Can you explain why?

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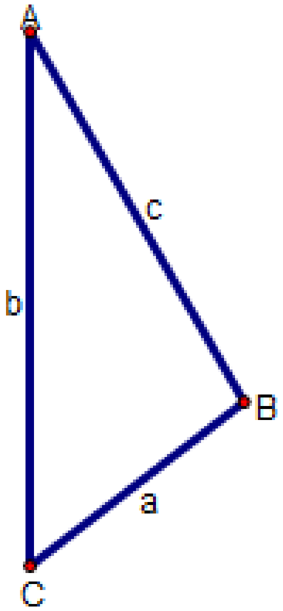
Investigate if there is a relationship between the measures of the sides and angles in a triangle.

Work in pairs.



(i) Measure the lengths of the sides a, b, and c in cm and the angles A,B,C in degrees

$|a| =$ $\quad$ $|b| =$ $\quad$ $|c| =$

$|\angle A| =$ $\quad$ $|\angle B| =$ $\quad$ $|\angle C| =$

Name the longest side: _______ Name the largest angle:___________

Name the shortest side: _______ Name the smallest angle: ___________

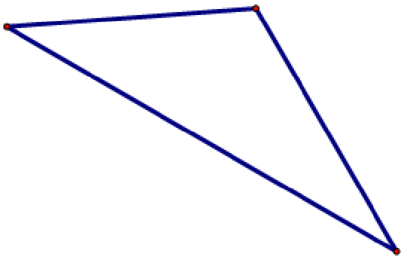
Name the median side: _______ Name the median angle: ___________

Of the two sides, largest and smallest, which has the largest angle opposite? ________________

Of the two sides, largest and median, which has the largest angle opposite? ________________

Of the two sides, median and smallest, which has the largest angle opposite? ________________

(ii) Label the angles and sides using the letters D, E, F and d, e, f, and fill in the measurements.



$|d| =$ $|e| =$ $|f| =$

$|\angle D| =$ $|\angle E| =$ $|\angle F| =$

Name the longest side: _______Name the largest angle:____________

Name the shortest side: ________Name the smallest angle: ___________

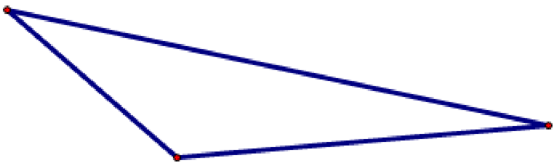
Name the median side: _________Name the median angle: ____________

Of the two sides, largest and smallest, which has the largest angle opposite it? ________________

Of the two sides, largest and median, which has the largest angle opposite it? ________________

Of the two sides, median and smallest, which has the largest angle opposite it? ________________

(iii) Label the angles and sides using the letters G, H, I and g, h, i and fill in the measurements



$|g| =$ $|h| =$ $|i| =$

$|\angle G| =$ $|\angle H| =$ $|\angle I| =$

Name the longest side: _______Name the largest angle:___________

Name the shortest side: _______Name the smallest angle: ___________

Name the median side: _______Name the median angle: ___________

Of the two sides, largest and smallest, which has the largest angle opposite it? ________________

Of the two sides, largest and median, which has the largest angle opposite it? ________________

Of the two sides, median and smallest, which has the largest angle opposite it? ________________

What pattern have you noticed regarding relationship between the measures of sides and angles in a triangle?

____

____

____

____

(iv)Draw any triangle, except an equilateral triangle, to see if the observed pattern continues.

(Why not draw an equilateral triangle? ____)

Label the sides and angles using the above convention. Measure the lengths of all the sides and angles in the triangle.

Length of side/cm			
Measure of angle opposite/^{0}			

Name the longest side: _______Name the largest angle:___________

Name the shortest side: _______Name the smallest angle: ___________

Name the median side: _______Name the median angle: ___________

Of the two sides, largest and smallest, which has the largest angle opposite it? ________________

Of the two sides, largest and median, which has the largest angle opposite it? ________________

Of the two sides, median and smallest, which has the largest angle opposite it? ________________

From the above triangles it appears that for any 2 sides in the triangles the angle opposite the larger (greater) of the 2 sides is ___________than the angle opposite the smaller (lesser)of the 2 sides.

Do the above examples prove that this is always the case? Explain

**Refer back to the question on the director's chair, and use what you have learned to answer the question.

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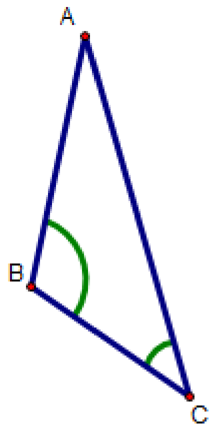
To Prove: In a triangle, the angle opposite the greater of 2 sides is greater than the angle opposite the lesser of 2 sides.



Given: |AC| > |AB| prove that |<ABC| > |<ACB|

- Using a compass construct a point D on AC such that |AD|=|AB|

What type of triangle is ABD? ____________________

- Shade in and write down the angles in triangle ABD which are equal.

_________________________________ (i)

Why are they equal? ____

<ADB is an ___________________ angle for triangle BDC.

Use a previous theorem to write down |<ADB| in terms of 2 angles in triangle BDC.

|<ADB| =____

Shade in those 2 angles in different colours.

Write down the relationship between |<ACB| and |< ADB| (bigger /smaller)

____ (ii)

Hence write down the relationship between |<ACB| and |< ABD| (using (i))

____

Hence write down the relationship between |<ACB| and |<ABC|

____

Hence write down the relationship between |<ABC| and |<ACB| (in this order)

____

Concept of "Converse" (students at the Relational level of the Van Hiele levels of geometric reasoning, can recognise the difference between a statement and its converse)

The **Converse** of "**If** A, **then** B" is the assertion "**If** B, **then** A".

For example, the converse of "If it is my car, then it's silver" is "If the car is silver, then its mine."

From this example we see that there is no guarantee that the converse of a true statement is true.

Given the statements below, fill in whether they are true/false, fill in their converses, and whether the converses are true or false.

Statement	**True/False**	**Converse**	**True/False**
If I live in Dublin, then I live in Ireland.			
A triangle is a polygon with three sides			
If an angle is a right angle then its measure is 90^0.			
If 3 points are collinear, then they lie on the same line.			
A square is figure with four right angles.			
In a triangle the angle opposite the greater of two sides is greater than the angle opposite the lesser of the two sides.			

Investigating the converse of theorem 7

Refer back to Pages 1 and 2, triangles (i), (ii), (iii), and (iv)

Fill in from (i)

$|<A| =$ $\qquad |<B| =$ $\qquad |<C| =$

$|a| =$ $\qquad |b| =$ $\qquad |c| =$

Of the two angles, largest and smallest, which has the largest side opposite it? ________________

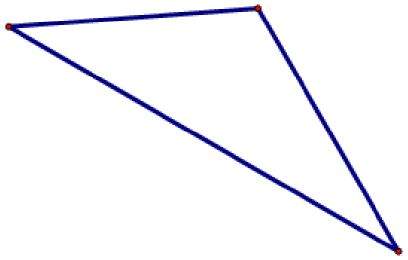
Of the two angles, largest and median, which has the largest side opposite it? ________________

Of the two angles, median and smallest, which has the largest side opposite it? ________________

Is the greater side opposite the greater angle?

Repeat this for triangles (ii), (iii), and (iv), on Pages 1 and 2

(ii) Label the angles and sides using the letters D, E, F and d, e, f, and fill in the measurements.



$|\angle D| =$ $\quad |\angle E| =$ $\quad |\angle F| =$

$|d| =$ $\quad |e| =$ $\quad |f| =$

Name the largest angle: __________ _Name the longest side: ________

Name the smallest angle: _________Name the shortest side: _________

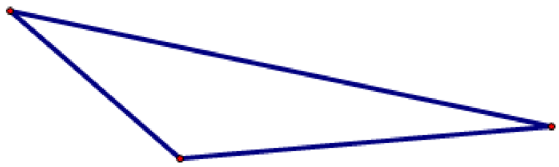
Name the median angle: __________Name the median side: ____________

Of the two angles, largest and smallest, which has the largest side opposite? _________________

Of the two angles, largest and median, which has the largest side opposite? _________________

Of the two angles, median and smallest, which has the largest side opposite? _________________

(iii) Label the angles and sides using the letters G, H, I and g, h, i and fill in the measurements



$|\angle G| =$ $\quad |\angle H| =$ $\quad |\angle I| =$

$|g| =$ $\quad |h| =$ $\quad |i| =$

Name the largest angle: __________ _Name the longest side: ________

Name the smallest angle: _________Name the shortest side: _________

Name the median angle: __________Name the median side: ____________

Of the two angles, largest and smallest, which has the largest side opposite it? _________________

Of the two angles, largest and median, which has the largest side opposite it? _________________

Of the two angles, median and smallest, which has the largest side opposite it? _________________

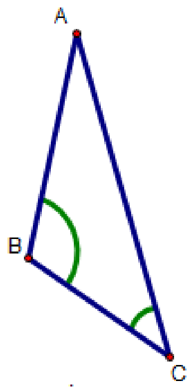
What pattern have you noticed regarding the relationship between the measures of angles and sides in a triangle?____

____

____

Proof of the converse of Theorem 7 – We use **proof by contradiction** (students at higher level leaving cert are expected to know the meaning of this term)

(Note on proof by contradiction: Assume a statement is not true and show that this assumption leads to a contradiction – called *reduction as absurdum* (reduction to absurdity) in Latin.)



To Prove: The side opposite the greater of two angles in a triangle is greater than the side opposite the lesser of two angles.

Given: |<ABC| > |<ACB|

To Prove: |AC| > |AB|

Proof: Assuming that **|AC| is not greater than |AB|,** what are the **only** other options for the relationship between **|AC|** and **|AB|?**

Option 1: ___________________________

Option 2: ___________________________

If option 1 is true draw the triangle which would represent option 1.

Hence what type of triangle is triangle ABC? ______________

Hence what is the relationship between the |<ABC| and |<ACB|? __________________

Is this in agreement with or does it contradict, what we were given? ____________________

Hence, can option 1, i.e. ___________________, be true? _______

If option 2 is true draw the triangle which would represent option 2.

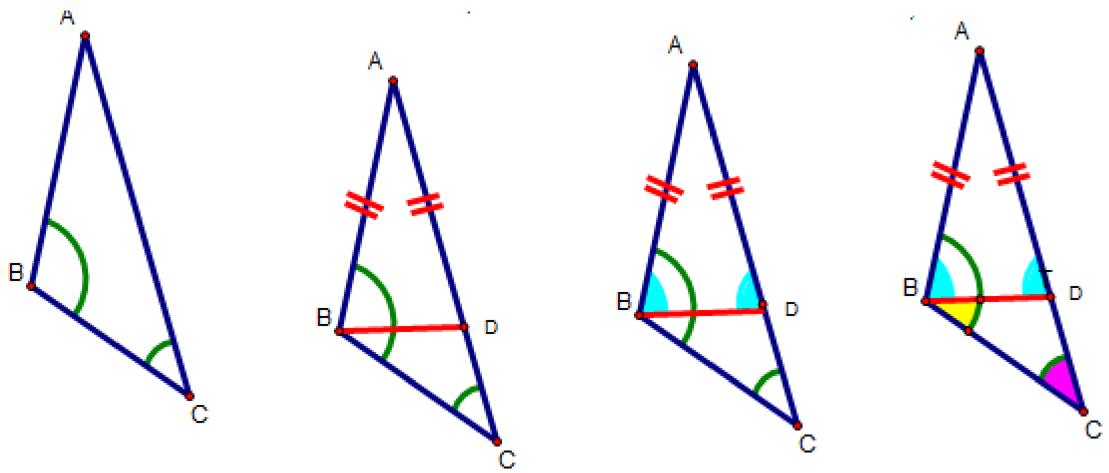
Using the theorem we proved earlier, what does this does this tell us about the relationship between |<ABC| and |<ACB| in this scenario? _________________________

Is this in agreement with or does it contradict, what we were given? ________________________

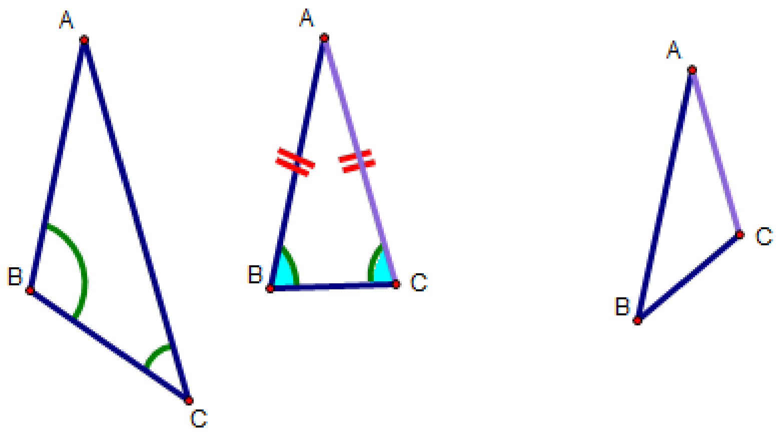
Hence, can option 2, i.e. _____________________________ , be true? ______

If there are only 3 options, which option/s are now possible for the relationship between **|AC| and |AB| given that |<ABC| > |<ACB|? ______________________________**

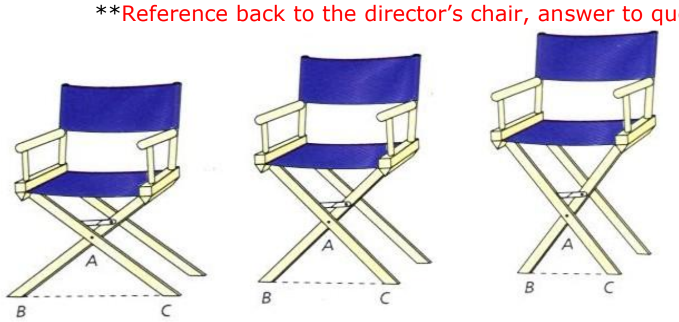
Teacher's board and students' copy for the proof of theorem 7 and its converse



Converse of theorem 7



**Reference back to the director's chair, answer to question



When the crossbar is positioned so that angle A is larger, the side BC of ΔABC is larger. The first chair is the most stable because its legs are farthest apart.