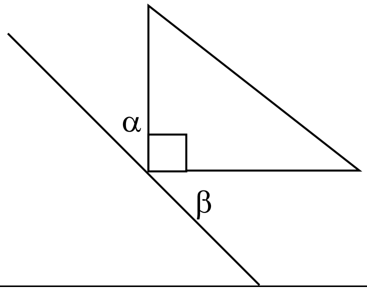
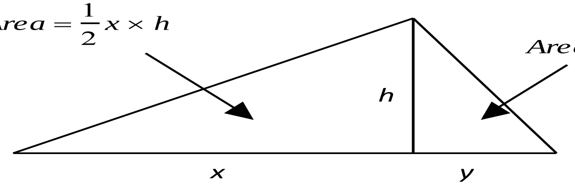
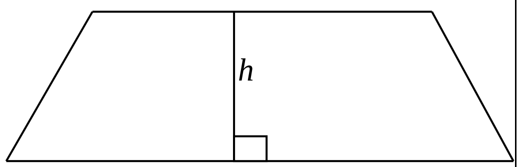
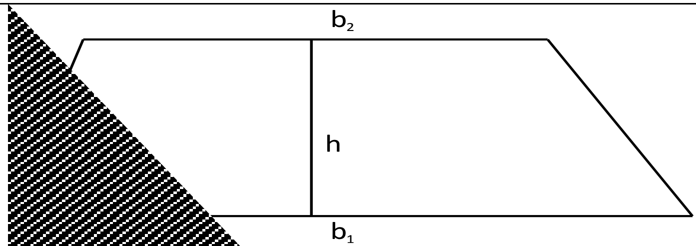
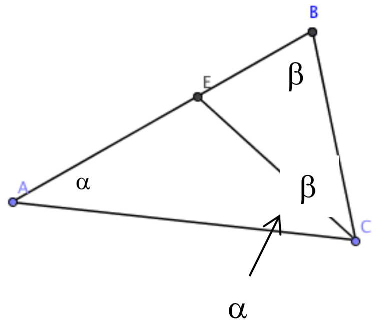
Problem Solving Questions

Solutions (Including Comments)

Question	Solution	Comments
1. True/False. In the diagram below the angles α and β are complementary. Justify your answer 	True as $\alpha + \beta = 90^0$	The idea is that the students should look up the definition of complementary angles. A nice development would be to get the students to create a GeoGebra file to illustrate the property of complementary angles.
2. Using proof by superposition show that the area of a triangle is $Area = \frac{1}{2}(base)(\perp\ height)$	The students could use a number of approaches. A simple one might be to choose a rectangle $(Area = base \times height)$. Now a diagonal divides this in two so the area of the resulting triangle(s) is $\left(\frac{base \times height}{2}\right)$. These triangles are right-angled Now this can be generalised by taking any triangle and constructing its perpendicular height. This gives two right-angled triangles of base *x* and *y* respectively. $(x + y)$ is the base of the big triangle	It would be worth discussing with the students why the diagonal divided a rectangle (or any parallelogram) into two triangular regions of equal area. The answer can be illustrated using the attached GeoGebra file.

	 $Total\ Area = \frac{1}{2}y \times h + \frac{1}{2}x \times h$ $= \frac{1}{2}h(x+y)$ $= \frac{1}{2}h(base)$	
3. A trapezoid is a four-sided object with two sides parallel to each other. An  example of a trapezoid is shown above.	 $Area\ of\ Trapezoid = Area\ of\ \Delta_1 + Area\ of\ \Delta_2$ $= \frac{1}{2}b_1 \times h + \frac{1}{2}b_2 h$ $= \frac{1}{2}h(b_1 + b_2)$	As in the problem above the question creates a nice link back to algebra through the use of common factors. While the Trapezoid (per se) is not on JC but as it has been defined here and as it is made up of two triangles it is a combination of 2D shapes listed in the JC syllabus it is a reasonable question, I think.
4. True/False. A triangle can have two obtuse angles. Justify your answer.	False. $A > 90^o, B > 90^0$ $A + B > 180^0$	The idea is that the students should look up the definition of obtuse angles. The link to inequalities should also be made

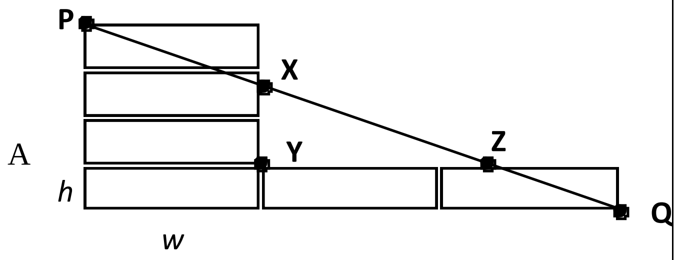
5. In the diagram below find the measure of the angle α if AB = AC and AE = CE = BC  	$\alpha+\beta+\beta=180^o$ $\beta=2\alpha$ $5\alpha=180^0$ $\alpha=36^0$	Ask the students do they think that the answer is reasonable. Check out using an accurate drawing and a protractor and then use GeoGebra.
6. If the length of a rectangle is halved and the width is tripled how is the area of the rectangle affected?	Original area is $A_1 = xy$, the new area is given by $A_2=\frac{x}{2}\times 3y$ $=\frac{3xy}{6}$ $=\frac{3A_1}{2}$ The area is reduced by a factor of 1.3.	This addresses the concept of proportional reasoning The students should also be asked to consider the effect of increasing the length is increased by two and the wide reduced by three.

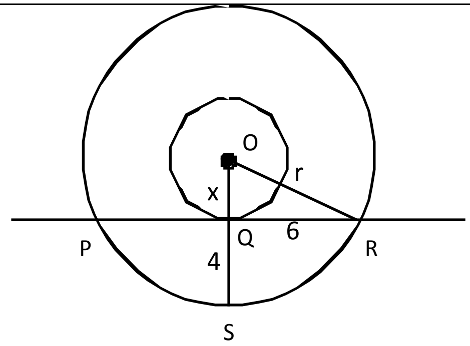
<table>
<tr>
<td>7. A mathematics test has 25 questions. Four points are given for each correct answer1 point is deducted for each incorrect answer. If Brad answered all questions and scored 35. How many questions did he answer incorrectly?</td>
<td>Say the student answers x questions correctly and y incorrectly. Then: $x + y = 25$ and $4x - y = 35$. Etc</td>
<td>Most students would use trial and error here. So use this opportunity to get them to structure their trial and error in an orderly fashion by making out a table such as
<table>
<tr><td>√</td><td>χ</td><td>Points</td></tr>
<tr><td>25</td><td>0</td><td>4(25)</td></tr>
<tr><td>24</td><td>1</td><td>4(24) + 1(1)</td></tr>
<tr><td>23</td><td>2</td><td>4(23) + 1(2)</td></tr>
</table>
Etc.</td>
</tr>
<tr>
<td>8. A playground has 3 entrances, each equally likely to be used. What is the probability that two children entering the playground will use the same entrance?</td>
<td>Each child has three choices and so the probability that both use the same entrance is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$</td>
<td>Ask the students how this question could be made more complicated. Such as, if there were more entrances or finding the probability of them entering using different entrances……</td>
</tr>
</table>

9. The yearly change in the yield of milk on a dairy farm over four consecutive years was as follows: An increase of *x*% in each of the first two years, followed by a decrease of *x*% in each of the next two years. Assuming that at least some milk was produced during the period described what has happened to the yield of milk from the beginning of the four-year period to the end?	Say the farm produces 100 units to begin with and assuming that the percentage change is 10% each time. At the end of year 1 $\textit{The yield} = 110$ and at the end of year 2, $Yield = 110 + \left(\frac{10}{100} \times 110\right)$ $= 121$ units At the end of year 3 $Yield = 121 - \left(\frac{10}{100} \times 121\right)$ $= 108.9$ units At the end of year 4 $Yield = 108.9 - \left(\frac{10}{100} \times 108.9\right)$ $= 98.01$ units The yield falls by 1.99% or $0.199 \times 10\%$ or more generally $0.199 \times x\%$	It might be best, to begin with, to deal with particular values of *x* (say 10%)

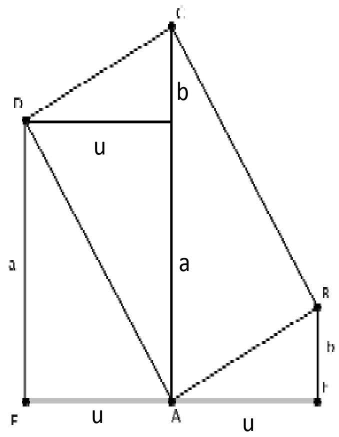
10. If the radius of a circle is decreased from 5 cm to 3 cm, by what percentage is (a) its circumference and (b) its area decreased	The original circle has circumference (C) given by $C = 2\pi \times 5 = 10\pi$, therefore the new circumference (D) is $D = 2\pi \times 3 = 6\pi$. Percentage decrease: $\frac{4\pi}{10\pi} \times 100 = 40\%$ The original circle has area (A), given by $A = \pi \times 5^2 = 25\pi$. The area of the new circle (B) is given by $B = \pi \times 3^2 = 9\pi$. Percentage decrease $\frac{16\pi}{25\pi} \times 100 = 64\%$	A better approach is to use proportional reasoning. As the circumference increases in proportion to the radius, if the radius falls by two fifths the circumference falls by two fifths or forty percent A better approach is to use proportional reasoning. As the Area increases in proportion to the square of the radius. if the radius falls by two fifths the circumference falls by two fifths squared The ratio of the areas will be nine is to twenty five and so the area of the new circle is 36% of the original area. The area falls by 64%
11. A pile of gold dust is divided among three prospectors. Calamity Jane and Wild Bill get $\frac{2}{5}$ and $\frac{1}{4}$ of the dust respectively. Bobby "Nugget" Smith gets the remaining 14 grams. How many grams does Calamity and Wild Bill each get?	Bobby gets $\frac{13}{20}$ of the total (*x*). Now $\frac{13}{20}x = 14$ $x = \frac{20 \times 14}{13}\, grams$ Jane gets $\frac{2}{5}\left(\frac{20 \times 14}{13}\right) grams$. Wild bill gets the rest.	It works better if Bobby gets 14 grams. I'd suggest you use 14 in class

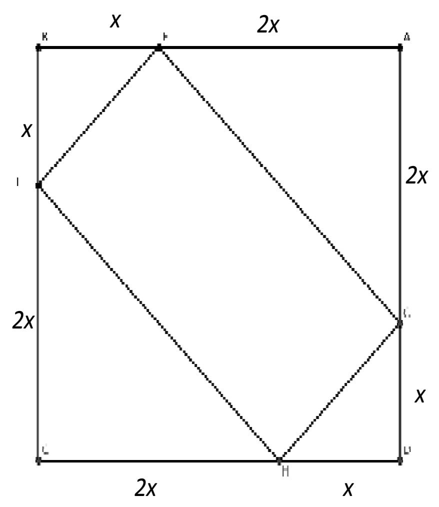
12. Which of the following must be an even integer? a. The average of two even integers b. The average of two prime numbers c. The average of two perfect squares d. The average of two multiples of 4 e. The average of three consecutive integers Explain your reasoning or show a counter example in each case	(a) May not be even-use a counter example $\frac{4+6}{2}=5$ (b) May not be even-use a counter example $\frac{2+3}{2}=\frac{5}{2}$ (c) May not be even-use a counter example: 9+16 is odd, for example, and the answer follows (d) Must be even:- Now we need to solve this in general. The sum of two multiples of four must look like $4n+4m$ where *m* and *n* are positive integers. Therefore $\frac{4m+4n}{2}=\frac{2(2m+2n)}{2}$, which is even $=2m+2n$ (e) Not necessarily even use a counter example $\frac{1+2+3}{2}=3$	(a) This problem offers a nice opportunity to look at general principles: The sum of two even numbers looks like $2m+2q$ where *m* and *q* are positive integers. The average is then $\frac{2m+2q}{2}=\frac{2(m+q)}{2}$ and we $=m+q$ don't' know anything about $m+q$ Note: The purpose of these questions is to get the students to see the power of the counter example. If an hypothesis fails once it is sufficient reason to discard it. On the other hand if one wants to prove something true in general a rigorous treatment is required (e) This is worth exploring further, as the students might explore any other properties of the sum of three consecutive integers

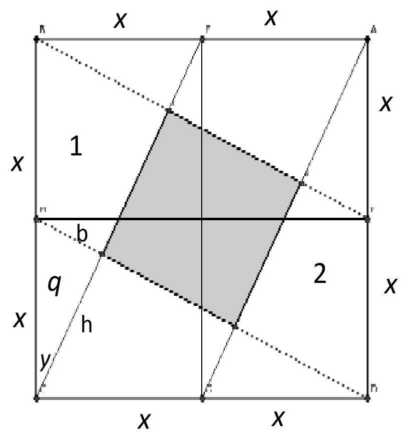
13. Six identical rectangles with height h and width w are arranged as shown. The line segment PQ intersects the vertical side of one rectangle at X and the horizontal side of another rectangle at Z. If the right-angled triangle XYZ is such that Y Z = 2XY, find the value of $\frac{h}{w}$.	The triangles PAQ and XYZ are similar and so $\frac{4h}{3w}=\frac{1}{2}$ $\frac{h}{w}=\frac{3}{8}$ 	
14. A gumball machine that randomly dispenses one gumball at a time contains 13 red, 5 blue, 1 white, and 9 green gumballs. What is the least number of gumballs that a customer must buy to guarantee that he/she receives 3 gumballs of the same colour?	The maximum possible number of gumballs dispensed before a third ball of either red blue or white must occur is 7 one such example is 2R,2B,W,2G whatever ball now emerges must be either R, G or B. And so the required number is 8.	This question prompts additional interesting ones. For example, what is the minimum possible number before three of the same colour may be dispensed or the number to guarantee that three of the same colour is dispensed one after the other?

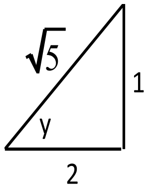
15. In the diagram, the two circles are centred at O. Point S is on the larger circle. Point Q is the point of intersection of OS and the smaller circle. Line segment PR is a chord of the larger circle and touches (that is, is tangent to) the smaller circle at Q. Note that OS is the perpendicular bisector of PR. If PR = 12 and QS = 4, find the length of the radius of the larger circle.	 We can see that $r = x + 4$ and $x^2 + 36 = r^2$ and so $(x+4)^2 = x^2 + 36$ $x^2 + 8x + 16 = x^2 + 36$ $8x = 20$ $x = \frac{20}{8} = \frac{5}{2}$ $r = 4 + \frac{5}{2}$ $= \frac{13}{2}$	The key ideas here are:- (a) A diameter at right angles to a chord bisects the chord (b) The use of Pythagoras having recognised that [OS] and [OR] are radii
16. A palindrome is a positive integer that is the same when read forwards or backwards. For example, 545 and 1331 are both palindromes. Find the difference between the smallest and largest three-digit palindromes.	The smallest three-digit palindrome is 101, the largest is 999	The problem provided interesting opportunities for conversations around what are the features of a three-digit number (it can't begin with 0, for example) and around the total number of both three digit numbers and palindromic three digit numbers etc.

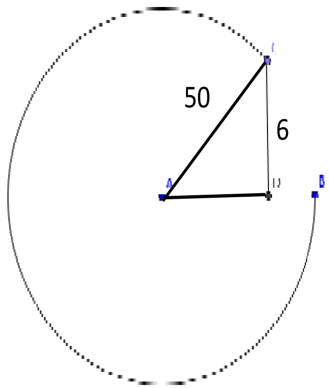
17. A 51 cm rod is built from 5 cm rods and 2 cm rods. All of the 5 cm rods must come first, and are followed by the 2 cm rods. For example, the rod could be made from seven 5 cm rods followed by eight 2 cm rods. How many ways are there to build the 51 cm rod?	The approach here involves recognising that the total length formed the 2cm rods is even and the remaining length is an odd multiple of five (and so ends in five). So the possible lengths are $5+46=1(5)+23(2)$ $15+36=3(5)+18(2)$ $25+26=5(5)+13(2)$ $35+16=7(5)+8(2)$ $45+6=15(5)+3(2)$	This question should lead to discussions about the properties of numbers generally: Sum and product of even numbers Multipes. How do we know if a number is odd (or even) How do we know (without division) that a number is a multiple of2, 3, 4 5 and so on.
18. Three pumpkins are weighed two at a time in all possible ways. The weights of the pairs of pumpkins are 12 kg, 13 kg and 15 kg. How much does the lightest pumpkin weigh?	The two bigger numbers must be consecutive positive integers as the sum of the lightest with these differs by 1. The middle number must be two greater than the smaller as when it is added to the larger the sum is three greater than when it is added to the smaller. Observation the leads to the numbers 5,7,8	
19. The sum of four numbers is *x*. Suppose that each of the four numbers is now increased by 1. These four new numbers are added together and then the sum is tripled. What is the value, in terms of *x*, of the number thus formed?	$a+b+c+d=x$ $3(a+1+b+1+c+1+d+1)=3(a+b+c+d+4)$ $=3(x+4)$ $=3x+12$	The idea here is to reinforce the link between algebra and number generally and that students realise that algebraic solutions are the most powerful because they hold in general

20. In the figure below, ABCD is a rectangle. The points A, F, and E lie on a straight line. The segments DF, BE, and CA are each perpendicular to FE. Denote the length of DF by *a* and the length of BE by *b*. Find the length of FE in terms of *a* and *b*.	 If the area of the rectangle is A then $A = 2\left(\frac{1}{2}u \times a + \frac{1}{2}u \times b\right)$ $= u(a+b)$ Therefore $u = \frac{A}{a+b}$ and $FE = 2$ $= \frac{2A}{a+b}$	The discussion points around this problem include recognising the congruent triangles that result from the construction:-drawing the horizontal line segment at right angles to the diagonal of the rectangle. These triangles can then be used to relate the area of the rectangle to that of the required triangles.
21. The average age of a group of mathematicians and computer scientists is 40. If the mathematicians' average age is 35 and the computer scientists' average age is 50, what is the ratio of the number of mathematicians to the number of computer scientists?	Say there are *x* Computer Scientists and *y* Mathematicians, therefore $\frac{x(50)+y(35)}{x+y} = 40$ $50x + 35y = 40x + 40y$ and $10x = 5y$ $\frac{x}{y} = \frac{5}{10} = \frac{1}{2}$	

22. A rectangle with unequal sides is placed in a square so that each vertex lies on a side of the square and divides the side in the ratio 1:2. Find the fraction of the area of the square that is covered by the rectangle.	 Area of the square is $(3x)\times(3x)=9x^2$. The area of the smaller triangles is $2\left(\frac{1}{2}x^2\right)=x^2$ while that of the larger triangles is $2\left(\frac{1}{2}4x^2\right)=4x^2$. So the fraction covered is $\frac{5x^2}{9x^2}=\frac{5}{9}$	
23. (a) How many 10-digit positive integers use each and every one of the ten digits 0, 1, _ _ _ , 9 once and once only? (b) How many 10-digit numbers are there?	(a) the required number is $9\times9\times8\times7\times6\times5\times4\times3\times2\times1$ (b) there are $9\times10\times10\times10\times10\times10\times10\times10\times10\times10$, ten-digit numbers	(a) The number can't begin with 0 and so there are only 9 choices for the first position. As 0 can be included in the second position and we have 9 choices, then 8 choices etc

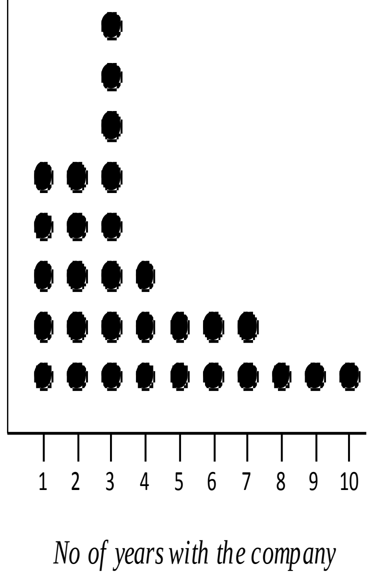
24. Richie's ***King Henry*** training shoes have 9 eyelets on each side, spaced 0.7 cm apart. Using the standard lacing shown below. Richie laces his right shoe so that there is 1.4 cm between the two parallel rows of eyelets. This leaves 15 cm of lace free on each side for tying. How long is the lace?	There are 16 lacings running diagonally each of length:- $x = \sqrt{1.4^2 + 0.7^2}$ $= 1.565cm$ Thee is also one horizontal lace of length 1.4 cm and so the total lengths is $15 + 15 + 8 \times 1.565 + 1.4 = 43.92$ cm	
25. What is the area of the shaded square shown if the larger square has sides of length 1?		It should be noted that E,F G and h are the midpoints of the sides of the outer square. Get the students to construct the figure using pen and paper and then GeoGebra.

	So the triangles have area $A = \frac{1}{2}(2x)(x) = x^2$ Now if we take two of these triangles we are left with the two quadrilaterals labelled 1 and 2 in the diagram. To find the area of either of these it requires that we take the area of two of the triangles labelled q in the diagram from the area of the large triangles. We can see that $\tan y = \frac{x}{2x} = \frac{1}{2}$ and the area of the triangle is $\frac{1}{2}(base)(height) = \frac{1}{2} b \times h$ $\tan y = \frac{1}{2} = \frac{b}{h}$, therefore $2b = h$ and $Area = b^2$ $\sin y = \frac{b}{x} = \frac{1}{\sqrt{5}}$ $b = \frac{x}{\sqrt{5}}$ Area of $q = \frac{x^2}{5}$. Therefore the area of quad 1 is given by $x^2 - \frac{2x^2}{5} = \frac{3x^2}{5}$. The area of the inner square is given by:- $4x^2 - \left(2x^2 - \frac{3x^2}{5} - \frac{3x^2}{5}\right) = \frac{4x^2}{5}$	

26. Without using a calculator or a computer, determine which of the two numbers 31^{11} or 17^{14} is larger?	$32^{11} > 31^{11}$ $32^{11} = (2 \times 16)^{11}$ $= 2^{11} \times 16^{11}$ $= 8 \times 2^4 \times 2^4 \times 16^{11}$ $= 8(16^{13})$ Now $17^{14} = 17(17^{13})$ as $17 > 8$ and $17^{13} > 16^{13}$ it follows that $17^{14} > 32^{11}$ and the result follows	
27. A machine-shop cutting tool has the shape of a notched circle, as shown. The radius of the circle is 50 cm, the length of CD is 6 cm, and that of BD is 2 cm. The angle CBD is a right angle. Find the distance from D to the centre of the circle (A).	 1. Find the horizontal distance AD $AD = \sqrt{50^2 - 6^2}$ $= 49.64$ 2. Now DB. $DB = (50 - 49.64)$	

28. The numbers 1447, 1005, and 1231 have something in common: each is a four-digit number beginning with 1 that has exactly two identical digits. How many such numbers are there?	The solution involves looking a number of mutually exclusive cases: Numbers where 1 is repeated (a) 1 first and second:- $1\times1\times9\times8$. The other cases (first and third and first and last also have the same number of possibilities). The total number is 3×72 (b) Matching numbers (other than1 second and third). We get $1\times9\times1\times8$. Now the other cases:- matching numbers second and last and third and last also yield these numbers of possibilities. So the total is $3(72)$. The grand total is $3(72)+3(72)=6(72)$	The main thing to consider here is that the use of diagrams to solve this type of problem is very useful. So if the students mark out four boxes and considers all of the ways that they can be filled without repeating any of the earlier cases

<table>
<tr>
<td>29. The diagram below shows the number of employees plotted against their length of service with a given company. The vertical axis indicates the number of employees but the scale was accidentally omitted from this graph. What percent of the company's employees have worked there for 5 years or more?</td>
<td>



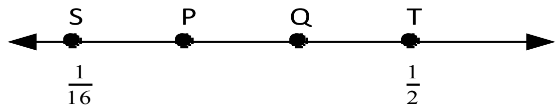
Let x represent the number of employees represented by each dot. Then the total number of employees is $32x$. The number serving for 5 years or more is $9x$. Therefore the percentage is $\frac{9x}{32x} \times 100$

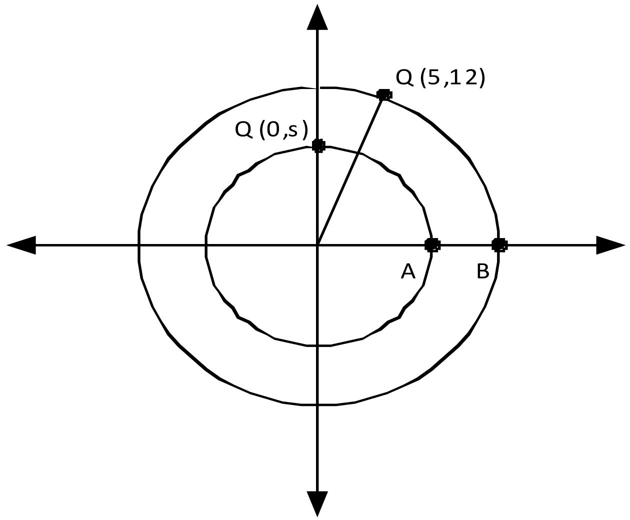
</td>
<td>The big issue here is to discuss why the value assigned to each dot is immaterial as long as it is consistently applied. Again underpinning the power of an algebraic approach.</td>
</tr>
</table>

30. What do the numbers 2335, 1446, and 1321 have something in common? How many such numbers are there?	They are four-digit numbers with exactly two repeated digits Again we should look at a diagram to explore the answer (a) The repeated digit can be first and second location. In such a case the number of outcomes is $9 \times 1 \times 9 \times 8$ always remembering that 0 can't appear first. Now we get the same number of outcomes if the repeated digits are in the first and third and first and last, giving $3\left(9 \times 1 \times 9 \times 8\right)$. (b) We can repeat the process for second and third and we have two cases: (i) where 0 is the repeated number $9 \times 1 \times 1 \times 8$ and (ii) where it is not $8 \times 9 \times 1 \times 8$ a matching number for second and last. (c) The only one remaining is where the matching numbers are third and last, there are two cases (i) where 0 is repeated $9 \times 8 \times 1 \times 1$, and (ii) where 0 is not repeated $9 \times 8 \times 1 \times 1$ filling the restricted locations first The total is arrived at by adding the individual outcomes	The most important thing about this problem is not the solution per se rather the discussion about the care that should be taken in describing how the numbers are constructed and the need to treat some cases separately.

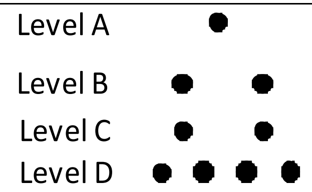
31. Can you find a rule to describe numbers in the sequence; 101, 104, 109, 116, … Find the next four terms in the sequence	Each number increases by an adjacent odd number each time:- $101+3=104$ $104+5=109$ $109+7=119$ The next four terms are:- $119+9=128$ $128+11=139$ $139+13=152$	
32. Mrs Gallagher bought a new plant for my garden and asked her children to guess the type and colour of the plant. Conor said it was a red rose, Sharon said it was a purple daisy, and Orla said it was a red dahlia. Each was correct in stating ***either*** the colour or the type of plant. Identify the plant bought by Mrs Gallagher? Explain your reasoning	As it can't be a red rose and the same time a red dahlia and if the colour is not red, then either Conor or Orla got both wrong, which disallowed by the assumption in the question So it's a red daisy as Sharon must have got the flower's name correct	
33. A pool has a volume of 2000 litres. Tommy starts filling the empty pool with water at a rate of 10 litres per minute. The pool springs a leak after 10 minutes and water leaks out at 1 litre per minute from then on. Beginning from the time when Tommy starts filling the empty pool, how long does it take to completely fill the pool?	The rate at which the pool fills after 10 minutes is 9 litres per minute. The volume to be filled is $2000-100=1900$ litres. The time taken from the time the pool springs a leak is therefore:- $\frac{1900}{9}=211.11\,min$ The total time is $211.11+10=221.11\,min$	

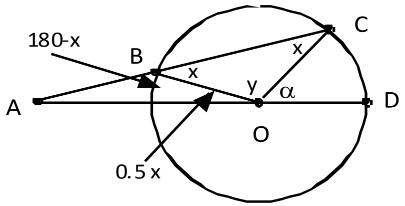
34. What is the surface area of a cube of side 5 cm. Three such cubes are joined together side-by side as shown. What is the surface area of the resulting cuboid?	Surface area of the cube is $6 \times \left(5^2\right) = 150cm^2$ The cuboid has dimensions $5 \times 15 \times 15$ and so the surface area is $2\left(5 \times 15\right) + 4\left(15^2\right) = 1050cm^2$	
35. In the diagram, a garden is enclosed by six straight fences. If the area of the garden is $168m^2$, what is the length of the fence around the garden		
36. The first four terms of a sequence are 1, 4, 2, and 3. Beginning with the fifth term in the sequence, each term is the sum of the previous four terms. Therefore, the fifth term is 10. What is the eighth term?	$T_5 = 1 + 4 + 2 + 3 = 10$ $T_6 = 4 + 2 + 3 + 10 = 19$ $T_7 = 2 + 3 + 10 + 19 = 34$ $T_8 = 3 + 10 + 19 + 34 = 66$	Sometimes, a solution just requires perseverance. This is a case in point
37. The set [illegible] contains the first 50 positive integers. After the multiples of 2 and the multiples of 3 are removed, how many integers remain in the set S?	There are 25 multiples (in fact there are 5 in each decade) of two there are also 16 multiples of three as $\frac{50}{3} = 16\frac{2}{3}$, therefore there are 41 multiples combined however there are a number of common multiples (6,12, 18, 24, 30,36, 42, 48) all of which are six apart. So the number remaining is $50 - 41 + 8 = 17$	So this is worth using wither to introduce or reinforce common multiples. Questions relating to the properties of the remaining numbers are also worth pursuing?

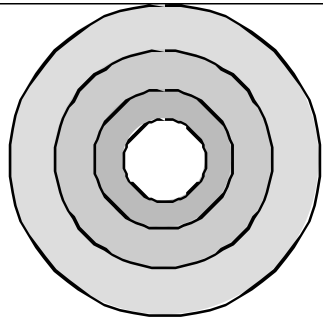
<table>
<tr>
<td>38. On the number line, points P and Q divide the line segment ST into three equal parts. What is the value at P?</td>
<td>


S and T are $\frac{7}{16}$ units apart and so the divisions are $\frac{7}{48}$ units wide. Therefore the value at P is $\frac{1}{16}+\frac{7}{48}=\frac{5}{24}$</td>
<td>Try and get the students to do this without the use of the calculator. The calculator gives them the correct answer
(0.5 – 1/16) = 7/16
7/16 divided by 3 = 7/48
P is 1/16 + 7/48
Q is 1/16 + 14/48
Ask is there any way to check answer. {1/16 + 3(7/48)} should give T which is ½.</td>
</tr>
<tr>
<td>39. Two circles are centred at the origin, as shown. The point $Q(5,12)$ is on the larger circle and the point $P(0,s)$ is on the smaller circle. If $|AB|=3$, what is the value of s?</td>
<td>


The radius length of the larger circle is $r=\sqrt{5^2+12^2}=13$.
The radius length of the inner circle is 10 i.e. $(13-3)$.
Therefore $s=10$</td>
<td></td>
</tr>
</table>

40. In the diagram, there are 26 levels, labelled [illegible]. There is one dot on level A. Each of levels [illegible] contains twice as many dots as the level immediately above. Each of levels [illegible] contains the same number of dots as the level immediately above. How many dots does level Z contain?	 There are 13 levels where the doubling takes place. Level B is Level 1 in this case and the number of dots is 2^1 so we'd expect $2^{13} = 8192$ dots on level Z (or level 13)	This is an example of exponential growth where the base is 2. This type of problem is particularly fascinating in that mitosis in biology obeys this type of behaviour-without the skips at every second level. A very nice question might be to ask the students to establish the number of dots on level O or S and so on. A particularly keen student might determine the total number of dots in the array.

41. In the figure below, B, C, D are points on the circle with centre O, and the lengths of the segments $\|AB\| = \|BO\|$. If the $\|\langle COD\| = \alpha$, find $\|\langle ABO\|$ in terms of α.	 $y = 180^o - 2x$ and the required angle is $180 - x$ $\alpha = 180 - \frac{x}{2} - y$ $y + 2x = 180$ $y = 180 - 2x$ and $\alpha = 180 - \frac{x}{2} - (180 - 2x)$ $\alpha = \frac{3x}{2}$ so the required angle is $180 - \frac{2\alpha}{3}$	A common oversight for students is that triangles comprised of the radii of the same triangle are isosceles. The most common error one can expect from students with this question however is that they will assume alternate angles where none exist
42. A hybrid car can run on petrol or on ethanol. It can drive for 495 kilometres on 45 litres of petrol and it can drive 280 kilometres on 14 litres of ethanol. If the price of petrol € 1.61 per litre, find the price per litre of ethanol which will give the same cost per kilometre as petrol?	The petrol car has a consumption of 11 kilometres per litre at a cost of $11 \times 1.61 =$ ó17.71 The ethanol car has a consumption of 20 kilometres per litre and so $20 \times cost / l = 17.71$ $cost / l = \frac{17.71}{20} =$ ó$0.886 l^{-1}$	

43. Determine which of the following statements is true for any two integers p and q where 11 divides into $2p+5q$ (a) 11 divides into $2p-5q$ (b) 11 divides into $(2p+5q)^2$ (c) 22 divides into $8p+20q$ (d) 11 does not divide into $20p+50q$ (e) $2p+5q$ divide into 11	(a) Not true as $2p-5q$ is not a multiple of ¸ (b) True as $(2p+5q)^2$ divides by $2p+5q$ (c) True as 22 is twice eleven and $8p+20q$ is $4(2p+5q)$ so 2 and 11 are factors of $8p+20q$ or 22 is a factor of $8p+20q$ (d) 11 does divide into $8p+20q$ as $8p+20q=4(2p+5q)$, so the statement is not true (e) It is only true if $2p+5q=11$ so it is not generally true	
44. The circles shown below are concentric and have radii of length 3 cm, 4cm, 5cm and 6 cm respectively. What is the probability that a random shot that hits the target at will hit the bull's eye (i.e. land in the innermost circle)?	 The area of the inner circle is 0.25 of the total area. $\frac{\pi 3^2}{\pi 6^2}=\frac{9}{36}=\frac{1}{4}$ Therefore the probability of a bull's-eye is 0.25 1 3 5 7 9 11 13 15 17 19	Once the marksman hits the target, the only point of interest is the area of the inner circle compared to the rest. The problem assumes a random distribution of shots on the target area, and perhaps the use of the word marksman isn't really appropriate! Follow up questions might include, what would the probability of hitting the outer ring? Etc. A nice extension question would be to ask the students to figure out the probability of hitting France when a dart is thrown at the map of mainland Europe.

45. Let *k* be a *integer*. Which of the following is always greater than *k*? (a) k^2+1 (b) $2k$ (c) k^{100} (d) $(k+1)^3$ Explain your reasoning	(a) k^2+1 is always greater than *k*. k^2 is a positive number greater than or equal to *k*. and so the result follows. (b) $2k$ isn't always greater than *k*. If k is negative $2k$ is less than *k*. (c) k^{100} is always greater than *k*. This is true as 100 is an even power and as *k* is a whole number and k^{100} is a positive whole number greater than *k*. (d) $(k+1)^3$ is also always greater than *k* for more or less the same reason(s)	(a) For what integer(s) is $k = k^2$
46. If the pattern shown continues, what will be (a) The first number in the 5th row (b) The last number in the 6th row (c) The middle number in the 7th row In which row will the number 289 appear?	1 3 5 7 9 11 13 15 17 19 The first no. in row two is $1+2$, the first number in row three is $3+4$ and row four is $7+6$. If the pattern continues, row 5 starts with $13+8=21$ The last number in row two is $1+4$ and row three is $5+6$, therefore the last number in row six is $29+12=41$ Well $17^2=289$ and the first number in the nth row is * $1+n(n-1)$. Now by trial and error and recognising that the 17th row begins with $1+17(16)=273$. There are 17 odd numbers in the 17th row and it ends in $273+16(2)=305$, remembering that it begins with 273 and there are 16 additional even numbers. On the other hand the numbers at the end obey the rule $(n-1)+n^2$ € and so the 17th row ends in $16+17^2=305$	This should be explored by observing the pattern:- Row 1: $1+1(0)$ Row 2: $1+2(1)$ Row 3: $1+3(2)$ Row 4: $1+4(3)$ € This should be explored by observing the pattern:- Row 1: $0+1^2$ Row 2: $1+2^2$ Row 3: $2+3^2$ Row 4: $3+4^2$